The 1969 Summer Seminar on

THE MATHEMATICAL THEORY OF SCATTERING

An Advanced Science Seminar on the subject "The Mathematical Theory of Scattering" was held at Northern Arizona University in Flagstaff, Arizona during the four week period from July 20 to August 16, 1969. The seminar was sponsored by the National Science Foundation and was arranged by the Rocky Mountain Mathematics Consortium. The Seminar Director was Professor Calvin H. Wilcox of the University of Arizona (now at the University of Denver).

The study of the scattering of beams of elementary particles (electrons, protons, photons, α-particles, etc.) by metallic foils and crystals has played and continues to play a fundamental role in the development of atomic and nuclear physics. With the invention of quantum mechanics in the 1920's it became possible to make quantitative predictions concerning the scattering of particles. The mathematical methods which have been developed for this purpose constitute the quantum theory of scattering.

Similarly, the study of the scattering of waves (acoustic, seismic, electromagnetic, etc.) by obstacles has played a fundamental role in the development of classical physics. The theory of such waves, which has been studied intensively from the eighteenth century to the present, may be called the classical theory of scattering.

Scattering theory is also of basic importance in a number of areas of applied physics. Thus, in theoretical chemistry it provides a theoretical framework for the study of chemical reactions and prediction of reaction rates. In electron optics it provides theoretical basis for the design of electron microscopes and the prediction of their performance. In electromagnetic theory it has numerous applications, such as the prediction of radar echos, prediction of transmission and reflection coefficients for wave guides, etc. The list of actual and potential applications could easily be lengthened.

When the classical and quantum mechanical scattering problems are formulated mathematically an underlying unity of structure is revealed. In both cases the states of the system (particle or wave) may be described by the vectors in a Hilbert space $\mathcal{H}$. The evolution of the system with time is described by a group of unitary operators $U(t) = \exp(itH$ where H is a selfadjoint operator on $\mathcal{H}$. If the scatterer (nucleus or obstacle) is absent the evolution of the system is described by a second (simpler) group of unitary operators $U_0(t) = \exp(itH_0)$. A central problem of the theory of scattering is to compare the behavior of these two groups of operators for $t \to \pm \infty$. This leads to the study of the wave operators W_+ and W_- which are defined by

$$W_{\pm} = \lim_{t \to \pm \infty} U^{-1}(t) U_0(t) = \lim_{t \to \pm \infty} \exp(-itH) \exp(itH_0).$$

During the last twenty years a substantial core of mathematical results has developed in this context and the main lines of a theory are now clear, although there are still many unsolved problems. The subject has begun to show the completeness and inner logic characteristic of a distinct branch of mathematics. It is this core which is called here the mathematical theory of scattering.

The main objective of the Seminar was to provide young mathematicians at both the predoctoral and postdoctoral levels with a survey of the basic literature, principal achievements and unsolved problems of the theory. A secondary objective was to stimulate research on the subject by bringing together a group of the most active researchers in the field.

The main part of the program consisted of four courses of lectures. Taken together, these were intended to provide a survey of the basic literature, principal achievements and unsolved problems of the mathematical theory of scattering. In addition, a research colloquium was held in which thirteen different participants presented lectures on recent research and current problems in the theory. Both the four lecture courses and the research colloquium extended through the four weeks of the Seminar.

The four basic courses and the lecturers who gave them were:

I. *The Abstract Theory of Scattering* by Professor Tosio Kato, University of California, Berkeley and Professor S. T. Kuroda, University of Tokyo.

II. *Representation Theory and Scattering Theory* by Professor Peter D. Lax, New York University and Professor Ralph S. Phillips, Stanford University.

III. *The Theory of the Coulomb Interaction* by Professor John D. Dollard, University of Rochester.

IV. *Eigenfunction Expansions and Scattering Theory* by Professor Norman Shenk, University of California, San Diego and Professor Dale Thoe, Purdue University.

Each course consisted of ten one-hour lectures. In addition, Professor Dollard gave three introductory lectures on the physical background and mathematical formulation of scattering theory.

This inaugural issue of the Rocky Mountain Journal of Mathematics contains four papers on the mathematical theory of scattering, based on the four basic seminar courses, together with a paper by Professor L. P. Horwitz and J. P. Marchand based on their talk in the research colloquium.

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6 MARCH, 1970