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GENERALIZATIONS OF THE FIRST AXIOM OF COUNTABILITY

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ABSTRACT. This paper is a comprehensive survey of the concepts which generalize first countability, of the relations among the concepts, and of the major examples found in the literature on the topic.

0. **Introduction.** There are several reasons which one may give to indicate the value of generalizing the first axiom of countability. Among these are the following:

(1) To weaken assumptions in important theorems. For example, a closed image of a metric space is metrizable, if the range is assumed to be first countable.

(2) To study important properties. For example, if a real valued function f is continuous upon restriction to each compact subspace of a space X, then f is continuous on X, if X is a k-space.

(3) To study sequences and their properties. For example, in first countable spaces every accumulation point of a subset A is the limit of a sequence in A.

The emphasis in this article is on giving a comprehensive list of concepts which have been introduced to generalize first countability, from different points of view, along with examples and references. The reader may often judge the value of a concept by considering the number of references pertaining to it, as listed in our references in Section 1.

The structure of this survey is as follows: In Section 1, we present a list of definitions of concepts which generalize first countability. For standard terminology we follow Kelley [199], Nagata [**290**], and Thron [379], and the reader may note that there are some remarks concerning notation and terminology at the beginning of Section 1. The reader is advised to skim Section 1 and refer to it as needed as he would a dictionary. The history of the subject is briefly surveyed in Section 2, along with some motivation for the concepts and some

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mention of unsolved problems. Section 3 gives implications and equivalences among the axioms discussed. In Section 4, we give a list of examples, all of which are both Hausdorff and not first countable. This list may serve the purpose of establishing which of the classes of spaces are distinct. The paper concludes with an exhaustive bibliography on the subject.

All spaces are understood to be Hausdorff spaces, unless otherwise stated, with the following exception: In Section 1 we define the concepts without assuming the Hausdorff axiom. It should be noted that in some of the original articles some separation axiom is taken as part of the definition of the concept. Thus, definitions in the original papers may at times differ, though they are equivalent upon assuming the Hausdorff property.

Finally, we would like to add that as in any other rapidly developing field, terminology tends to vary, and we hope this survey will serve as an incentive for standardization.

1. **The Concepts.** In this section we give what we believe to be a comprehensive list of the concepts found in the literature which generalize first countability. First, however, it is desirable for us to discuss some terminology used in this paper.

The following conventions will be used unless stated otherwise: The word mapping will mean a continuous surjection; the letter X will stand for a topological space (see the remarks concerning the assumption of the Hausdorff axiom in the Introduction); N will stand for the set of positive integers, while letters such as i, j, k, m, n will represent elements of N, and c will represent the cardinality of the continuum. The symbol "iff" is an abbreviation for "if and only if"; "+" will mean that two (or more) properties are held simultaneously; "=" will mean that two classes of spaces are equivalent. The boundary, closure, and interior of a set, will be denoted by "Bdy", "Cl", and "Int", respectively.

The words accumulation point and cluster point will be used in the following senses: A point x is an *accumulation point* (or synonymously *limit point*) of a set A if every neighborhood U of x meets A in a point distinct from x. A point x is a *cluster point* of a sequence $\{x_n\}$ if for every neighborhood U of x and every positive integer m, there exists an integer $n > m$ such that $x_n \in U$. By extension, the term cluster point will also be used in cases involving nets and filters in the usual sense. (This terminology is consistent with that used in most topology textbooks.) A sequence (or net) is *eventually in a set* U if U contains all but finitely many elements of the sequence (or net). Following Michael [**267**], we say that a filter base $\mathcal{F}$ *converges to a set* A (where

A is permitted to be a one point set) if every neighborhood of A contains some $F \in \mathcal{F}$. A sequence $\{A_n\}$ of sets is *decreasing* if $A_n \supset A_{n+1}$ for all n in N. A decreasing sequence $\{A_n\}$ of non-empty sets is a *k-sequence* (respectively, *q-sequence*) if it converges to some compact (respectively, countably compact) set which is contained in A_n for all n. In the following, a slash, /, separates the major references from the minor references.

A space X is an *absolute Fréchet-Uryson* space iff whenever $x \in X$, $M \subset \beta X$ and $x \in \mathrm{Cl}_{\beta X} M$, there exists a sequence of points in M converging to x. A bi-sequential space is absolute Fréchet-Uryson, and such a space is countably bi-sequential. Arhangel'skii [31].

For a space X and for any class $\mathcal{C}$ of subsets of X, an accumulation point x of a set M is *accessible by $\mathcal{C}$-sets* provided x is an accumulation point of a $\mathcal{C}$-set lying in $M \cup \{x\}$. See: approximately accessible by $\mathcal{C}$-sets, property H, property K, and also accessibility space. Whyburn [399]/[346].

A space X is an *accessibility* space iff every accumulation point x of a set M in X is approximately accessible by closed sets, that is, there is a closed set C such that x is an accumulation point of C but not of $C - M$. For a T_1-space X, X is an accessibility space iff it is a pseudo-open space; property H implies accessibility space; and for a regular space X, X is an accessibility space iff it has property H. Another related concept is Aull's Z space. Whyburn [399]/[44, 238, 346, 352, 354, 356].

A space X is said to be *accumulation complete* iff each sequence that has a point x as a cluster point, has a subsequence that converges to x. A space being accumulation complete is equivalent to every countable subspace being Fréchet. This concept has also been called subsequential in [373]. An accumulation complete space is an H_2 space. [175, 267, 323, 354, 373].

A space satisfying *property α* is the former terminology of Whyburn in [398] for a k'-space.

For a space X and for any class $\mathcal{C}$ of subsets of X, an accumulation point x of a set M is *approximately accessible* by $\mathcal{C}$-sets provided there exists a $\mathcal{C}$-set C having x as an accumulation point and such that all points of C in some neighborhood of x lie in $M \cup \{x\}$. See also: accessible by $\mathcal{C}$-sets, accessibility space. Whyburn [399].

A space X is a *b_R-space* iff every real-valued function whose restriction to each relatively pseudocompact subset K is continuous on K, is continuous on X, where a subset K is said to be relatively pseudocompact in X iff each continuous real-valued function on X is bounded on K. Noble [302].

The definition of a b_R*-*space* is analogous to that of a b_R-space with the term "relatively pseudocompact subset" replaced by "subset whose product with each pseudocompact space is relatively pseudocompact in the product". The class of spaces which are b_R* is between the classes of spaces which are k_R and b_R. Noble [302].

A space X is a *bi-k* space iff whenever a filter base $\mathcal{F}$ has a cluster point x in X, there exists a k-sequence $\{A_n\}$ in X such that F intersects A_n for all n and all $F \in \mathcal{F}$. This terminology was formerly used for the concept of a bi-quasi-k space by Nagata in [292]. Michael [267]/[31, 170, 312, 336, 337, 351, 353].

The definition of a *bi-quasi-k* space is the same as the definition of a bi-k space with the term "k-sequence" replaced by "q-sequence". This concept was formerly called a bi-k space by Nagata in [292]. Michael [267], Nagata [292]/[312, 336, 337].

A space X is a *bi-sequential* space iff whenever a filter base $\mathcal{F}$ has a cluster point x in X, then there is a decreasing sequence $\{A_n\}$ of sets in X converging to x and such that F intersects A_n for all n and for all $F \in \mathcal{F}$. Michael [267], Arhangel'skii [31], Malyhin [233], Olson [312]/[228, 235, 236, 336, 337, 351, 352, 353].

The terminology *c-space* has been used for at least three concepts. (1) A space X is a c-space iff for each non-isolated point x of X and each sequence $\{U_n\}$ of neighborhoods of x, there exists a set $\{x_n \mid n \in N\}$ which is not closed and is such that $x_n \in U_n$ for every n. Every k-space is a c-space in this sense. Chaber [87]/[355]. (2) A space is a c-space iff the closed sets are exactly those for which the intersection with every compact closed set is compact. Assuming the Hausdorff property, this is equivalent to the concept of a k-space. de Groot [140]/[17, 18, 45, 327, 339]. (3) This terminology is also in use [331, 122, 335, 336] for spaces determined by countable subsets. See: countable tightness.

A space satisfies *condition* (C_0) iff each sequence whose closure is countably compact has a subsequence whose closure is compact. This is implied by condition (k_0) of Chiba. Tanaka [375].

A space being of *type* $\mathcal{C}$ is Noble's [304] terminology for a space of point countable type.

A space X is of *type* $\mathcal{C}^*$ iff there exists a cover $\mathcal{C}_0$ of subsets of X which are compact + first countable + have countable character in X and $\mathcal{C}_0$ has the following property: for each x in X, each C in $\mathcal{C}_0$ for which x is in C, and each neighborhood U of x, there exists a C' in $\mathcal{C}_0$ with $x \in C' \subset U \cap C$. For Hausdorff spaces, this is equivalent to first countability (by (d) of (20) of Section 3). Noble [304].

The *character* of a set A in a space X, denoted by $\chi(A, X)$, is the least cardinality of a collection $\mathcal{B}$ of open neighborhoods of A having the property that every neighborhood of A contains some member of $\mathcal{B}$. The character of a space X, denoted by $\chi(X)$, is the supremum of the characters of all points of X. Other terminologies are: local character, local weight, point character, pointwise weight, weight; compare: pseudo character. This concept was discussed in the 1929 memoir of Alexandroff and Urysohn [**2**]. A few recent references are: [**26, 177, 187, 195, 247, 250, 251, 282, 283, 315, 316, 318, 322**].

The terminology *closure-countable* is used by Wilansky in [**406**, p. 292] for the concept of countable tightness.

The terminology *closure-sequential* is that of Wilansky in [**406**, p. 292] for a Fréchet space.

The terminology *compactly-generated* space is in common use among algebraic topologists (e.g., Spanier in [**360**]) and others (e.g., Herrlich [**170**]) for a k-space.

A space X has *countable tightness* iff whenever a point x is in the closure of a set A in X, there is a countable subset C of A such that x is in the closure of C. Equivalent conditions are: the closure of each set in X is the union of the closures of its countable subsets; and if A is a set in X and Cl $C \subset A$ for every countable subset C of A, then A is closed in X. The latter condition is the version in which this concept was first introduced by Moore and Mrowka in [**273**] under the name: a space determined by countable subsets. This terminology is in common use. Other names for this concept are: c-space, closure-countable space, countably generated space, countably accessible space (and formerly, space with an $\aleph_0$-topology [**219**]). The concept of tightness, for arbitrary cardinality, was introduced by Arhangel'skii and Ponomarev in [**35**] and has been discussed frequently in Arhangel'skii's recent work. Compactification: [**31, 233**], examples: [**126, 188**], general theory: [**54, 187, 286**], lattice of topologies: [**218, 219, 221**], relation to mappings: [**267, 268, 335, 353**], relation to products: [**31, 212, 233**], relations to other cardinal functions: [**28, 29, 30, 31, 32, 33, 35, 250, 342, 343, 344**], other: [**122, 125, 330, 331, 334, 335, 336, 406**].

Countably accessible space is the terminology of [**218, 221**] for a space of countable tightness.

A space X is a *countably bi-k* space iff whenever $\{F_n\}$ is a decreasing sequence of sets in X, having x as a common accumulation point, then there is a k-sequence $\{A_n\}$ such that $x \in \mathrm{Cl}(A_n \cap F_n)$ for all n. Michael [**267**]/[**31, 312, 336, 351, 353**]. A somewhat related property is mentioned in [**228**, p. 46].

The definition of a *countably bi-quasi-k* space is the same as the definition of a countably bi-k space with the term "k-sequence" replaced by "q-sequence". Michael [**267**]/[**278, 312, 336, 355**].

A space X is a *countably bi-sequential* space iff whenever $\{F_n\}$ is a decreasing sequence of sets in X having x as a common accumulation point, there exists a decreasing sequence $\{A_n\}$ which converges to x and such that A_n intersects F_n for all n. Equivalently, the conclusion of the preceding sentence may read: there exist points $x_n \in F_n$ such that $x_n \to x$. The latter version was called strongly Fréchet in [352]. Michael [267], Siwiec [352], Arhangel'skii [31], Malyhin [233], Olson [312]/[336, 351, 353, 355].

The terminology *countably generated* space is that of Franklin and Kohli [125] for a space of countable tightness.

A space X is *determined by countable closed subsets* iff every set A in X which contains the closure of every subset B of A such that Cl B is countable, is closed in X. Moore and Mrowka [273], other: [122, 187, 188, 286].

For the concept of a space *determined by countable subsets*, see: countable tightness.

A space being *determined by sequences* is the terminology of Moore and Mrowka [273] for a sequential space.

A space X is a *DN-space* iff for each x in X there is a net in $X - \{x\}$ that converges to x and has the property that its range is a discrete subspace of X. Anderson [4].

The terminology *E-space* is that of McDougle [181] for a Fréchet space.

For the concept of an *E_0-space*, see: space in which each point is a G_δ-set.

A space X is an *E_1-space* iff every point of X is a countable intersection of closed neighborhoods. Aull [37]/[81, 124, 349, 350, 355].

Hausdorff's terminology for a space satisfying the *first axiom of countability* is nowadays commonly shortened to a first countable space.

A space X is a *first countable* space iff it has countable character, that is, every point has a countable open base for its neighborhoods. Other terminology: space satisfying the first axiom of countability, space with a locally countable base, locally separable space, weakly separable space.

A space X is *Fréchet* iff whenever a point x is an accumulation point of a set A, there is a sequence in A which converges to x. This concept has also been called: closure-sequential space, E-space, Fréchet-Urysohn space, *FU* space. The latter two terms are standard among Russian topologists. Otherwise the term Fréchet space is standard, though the Russian terminology might be used where confusion is possible with another meaning of Fréchet space. For Hausdorff spaces, the following are equivalent: Fréchet space, hereditarily k-space (i.e., every subspace is a k-space), space with property K. The major references are Franklin's [118 and 120].

Characterizations: [13, 15, 19, 23, 44, 102, 118, 146, 157, 175, 187, 225, 233, 248, 250, 312, 337, 351, 352, 353, 370, 371].

Compact spaces: [35, 41, 108, 110, 111, 239].

Dimension: [414].

Examples: [4, 9, 15, 30, 34, 39, 41, 49, 102, 112, 119, 120, 126, 128, 133, 166, 192, 212, 233, 244, 267, 304, 310, 312, 313, 358].

Function spaces: [102, 143, 246, 248].

General theory: [54, 121, 187, 190].

Mappings: [13, 15, 19, 23, 54, 117, 118, 127, 146, 150, 154, 157, 168, 205, 222, 223, 232, 238, 243, 244, 250, 267, 304, 312, 336, 337, 346, 351, 352, 353, 355, 370, 371, 413].

Metrization: [108, 141, 154, 239, 354].

Modifications of a Fréchet space: [34, 157, 168, 205, 206, 247, 249, 250, 253, 254, 287].

Paracompactness: [63, 64, 212].

Products: [31, 118, 120, 127, 143, 154, 155, 212, 238, 267, 304, 310, 313].

Relations to other generalizations of first countability: [18, 19, 23, 31, 34, 39, 41, 44, 102, 117, 118, 141, 157, 166, 168, 175, 182, 187, 190, 192, 233, 244, 246, 247, 248, 249, 250, 253, 346, 352, 358, 399].

Survey remarks: [18, 30, 267, 336, 399].

Other: [4, 30, 31, 33, 43, 50, 109, 128, 138, 192, 236, 330, 341, 380, 406].

The term *Fréchet-Urysohn* space is the usual terminology for a Fréchet space among Russian mathematicians.

An *FU-space* is an abbreviation for Fréchet-Urysohn space.

A space in which *each point is a G_δ-set,* that is, a countable intersection of open sets, may also be referred to as a space having countable pseudo-character. The term E_0-space is also in common use for this concept, but the term G_δ-space, which has been used, is unsuitable due to its use for other purposes. Compare: E_1-space, generalized G_δ space, space of pseudopoint-countable type. Since pseudo-character appears in the Alexandroff and Urysohn memoir of 1929 [2], the following are only some recent references. Conditions for equality of character with pseudo-character: [1, 2, 10, 166, 167, 195, 239, 253, 267, 284, 315], relation to other generalizations of first countability: [37, 42, 124, 141, 166, 167, 195, 228, 253, 267, 312, 352, 353, 375, 386], relation to other cardinality functions: [32, 111, 148, 186, 342], relation to paracompactness and other compactness conditions: [1, 2, 194, 387, 388], realcompact: [**164**, **406**], function spaces: [**6**, 144, **145**, **164**], examples: [2, **6**, 37, **124**], other: [139, 283, 330].

The term *G_δ-space* has been used by some mathematicians [**6**, **164**, **406**, p. 163] for a space in which each point is a G_δ-set. This term has also been used for at least two other unrelated concepts (spaces in which each closed subset is a G_δ-set; spaces which are G_δ subsets of their Stone-Cech compactification).

A (completely regular) space X is a *generalized G_δ space* iff each point x of X is either an isolated point of X or is such that there exists a real-valued function continuous on $X - \{x\}$ which has no continuous extension to all of X. Heider [**164**].

A space satisfying the *gf-axiom of countability* is alternate terminology of Arhangel'skii in [18] for a space satisfying the weak first axiom of countability.

A space is said to have *property H* iff every accumulation point of a subset is accessible by closed sets. A space with property H is an accessibility space and for regular spaces these concepts coincide. Also compare a Z space. Whyburn [**397**, **399**] / [**346**].

A space X is an *H_1 space* iff every countably compact subset is sequentially compact; equivalently, whenever S is a sequence in X having no convergent subsequence, there exists a subsequence T of S such that the range of T has no accumulation point. Both H_2 spaces and Z spaces are H_1 spaces. Aull [39] and Meyer [**254**].

A space X is an *H_2 space* iff whenever S is a sequence in X having no convergent subsequence, the range of S has no accumulation point. All sequential spaces and all accumulation complete spaces are H_2 spaces. Aull [39].

The terminology *k-point* has been used with different meanings. The oldest is due to Alexandroff (and Urysohn) in [2] (see also [**15**, **108**, **213**]). A point x of a space X is a κ-point iff it is isolated or there exists a sequence of distinct points of X converging to x. Tamano in [374, p. 230] called a point x of a space X a k-point iff whenever x is an accumulation point of a subset A of X, there is a compact set K in X such that x is also an accumulation point of $A \cap K$. A space X is a k'-space iff every point of X is a k-point in this sense. For a third meaning, Noble in [**301**, p. 391] calls a point x of a space X a k-point iff for any set U which contains x and such that $U \cup K$ is open in K for every compact subset K of X, is itself a neighborhood of x. A space X is a k-space iff every point of X is a k-point in this sense.

A space X is a *k-space* iff every subset of X whose intersection with every compact set K is closed in K, is closed in X. Definitions vary because the Hausdorff axiom is often assumed. For the origins of this important concept see [8, 85]. A k-space is also commonly called a compactly-generated space; less commonly: K-space, k_3-space (by Fuller), Kelley space (by the French), space of class $\mathfrak{S}$. Compare de Groot's notion of a c-space. See also k-point.

General references on k-spaces: Arhangel'skii [15] and Steenrod [362].

Textbooks: Kelley [199], Wilansky [406], Willard [408].

Early articles: [8, 135, 257, 325, 94, 96, 199, 274, 275].

Algebraic topology: [68, 134, 360, 362, 394].

Cardinality functions: [30, 34, 189, 342, 343, 344].

Category: [123, 129, 130, 134, 169, 170, 172, 187, 326, 327, 339, 362].

Characterizations: [11, 15, 17, 18, 23, 96, 187, 226, 244, 260, 267, 274, 292, 351, 353, 356, 362, 370].

Examples: [15, 48, 56, 65, 77, 165, 210, 263, 270, 312, 331, 352, 353, 378, 404].

Exponential topology: [59, 92, 197, 198, 369].

Function spaces: [8, 48, 75, 96, 135, 145, 260, 275, 300, 325, 362, 363, 390].

General theories: [121, 187, 240, 286, 389].

Mappings: [11, 15, 16, 17, 18, 19, 22, 23, 24, 56, 66, 68, 87, 105, 125, 133, 146, 147, 165, 178, 187, 211, 226, 244, 260, 261, 262, 267, 274, 275, 277, 292, 299, 304, 337, 338, 348, 351, 353, 355, 356, 362, 370, 374, 377, 395, 398, 409].

Metrization: [78].

Paracompactness and normality: [36, 64, 65, 78, 92, 347].

Products: [15, 22, 47, 48, 59, 74, 75, 92, 94, 95, 97, 98, 178, 179, 261, 262, 275, 299, 301, 304, 307, 308, 309, 362, 363, 365, 374, 376, 378].

Products of pseudo-compact spaces: [98, 178, 301, 364, 374].

Realcompactness: [97, 98].

Relations to other generalizations of first countability: [11, 15, 18, 19, 23, 34, 44, 87, 98, 132, 146, 166, 189, 213, 244, 261, 263, 267, 270, 292, 312, 331, 333, 338, 342, 346, 352, 353, 356, 375, 398, 399].

Subspaces: [15, 19, 23, 362, 378, 392, 393].

Survey remarks: [18, 30, 267, 336, 339, 362].

Topological groups: [25, 193, 280, 281, 305].

Other: [21, 43, 50, 56, 57, 62, 103, 130, 131, 171, 210, 257, 330, 404, 405, 412].

A subset A of a space X has *property* (k) iff a subset of A is closed in A whenever it intersects every compact subset K of X in a set closed in $A \cap K$. Compare the concept of a k-space. Weddington [393].

The notation *K-space* is that of D. E. Cohen for a k-space.

A space is said to have *property* K iff every accumulation point of a set is accessible by compact sets. This concept has been called a k_1-

space by Fuller. It is equivalent to a space being hereditarily a k-space [23, Proposition 1, and 399], and assuming the Hausdorff axiom, it is also equivalent to the space being a Fréchet space. Halfar [150]/[23, 46, 133, 224, 244, 330, 399].

A space X is a *k'-space* iff whenever a point x is an accumulation point of a set A in X, x is an accumulation point of $A \cap K$ for some compact set K. This concept seems to have been discussed originally in 1946 [8]. It has also been called: property α (by Whyburn), k_1-space (by Arhangel'skii), and k_2-space (by Fuller). Compare: k_n-space, k-point. Arhangel'skii [11, 14, 15]/products: [47], subspaces: [393], relation to paracompactness: [64, 234], relation to mappings: [8, 66, 133, 267, 353, 356], other: [23, 34, 42, 43, 121, 132, 166, 187, 244, 352, 374, 398, 399].

The terminology *k'-space* is also that of Comfort [98] for a k_R-space.

A space satisfies *condition* (k_0) iff each sequence having a cluster point has a subsequence which is contained in a compact set. Chiba [88].

A space X is defined to be a *k_0'-space* by the condition that each sequence having a cluster point x has a subsequence which is contained in a compact set such that x is a cluster point of the subsequence.

The terminology *k_1-space* has been used by Arhangel'skii to mean k'-space (see k_n-space), and has been used by Fuller to mean a space with property K.

The terminology *k_2-space* has been used by Arhangel'skii in the sense of the definition given below for a k_n-space, and has been used by Fuller to mean a k'-space.

The terminology *k_3-space* has been used in the sense below by Arhangel'skii, and has been used by Fuller to mean a k-space.

For a positive integer n, a space X is a *k_n-space* iff for every subset M of X, $\mathrm{Cl}(M)_{k_n} = \mathrm{Cl}\, M$, where $\mathrm{Cl}(M)_{k_1}$ is the set of all points x for which there exists a compact set K such that $x \in \mathrm{Cl}(K \cap M)$, and $\mathrm{Cl}(M)_{k_{n+1}} = \mathrm{Cl}(\mathrm{Cl}(M)_{k_n})_{k_1}$. Arhangel'skii [14, 15, p. 52].

A space X is a *k_R-space* iff every real-valued function on X, whose restriction to every compact set is continuous, is continuous on X. Complete regularity of the space X is usually assumed. This concept has been called a k'-space by Comfort in [98]. Michael [263, 270], Comfort [98], Noble [301, 304]/[18, p. 154, 76, 178, 180, 300, 302, 305, 325, 390].

The terminology *Kelley* space is that of [134, 326, 339] and others for a k-space.

A space is defined to be a *KFC-space* by the condition that every compact subspace is first countable. Clearly spaces in which every compact subspace is metrizable [351, 353], and spaces in which each point is a G_δ-set (by 20e of Section 3) are KFC-spaces. [**70**].

An $\mathcal{L}$-*space* is defined as follows: Let X be a set and $\mathcal{C}$ a collection of pairs (S, x) where S is a sequence in X and x is a point in X, with S said to converge to x, satisfying: (1) if S is a sequence with constant value x, then $(S, x) \in \mathcal{C}$, (2) if $(S, x) \in \mathcal{C}$ and R is a subsequence of S, then $(R, x) \in \mathcal{C}$, and (3) if $(S, x) \in \mathcal{C}$ and $(S, y) \in \mathcal{C}$, then $x = y$. Then $(X, \mathcal{C})$ is an $\mathcal{L}$-space. Notice that $(X, \mathcal{C})$ is not a topological concept and see the discussion of this concept in Section 2. [**39, 61, 106, 137, 176, 208, 214, 382, 400**].

An $\mathcal{L}^*$-*space* is an $\mathcal{L}$-space, $(X, \mathcal{C})$ satisfying the additional condition: if $(S, x) \notin \mathcal{C}$ then there is a subsequence R of S such that for any subsequence T of R, $(T, x) \notin \mathcal{C}$. [**101, 106, 137, 138, 208, 214, 241**].

The terminology *local character* is that of [**250**] for the character of a space.

A space with a *locally countable basis* is the terminology of McDougle for a first countable space.

Locally separable space is the terminology of Vaidyanathaswamy [**382**] and Balachandran [**51**] for a first countable space.

The terminology *local weight* is that of [**127, 187, 230**] for the character of a space.

A space X is *maximally resolvable* iff it has isolated points or is the union of $\Delta(X)$ disjoint sets each of which intersects each non-empty open subset of X in at least $\Delta(X)$ points, where $\Delta(X)$ is the dispersion character of X, i.e., the minimum cardinality of the non-empty open subsets of X. All first countable spaces and all regular spaces of point countable type are maximally resolvable. Ceder [**84, 85**], Ceder and Pearson [**86**], El'kin [**113, 114**], Pearson [**320**].

A space is said to be a *mosaic* space iff it has a cover $\mathcal{K}$ of compact metrizable subspaces and has the (weak) topology: A set U is open iff $U \cap K$ is open in K for every $K \in \mathcal{K}$. Assuming the Hausdorff axiom, the class of sequential spaces and the class of mosaic spaces coincide. Davison [**102**], Lavallee [**224**].

A space X belongs to *class* $\mathfrak{N}$ iff the projection mapping of $X \times Y$ onto X is a closed mapping for every countably compact space Y. Spaces which are $k + S_4$ spaces (thus also sequential spaces) belong to $\mathfrak{N}$. Isiwata [**182**]/[**117, 152, 212, 213, 288, 345**].

A space X is *o-metrizable* by an o-metric d iff d is a nonnegative real-valued function on $X \times X$ such that (1) $d(x, y) = 0$ iff $x = y$, and (2) a subset F of X is closed iff for each point x not in F, $d(x, F) > 0$. This concept has also been called generalized metrizable and g-metrizable. For T_1-spaces, o-metrizability is equivalent to the space satisfying the weak first axiom of countability. All the papers of Nedev in the bibliography are on o-metrizability. See also the references for the weak first axiom of countability.

The terminology *point character* is that of [247] for the character of a space.

A space X is of *point countable type* or pointwise countable type iff each point is contained in a compact set having countable character in X. Compare pseudopoint-countable type. This concept of Arhangel'skii was introduced in [12] and, with proofs, in [15]. Major results may be found in: Arhangel'skii [12, 15, 33], Borges [70], Filippov [115, 116], Wicke [401]. Results on mappings: [12, 15, 88, 91, 93, 115, 116, 266, 267, 268, 277, 304, 312, 319, 336, 337, 351, 401, 403], relations to other generalizations of first countability: [12, 15, 33, 115, 116, 161, 166, 167, 228, 261, 312, 351, 352, 376, 386], cardinality functions: [28, 31, 33, 342, 343, 344], metrizability: [12, 15, 70], products: [12, 15, 33, 304, 376], βX: [15, 386], exponential topology: [92, 300], dimension: [319], topological group: [319], survey remarks: [12, 15, 18, 161, 267, 277, 403], other: [24, 27, 300, 353].

Pointwise weight is the terminology of [262] for the character of a space.

The *pseudo-character* of a set A in a space X, denoted by $\Psi(A, X)$, is the least cardinality of a collection of neighborhoods of the set A whose intersection is A. The pseudo-character of a space X, denoted by $\Psi(X)$, is the supremum of the pseudo-characters of the points of the space X. This concept has been discussed in the 1929 memoir of Alexandroff and Urysohn [2]. The terminology pseudo-weight has been used at times in place of pseudo-character. See: space in which each point is a G_δ-set.

A space X is a *pseudo-open* space iff every quotient mapping onto the space is a pseudo-open mapping. Whyburn defined the concept of an accessibility space and proved that for T_1-spaces, these concepts coincide. Shirley [346].

A space X is of *pseudopoint-countable type* iff each point is contained in a compact set of countable pseudo-character in X. Vaughan [386].

The terminology *pseudo-weight* is that of [1, 139] for pseudo-character.

A space X is said to be a *q-space* iff every point of x is a q-point. A point x is a q-point iff x has a sequence $\{U_n\}$ of neighborhoods such that if $x_n \in U_n$ for all n, then the sequence $\{x_n\}$ has a cluster point x'. Assuming regularity, the concept of a q-space and a strict q space coincide. Assuming paracompactness, the following are equivalent: q-space, strict q space, r-space, space of point countable type. On mappings: [87, 88, **181**, **259**, **278**, **291**, **332**, **337**], metrizability: [**259**, **278**], paracompactness: [73, **385**], realcompactness: [**183**, **230**], βX: [**183**], products: [376], surveys: [**161**, **267**, **277**, **336**], other: [**70**, **166**, **173**, **227**, **228**, **261**, **312**, **377**, **403**].

A space X is a *quasi-k* space iff every subset whose intersection with every countably compact set C is closed in C, is closed in X. The concept was introduced by Nagata [292]. On mappings: [**54**, **146**, **181**, **213**, **278**, **279**, **292**, **308**, **337**, **355**], products: [**308**, **376**], general theory: [**54**], survey remarks: [**267**, **277**, **336**].

A space X is an *r-space* iff every point x of X is an r-point. A point x is an r-point iff x has a sequence $\{U_n\}$ of neighborhoods such that if $x_n \in U_n$ for all n, then the sequence $\{x_n\}$ is contained in a compact set. [**234**, **260**, **318**]/[**70**, **161**, **166**, **228**, **357**].

A space X is an *r_0-space* iff every point x of X has a sequence $\{U_n\}$ of neighborhoods such that if $x_n \in U_n$ for all n, then the sequence $\{x_n\}$ has a subsequence with compact closure. Rishel [337].

A space of *class* $\mathfrak{S}$ is the terminology of Morita in [274] for a k-space.

A space X is an S_3 space iff every sequentially compact subset is closed in X. Aull [39].

A space X is an S_4 space iff every countably compact subset is closed in X. The class of S_4 spaces includes the classes of sequential spaces and Z spaces. Aull [39]/[**102**, **376**].

A space X is an *s_R-space* iff every real-valued function on X, whose restriction to every convergent sequence (with its limit) is continuous, is continuous on X; equivalently, every real-valued sequentially continuous function on X is continuous. Noble [304]. Notice that if the words "real-valued" are omitted, this yields a characterization of a sequential space [53]. Vidossich in [388] discusses a similar concept with the words "countable subset" replacing "convergent sequence".

A space X is a *sequential* space iff every sequentially open set is open. Equivalently, every sequentially closed set is closed. Assuming the Hausdorff axiom, the following are equivalent: (1) sequential space, (2) mosaic space, (3) sequential in the sense of Dudley [**106**], i.e., a space determined by an $\mathcal{L}^*$-space of Fréchet in the sequential manner (as discussed in Section 2). A sequential space has also been referred to as a space determined by sequences, and a sequentially generated space. Closely related to the concept of a sequential space is the concept of a Fréchet space. Section 2 discusses some of the relationships.

Major references: Franklin [**118, 120**], Davison [**102**].

Early articles: [**61, 102, 241**].

Cardinality functions: [**26, 30, 32, 34, 249, 251, 253, 254, 322, 343, 344**].

Category: [**52, 169, 170, 172, 187, 346**].

Characterizations: [**34, 50, 52, 61, 102, 118, 146, 187, 351, 352, 353, 370, 371**].

Compact spaces: [**30, 32, 102, 122, 239, 254, 285, 287, 329, 380**].

Convergence structure: [**55, 61, 101, 106, 138, 208, 272, 400**].

Examples: [**34, 39, 41, 101, 102, 117, 118, 119, 120, 122, 126, 188, 189, 192, 216, 245, 324**].

Function spaces: [**101, 102, 106, 245, 246, 248, 251, 273, 300**].

General theory: [**54, 121, 187, 240, 286**].

Mappings: [**53, 54, 102, 118, 125, 146, 168, 178, 187, 212, 224, 229, 241, 250, 267, 304, 306, 307, 337, 346, 351, 352, 353, 370, 371, 375, 377**].

Modifications of the sequential space concept: [**34, 142, 157, 168, 172, 188, 205, 212, 247, 249, 250, 251, 253, 272, 273, 352, 358**].

Paracompactness and normality: [**36, 63, 64, 215, 347**].

Products: [**62, 102, 118, 119, 120, 178, 212, 215, 262, 267, 304, 364, 375, 376, 377**].

Relations to other generalizations of first countability: [**26, 30, 34, 39, 41, 102, 117, 118, 120, 126, 157, 175, 187, 188, 224, 245, 246, 247, 248, 249, 253, 267, 273, 322, 330, 331, 336, 337, 341, 376, 399**].

Survey remarks: [**245, 267, 336**].

Topological group: [**25, 329**].

Other: [**166, 191, 192, 255, 330, 341, 354, 364, 378, 391**].

A set F is a *sequentially closed* set in a space X iff every sequence which converges to a point in $X - F$ takes on at most finitely many values in F.

Sequentially generated space is the terminology of Franklin and Kohli [**125**] for a sequential space.

A set U is a *sequentially open* set in a space X iff every sequence which converges to a point of U is eventually in U.

A space X is said to be a *singly bi-k* space iff whenever a point $x \in \text{Cl}\, F$ there exists a k-sequence $\{A_n\}$ in X such that $x \in \text{Cl}(F \cap A_n)$ for all n. Michael [**267**]/[**31, 312, 336, 337, 351, 353**].

The definition of a *singly bi-quasi-k* space is that of a singly *bi-k* space with the term "k-sequence" replaced by "q-sequence". With a slightly different definition, a singly bi-quasi-k space was referred to as property-(P) in [332]. Michael [**267**]/[**278, 312, 332, 336, 337**].

A space X is said to be a *strict* q space iff every point is contained in a countably compact set of countable character in X. Assuming regularity, the concept of a strict q space is equivalent to that of a q-space. Michael [267].

A space X is said to be a *strong accessibility* space iff whenever $\{A_n\}$ is a decreasing sequence of subsets of X each having x as a common accumulation point, there exists a closed set C such that x is an accumulation point of C but, for each n, x is not an accumulation point of $C - A_n$. Siwiec [352, 353, 355].

Strongly Fréchet space was the terminology for a countably bisequential space in [352]. This term has also been used in [31, 233]. An abbreviation used in [31] is SFU-space.

A space X is *strongly* k' iff whenever $\{A_n\}$ is a decreasing sequence of sets each having x as a common accumulation point, there exists a compact set K such that $x \in \text{Cl}(K \cap A_n)$ for all n. Siwiec [352]/[267, 336, 353].

Subsequential is the terminology of Tall [373] for an accumulation complete space.

For the concept of *tightness* of a space, see: countable tightness.

The terminology *very k* space is that of Arhangel'skii [19, 23, 33] for a k-space every subspace of which is also a k-space. Arhangel'skii showed that for Hausdorff spaces this concept coincides with Fréchet space. Compare: property K.

A space X satisfies the *weak first axiom of countability* iff for each point x of X there exists a countable collection T_x of sets $Q(x)$ containing x, such that a set U is open in X iff for each x in U there exists a $Q(x) \in T_x$ for which $Q(x) \subset U$. This concept is also called the *gf*-axiom of countability. For T_1-spaces, spaces satisfying the weak first axiom of countability coincide with the o-metrizable spaces. Arhangel'skii [18, p. 129]/[79, 330, 353, 354, 355]. Also see the references of Nedev for o-metrizability.

A space X is a *weakly-k* space iff every set whose intersection with every compact set is finite, is closed. Rishel [333, 334], House [174].

The terminology *weakly separable* space is that of Whyburn [396] for a first countable space.

A space X is a *Z space* iff for each point x in X and each subset M of X such that x is an accumulation point of M, there exists a subset P of M such that x is the unique accumulation point of P. Every Fréchet space is a Z space and every Z space is both an H_1 space and a space satisfying property H. Aull [39], Kannan [191].

2. **A Brief Motivational and Historical Survey.** Riesz in 1906, Hausdorff in 1914, and Root in 1914 were the first to consider in one way or another the very natural topological idea of a countable base for the neighborhoods of a point. Though the concept is basic, a more important reason for the interest in spaces satisfying the first axiom

of countability, or *first countable* spaces as they are more commonly called today, is that "sequences are adequate". What we mean by this is that a point x is in the closure of a set A iff there exists a sequence in A which converges to x. This is certainly a familiar and commonly used property of metric spaces and more generally of first countable spaces. It is clear that a first countable space has this property. However, we may see this in a round-about manner by considering the following condition (which T'ong called condition c) in [**381**]): Every point x of the space X has a sequence $\{N_i\}$ of neighborhoods of x with the property that if a point x_i is chosen from each N_i, then the sequence $\{x_i\}$ converges to x. The reader may verify that this condition is equivalent to X being first countable, and that the condition implies that sequences are adequate in the sense described above. (This simple property is probably well known, but does not seem to appear in textbooks.)

The reader is probably aware that what we have given as the meaning of "sequences are adequate" is, in fact, the definition of a *Fréchet* space (in the terminology of American general topologists). Of course, in a first countable space sequences are also adequate in another equally important sense: the sequentially open sets are identically the open sets. This is the defining condition of a *sequential* space. We have the following implications among these fundamental concepts:

$$\text{first countable} \longrightarrow \text{Fréchet} \longrightarrow \text{sequential.}$$

Sequential and Fréchet spaces are closely related as the following analogies show. For a sequential space, every sequentially closed set is closed; for a Fréchet space, the sequential closure is the closure. Also, for a sequential space, whenever A is a non-closed set, some accumulation point of A is the limit of a sequence in A; for a Fréchet space, whenever A is a non-closed set, every accumulation point of A is the limit of a sequence in A. (These conditions actually characterize sequential and Fréchet spaces.)

The idea that sequences are adequate is in the foreground in the definitions given by Fréchet in 1906. The structures which he discussed are not, however, topological spaces as we understand them today. There are two basic ways in which we may translate his *$\mathcal{L}$- and $\mathcal{L}^*$-spaces* into the current idea of a "topology". (Due to Garrett Birkhoff (with acknowledgment to Baer) [**61**], and Kisyński [**208**]. The most useful recent references for topology are Dudley [**106**], and P. Meyer [**250**]. See also Kelley's text [**199**, end of Chapter 2].) Let X

be a set and $\mathcal{C}$ a collection of pairs (S, x) where S is a sequence in X, and x is a point in X, with S said to be converging to x. The collection $\mathcal{C}$ is to satisfy the condition that if $(S, x) \in \mathcal{C}$ and S' is a cofinal (at most finitely many elements have been removed) subsequence of S, then $(S', x) \in \mathcal{C}$. (Fréchet's $\mathcal{L}$- and $\mathcal{L}^*$-spaces assume additional axioms — see Section 1 for definitions of these and other concepts not defined here.) Then a topology τ may be defined, in the sequential manner, by: a subset U of X is in τ iff whenever $(S, x) \in \mathcal{C}$ and $x \in U$, then S is eventually in U. This collection τ, which might also be denoted $\tau(\mathcal{C})$ having been derived from the collection $\mathcal{C}$, is easily seen to be a topology and the space $(X, \tau(\mathcal{C}))$ is sequential. If $(X, \mathcal{C})$ is an $\mathcal{L}$-space of Fréchet, then $(X, \tau(\mathcal{C}))$ is also a T_1-space (but not necessarily Hausdorff). Conversely, if (X, τ) is a topological space, then a collection $\mathcal{C} = \mathcal{C}(\tau)$ may be defined by: $(S, x) \in \mathcal{C}$ iff whenever $x \in U \in \tau$, S is eventually in U. If $(X, \mathcal{C})$ is an $\mathcal{L}^*$-space of Fréchet, then $(X, \tau(\mathcal{C}))$ is a T_1 sequential space and $\mathcal{C}(\tau(\mathcal{C})) = \mathcal{C}$. And if (X, τ) is a Hausdorff sequential space, then $(X, \mathcal{C}(\tau))$ is an $\mathcal{L}^*$-space such that $\tau(\mathcal{C}(\tau)) = \tau$. A collection τ' may also be defined on X in the Fréchet manner, by: a subset U of X is in τ' iff $U = X - \mathrm{Cl}(X - U)$ where a point x is in the closure of a set F, $\mathrm{Cl}(F)$, iff there exists a pair $(S, x) \in \mathcal{C}$ such that $S \subset F$. It is well known [217, p. 185] that τ' might not be a topology (the closure operator need not be idempotent — though (X, τ') is a closure space in the terminology of Cech [82]), but if τ' is a topology, then $\tau' = \tau$ and the topology is Fréchet.

Conway [**101**] points out that if (X, τ) is the complete metric space l^1 of all summable (i.e., having finite sum) real sequences (note that τ is the topology obtained in the sequential manner from $\mathcal{C}(\tau)$), then there is a non-sequential topology τ'' on X, which is thus distinct from τ, but for which $\mathcal{C}(\tau) = \mathcal{C}(\tau'')$. Thus, though a topology uniquely determines the collection of convergent sequences, a collection of convergent sequences determines a "natural" sequential (that obtained in the sequential manner above) topology, but it may not be the only topology having the same collection of convergent sequences. ("Sequences are not adequate" in determining a unique topology.)

The last decade has brought a renewed interest in these spaces from a different viewpoint — that of *mappings.* Ponomarev [**321**] and Hanai [**153**] around 1960 discovered independently that a space is first countable iff it is an open (all mappings are understood to be continuous and onto) image of a metric space; thus yielding a theorem which links a first countable space with the important concepts of a metric space and an open mapping in a particularly interesting manner. Subsequently, Arhangel'skii [**13**] and Franklin [**118**] have characterized Fréchet and sequential spaces as pseudo-open (equivalently, hereditarily quotient) and quotient images of metric spaces, respectively. Bi-sequential and countably bi-sequential spaces have also been defined (see Section 1) and characterized as bi-quotient and countably bi-quotient images of metric spaces, respectively. As a result, the known relations among these mapping concepts immediately yields the following fundamental sequence of implications:

$$\text{first countable} \rightarrow \text{bi-sequential} \rightarrow \text{countably bi-sequential} \rightarrow \text{Fréchet} \rightarrow \text{sequential.}$$

It may be of interest to point out that in naming a bi-sequential space, E. Michael [267] chose to use the prefix "bi" to indicate a nice property which this concept shares with first countability. This being that the product of two (or even a countable number of) spaces of this type is again a space of this type. The property of being a bi-sequential space is also nice because it is hereditary (i.e., it is preserved by arbitrary subsets). Sequential spaces unfortunately have neither of these two properties. The interested reader may consult Michael's paper. (For a survey of the mappings discussed above see Siwiec and Mancuso [356]; for a survey of these and other characterizations of images of spaces, see Michael [267] and Siwiec [353].)

Having discussed images of metric spaces we may point out that an interesting unsolved problem is to characterize closed images and perfect images of first countable spaces. First recall that a perfect image of a metric space is metrizable and that a closed image of a metric space has been characterized as a Fréchet space with an additional condition. (See Lasnev [223], and the discussion of spaces of point countable type and q-spaces which follows.) Since a perfect mapping is a bi-quotient mapping, and a bi-quotient image of a bi-sequential space is again bi-sequential, we have:

$$\text{first countable} \longrightarrow \text{perfect image of a first countable space} \longrightarrow \text{bi-sequential.}$$

Example 1 of Section 4 may be shown to be a perfect image of a first countable space, and of course it is not itself first countable. On the other hand, a slight modification of this example yields an example which is bi-sequential without being a perfect image of any first countable space. Specifically, the example is the one-point compactification of a discrete space of cardinality exp exp $\aleph_0$. As in example 1, this space is bi-sequential. If it were a perfect image of a first countable (Hausdorff) space Z, then Z would be compact because of the perfect mapping. However, noting Arhangel'skii's recent discovery [**26**, see also **322** and **340**] of the solution to the well-known problem of Alexandroff and Urysohn, that a space which is compact + first countable (+ Hausdorff) has cardinality less than or equal to the cardinality of the continuum, c, we may apply this result to determine that Z has cardinality $\leqq c$. Thus its image, the space with which we are concerned, would have cardinality $\leqq c$, contradicting our assumption. We then have a bi-sequential + compact space which is not a perfect image of any first countable space.

In 1970 Whyburn defined (see Section 1) and characterized an *accessibility* space as one for which every quotient mapping onto the space is a pseudo-open mapping. Likewise a strong accessibility space has the property that every quotient mapping onto the space is countably bi-quotient. Half of the following is then an immediate corollary: A space is Fréchet (respectively, countably bi-sequential) iff it is sequential + accessibility (respectively, strong accessibility).

Convergent sequences have been central to the discussion thus far. But the set of values of a convergent sequence together with its limit (or one of its limits) forms a compact set, so that not surprisingly, first countable spaces have "nice" properties associated with compact sets. Actually, we have already made use of compactness, indirectly, in our discussion of perfect mappings, and we have stated that a compact first countable (Hausdorff) space has cardinality $\leqq c$. However, the most useful generalization of first countability which deals with compact sets is that of a *k-space.* (Compare de Groot's [**140**] notion of a c-space for non-Hausdorff spaces.) Both a Fréchet space and a sequential space are easily seen to be a k-space, but a Fréchet space is actually hereditarily so. The property of being a k-space is not hereditary as may be seen by considering a completely regular non-k space and its compactification. In 1966 Arhangel'skii [**23** and **19**] showed the converse, that is, a (Hausdorff) space which is hereditarily a k-space is necessarily a Fréchet space. (This was unexpected because a k-space is associated with compact sets and a Fréchet space with convergent sequences. M. Rudin [**244** and **399**] independently discovered this same result.) The concept of a k-space is of use in many other ways; we mention two uses involving mappings: First, every function on a k-space X, whose restriction to every compact subset of X is continuous, is itself continuous on X (in fact, this characterizes a k-space; compare the concept of a k_R-space). Second, D. E. Cohen [**94**] characterized a k-space as a quotient image of a locally compact space. (This may be compared with a characterization of a sequential space which we have discussed above. See also [**267**, **353**].) Steenrod in [362] has suggested k-spaces as a "convenient category" of spaces for algebraic topology, in the sense that it is "large enough to contain all of the particular spaces arising in practice" and has good properties. But the k-space assumption is convenient in most of general topology for the same reasons. In particular, one would commonly wish to exclude pseudo-finite spaces (spaces in which every compact subset is finite) from consideration. By making the assumption of a (Hausdorff) k-space one does in fact eliminate these. The reader may wish to examine the examples of non-k-spaces found in Section 4, of which the most interesting are examples 39 and 69.

Another useful and related idea is that of a *k'-space*. (In fact, it was thought at one time that a k-space had the k'-space property; see Arhangel'skii [15, p. 31].) Recalling that first countable spaces, even metric spaces, need not be locally compact, we may now consider the following relations:

$$(1)\qquad \begin{array}{ccccccccc}
\text{locally compact} & & \longrightarrow & & \text{strongly } k' & \longrightarrow & k' & \longrightarrow & k \\
 & & & & \uparrow & & \uparrow & & \uparrow \\
\text{first countable} & \rightarrow & \text{bi-sequential} & \rightarrow & \text{countably bi-sequential} & \rightarrow & \text{Fréchet} & \rightarrow & \text{sequential.}
\end{array}$$

The concepts in this diagram are related to an amazing extent. (i) Locally compact spaces may be defined (namely, characterized) so that the five concepts located in the central region of this diagram and local compactness have definitions which are completely analogous [352]. (ii) The upper sequence of classes of spaces has been characterized by mapping conditions completely analogous to that of the lower sequence (see tables (22) and (23) in Section 3).

Another class of spaces which contains the class of first countable spaces and in whose definition compactness plays a role are Arhangel'skii's spaces of *point countable type*. This class of spaces also generalizes locally compact Hausdorff spaces, spaces which are perfect preimages of metric spaces, and Cech-complete spaces (completely regular spaces which are G_δ subsets of their Stone-Cech compactification). However, this class is not related to that of the k'-spaces (examples 4 and 66 in Section 4). These spaces have been especially useful in the study of perfect preimages of metric spaces, known as paracompact M-spaces, on which there has come to be an extensive literature. In particular Wicke [401] has characterized spaces of point countable type as open images of paracompact M-spaces. (Wicke's theorem requires that the image space be a T_0-space, but Michael [**266**] has shown that this assumption is not needed. Wicke's theorem is actually true if the words "open mapping" are replaced by "almost-open mapping" or by "P_2-mapping". For these matters see [**353**]. See also Chiba [**88**] and Coban [**93**, Theorem 14].)

Two other important "mapping theorems" are known for spaces of point countable type. (1) A closed image of a metric space is metrizable if the image is of point countable type. (This result of Arhangel'skii [**12**] improves a well-known theorem.) (2) Filippov's theorem [**115**]: a quotient image of a space with a point countable base has a point countable base if the image is of point countable type and if preimages of points are second countable subspaces of the domain. (These results in turn have been improved by Michael — see below.) In regard to a "problem" stated above, Vaughan (and Coban) [**386**] have pointed out that a perfect image of a first countable space need not be of point countable type nor even of pseudopoint-countable type. (See also [**353**, p. 135].)

C. J. R. Borges proved a very nice theorem for spaces of point countable type. He showed that if X is the adjunction space of two metrizable spaces, then X is itself metrizable if it is of point countable type. Additionally, he proved a sum theorem for metrizability: If X is of point countable type and if X is dominated (in the sense of Michael) by a collection of metrizable subspaces, then X is metrizable. These and related results appear in [**70**].

Since a Hausdorff space of point countable type is a k-space (this is more difficult to prove than one would expect [**15**, p. 37]) and since both spaces of point countable type and k-spaces have been characterized as images of Hausdorff paracompact M-spaces under different mappings (also compare the result of D. Cohen mentioned above), Michael [**267**] has defined and studied some intermediate classes of spaces, yielding:

$$\begin{array}{c}\text{point}\\ \text{countable}\\ \text{type}\end{array} \longrightarrow \text{bi-}k \longrightarrow \begin{array}{c}\text{countably}\\ \text{bi-}k\end{array} \longrightarrow \begin{array}{c}\text{singly}\\ \text{bi-}k\end{array} \longrightarrow k.$$

This sequence of concepts also fits directly above diagram (1) given previously with additional implications in an upward direction. (See diagram (2) in Section 3.) Michael [**267**] has found that the mapping theorems regarding metrizability and point countable base quoted above for the case of point countable type are in fact true assuming only countably bi-k (or even a weaker condition).

With what we have seen, a question arises whether in fact countably bi-k, say, is a "nicer" condition than point countable type. In fact in 1964, almost simultaneously with Arhangel'skii's introduction of spaces of point countable type, Michael introduced q-spaces and proved that a closed image of a metric space is metrizable if the image is a q-space [**267, 259**]. Thus q-space is another concept which challenges the importance of point countable type. It appears difficult to determine the relative value of these concepts; however it might be pointed out that in the presence of paracompactness, q-spaces and spaces of point countable type are equivalent. Thus the result of Michael concerning the metrizability of a closed image of a metric space is essentially the same as the result of Arhangel'skii quoted before.

(This paragraph is an aside which the reader may omit.) Several other, somewhat related, though less important, concepts have been introduced in efforts to find the weakest conditions needed to prove certain interesting results. Michael's q-space is one example, another is condition (k_0), which was introduced as a condition for the countable product of M-spaces to again be an M-space. Let us pause to examine this more carefully. An M-space (respectively paracompact M-space) is a quasi-perfect (respectively perfect) preimage of a metric space as we have partially mentioned above. This in itself indicates than an M-space is a "nice" concept. However, the product of two M-spaces need not be an M-space unless the M-spaces also satisfy condition (k_0). Thus the question arises whether a "nicer" concept is: M-space satisfying condition (k_0). Probably not, but we do have the question here and elsewhere. In particular it is clear from the definitions (which we need not give in this aside) that for spaces satisfying condition (k_0), (1) countably compact = space of class $\mathcal{C}^*$ of Noble [**301**], (2) M-space = space of class $\mathfrak{C}$ of Ishii, Tsuda, and Kunugi as Chiba [**88**] points out, and (3) q-space = r_0-space. Note also that (1) implies (2) implies (3), and r_0-space implies condition (k_0). Now returning to Michael's q-space and related concepts we point out to the reader that in Michael's paper [**267**] the "nicer" concepts seem to be those in our diagram (2) of Section 3 though various weaker properties are also given in his paper which suffice for proving some results (strict $q - q$, point countable type — r, countably bi-k — property (a) of Michael's Proposition 4.E.5, etc.). Thus again it is not clear which concepts are really "nicer".

Another concept which has been considered for some time, though namelessly until recently, is *accumulation complete*. This is just the Fréchet property for countable subsets; thus a countable space is accumulation complete iff it is Fréchet. But a space need not be countable for this equivalence to hold. Not only will a sequential space do, but a space need only have *countable tightness*. Thus, if a space has countable tightness and every countable subset is Fréchet, then the space is Fréchet (there are related results stated in Section 3). This and a like condition (spaces determined by countable closed subsets) were first introduced by Moore and Mrowka in 1964 [273] and have been reintroduced in recent years under other names. For Hausdorff spaces we have:

sequential ⟶ determined by countable closed subsets ⟶ countable tightness.

If a space is compact Hausdorff and determined by countable closed subsets, then the space is sequential [273].

We now state some open problems on this topic. (1) (Franklin) Does Hausdorff + compact + countable tightness imply sequential? (2) (Franklin and Rajagopalan [126]) Does regular + sequentially compact + countable tightness imply sequential? (Compare example 45 and notice that as pointed out in [126], the Proposition of [122] is false and Theorem B is an open problem. However if the answer to (2) is positive then the answer to (1) is also positive for any space of cardinality $< 2^{\aleph_1}$, by the Corollary of [122].) (3) (Franklin [122]) Does regular + countable tightness imply determined by countable closed subsets? (Compare example 44). (4) Does Hausdorff + countable tightness (or even a countable space) + k_0 imply k-space? (The answer is positive if k_0 is replaced by k_0' in this statement.) (5) For Hausdorff spaces, is hereditarily k_0 equivalent to accumulation complete? If the answer to (4) is affirmative, then this answer is also affirmative because if S is a sequence with accumulation point p in a hereditarily k_0 Hausdorff space X, then $S \cup \{p\}$ is a Hausdorff k_0 countable space, thus a k-space hereditarily. So that, by the result of Arhangel'skii and Rudin, $S \cup \{p\}$ is Fréchet. Thus there exists a sequence in S, and so in X, which converges to p. (6) Consider a space X having the following property: Whenever a point x is an accumulation point of a set A in X there exists a subset C of A such that x is an accumulation point of C and C has countable closure. This property is clearly "between" the properties of being Fréchet and of being determined by countable closed subsets. Is the property in fact held by spaces which are determined by countable closed subsets? It may easily be seen that a space having property H and having countable tightness satisfies this property. (While this paper was in preprint form, V. Kannan has written that he has some solutions to the above problems. In regard to the third problem, he has a regular space with countable tightness which is not determined by countable closed subsets. The example uses elementary measure theory. Regarding the fourth problem he has a countable Hausdorff k_0 space which is not a k space. And regarding the fifth problem, he has proved the statement affirmatively.)

Considering cardinal numbers other than $\aleph_0$ is yet another means of generalizing first countability. We lead into this discussion by first discussing countable pseudo-character. Clearly every *E_0-space* (i.e., a space in which each point is a G_δ-set) is a T_1-space, and every regular E_0-space is an E_1-space (though there exists a homogeneous Hausdorff E_0-space which is not an E_1-space [**124**]). (Anderson [**6**] points out that a completely-regular space is an E_0-space iff each point is a zero-set for some real-valued continuous function.) These two conditions have the advantage of usually being easily verifiable for a given example of a space, and additionally, spaces with a G_δ-diagonal (i.e., the diagonal is a G_δ-subset of the product of the space with itself) are E_0-spaces. Since for E_0-spaces or E_1-spaces certain pairs of conditions become equivalent — these may be seen in Section 3, but as an example we mention that an E_0-space of point countable type is first countable — these properties are useful for determining the conditions that particular examples satisfy.

The result just quoted concerning point countable type is interesting — and it also generalizes to higher cardinality. Its history goes back to about the 1920's when Alexandroff and Urysohn in their memoir and also Chittenden [**89**] proved that a locally compact Hausdorff space is first countable iff each point is a G_δ-set. Some of the relations among *cardinality concepts* are given in Section 3, and we have already stated Arhangel'skii's solution of the Alexandroff and Urysohn problem. However, Arhangel'skii proved more generally, that every Hausdorff space has cardinality strictly less than the Lindelof degree times the character of the space, where the Lindelof degree is the smallest cardinal number m such that every open cover of the space has a subcover of cardinality m or less. (See also Comfort's survey [**99**].)

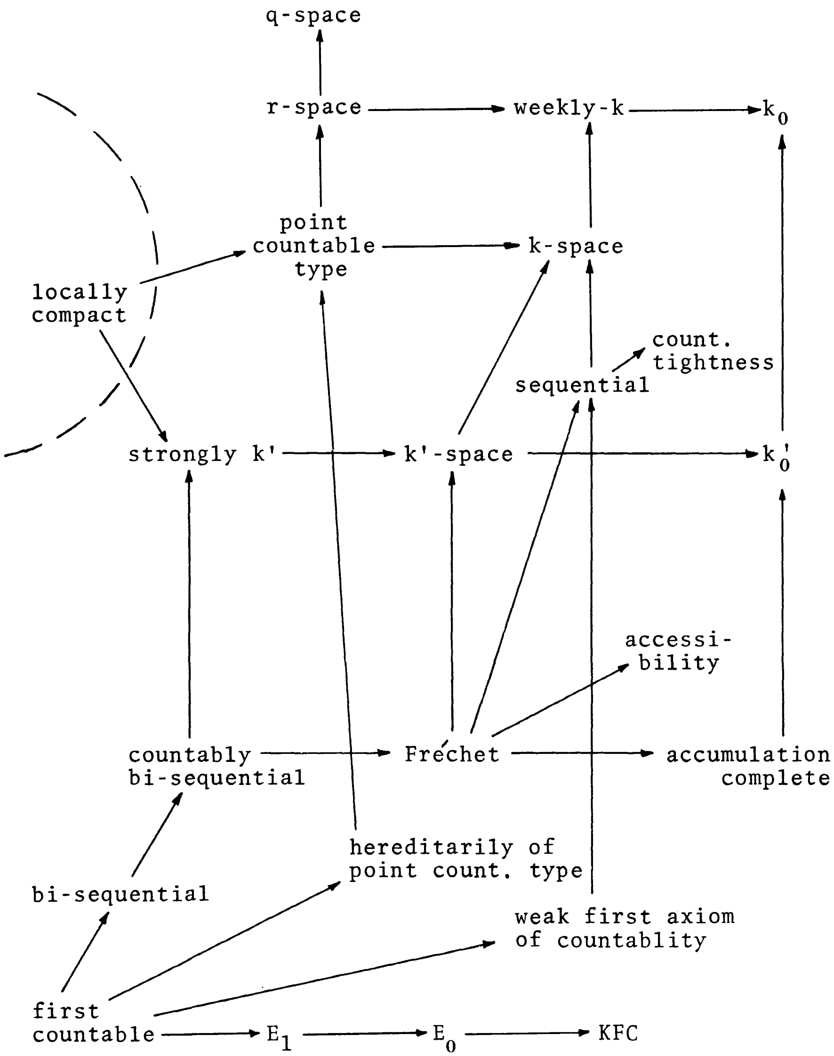
Fréchet and sequential spaces, as well as other spaces discussed in this survey, have also been considered with arbitrary cardinality (For example, Meyer has shown that every space having countable tightness is $2^{\aleph_0}$-Fréchet, and has pointed out that example 51 shows that the converse is false.) These will not be considered in this survey, but the reader may refer to the works of Meyer, and also [**26**, **28**, **30**, **34**, **127**, **142**, **169**, **186**, **187**, **204**, **272**].

As our final consideration we will discuss a space satisfying the *weak first axiom of countability* whose definition seems to indicate that the concept deserves more attention than it has received. This means of generating a topology does have some interesting properties. (1) Every accumulation complete space which satisfies the weak first axiom of countability is first countable. (2) A space which satisfies the weak first axiom of countability hereditarily is first countable. (See also [**354** or **353** IV D].) Means of weakening metrizability by modifying the concept of a metric have also been attempted for some time. A space which has an *o*-metric is in fact equivalent to a space satisfying the weak first axiom of countability. Nedev has done considerable work on this topic from the point of view of *o*-metrizability.

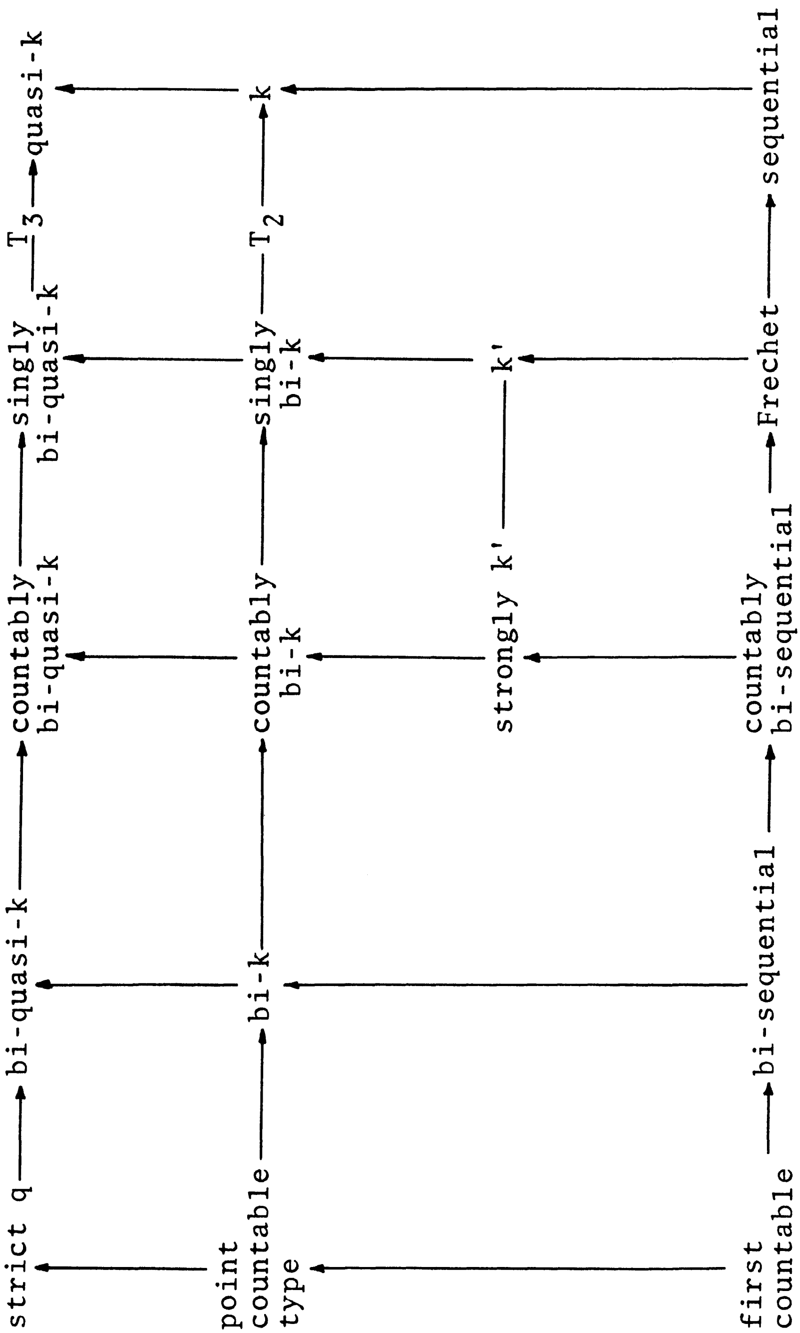
For categorical approaches to the concepts in this survey see: Franklin [**123**], Herrlich [**170**], Herrlich and Strecker [**172**], and Kannan [**187**].

3. **Some Relations Among the Concepts.** For the greater ease of the reader in finding and also in comparing results, we present much of this section in an informal "diagram" form. The statements in (8) and elsewhere are understood in the sense of the following example: If a space X has countable tightness, then X is an accumulation complete space iff X is a Fréchet space. The reader should note that many of the following results are true only with the assumption of the Hausdorff axiom.

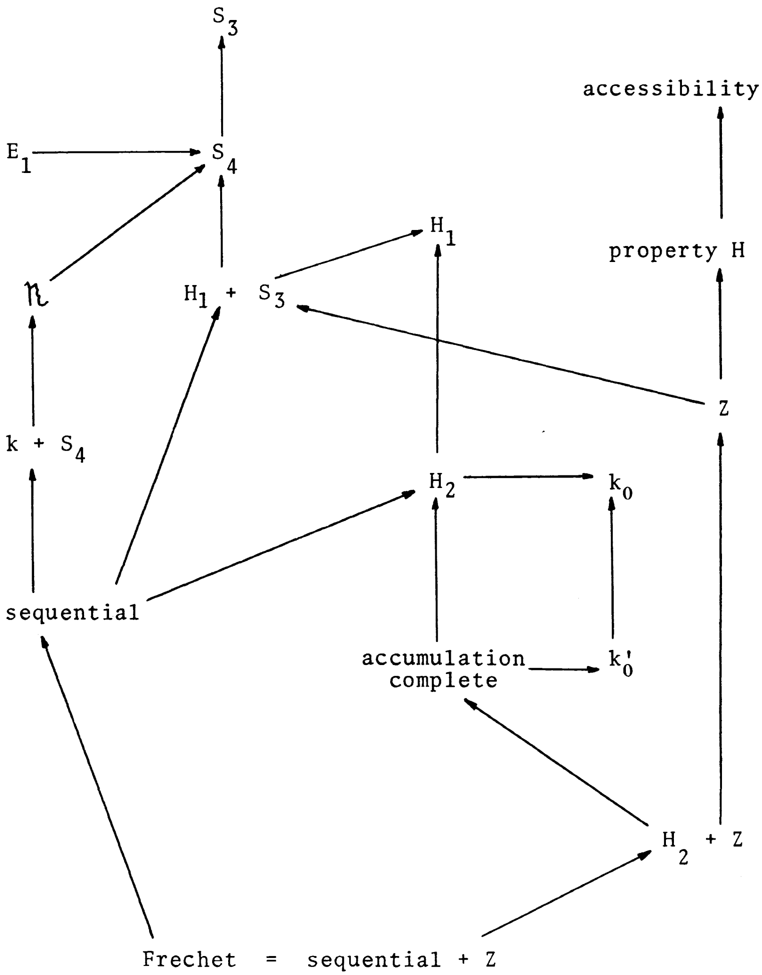
A few of the results stated have not appeared in the literature and proofs of these are indicated at the end of the section. There the reader may also find credits for the known results. (Charts (22) and (23) are of interest in this survey because they indicate characterizations of some of the classes of spaces discussed here. However, it does not seem justified to define the large number of mapping concepts given in these charts. The reader interested in the subject may consult the references which are suggested after the charts.)



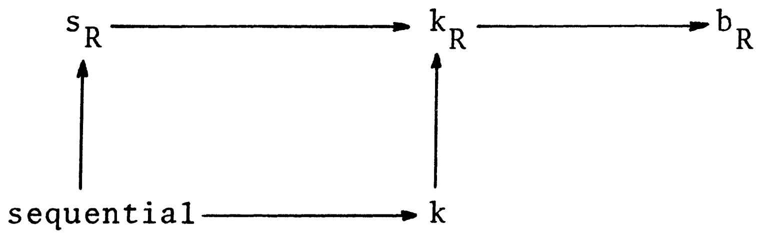
A general diagram. (1)



A diagram of *k* and related spaces. (2)
(See (13) and (14) in regard to this diagram.)



A diagram of concepts defined by sequences (and some related concepts). (4)



A diagram of concepts defined by functions. (3)

A non-discrete pseudo-finite space (a space in which each compact subset is finite) is not weakly-k (thus also not a k-space). (5)

A completely regular space with a non-isolated P-point (in the sense of Gillman and Jerison [**136**]), or a space with a non-isolated point such that every G_δ containing the point is a neighborhood of the point, does not have countable tightness. (6)

If a space X has countable tightness and every countable subspace of X has property P, then X also has property P, for P being: weakly-k, k', Fréchet, countably bi-sequential. (7)

$$\text{countable tightness} \longrightarrow \begin{cases} k_0 & = \text{weakly-}k \\ k_0' & = k' \\ \text{accumulation complete} & = \text{Fréchet} \end{cases} \qquad (8)$$

determined by countable closed subsets + k = sequential (9)

hereditarily k_0' = accumulation complete
hereditarily k = Fréchet (10)

$$\text{accessibility} \longrightarrow \begin{cases} \text{quasi-}k & = \text{singly bi-quasi-}k \\ k_0' \ (+\ \text{regular}) & = \text{accumulation complete} \\ k & = k' \\ \text{sequential} & = \text{Fréchet} \end{cases} \qquad (11)$$

strong accessibility $\longrightarrow$
- k = strongly k'
- sequential = countably bi-sequential (12)

E_0 or KFC $\longrightarrow$
- k_0' = accumulation complete
- k = sequential
- singly bi-k = Fréchet
- k' = Fréchet
- countably bi-k = countably bi-sequential
- strongly k' = countably bi-sequential
- bi-k = bi-sequential
- point countable type = first countable

$E_0 + q +$ regular $\longrightarrow$ first countable

(See diagram (2) in regard to the above results and also compare (20e).) (13)

$E_1 \longrightarrow$
- quasi-k = sequential
- singly bi-quasi-k = Fréchet
- countably bi-quasi-k = countably bi-sequential
- bi-quasi-k = bi-sequential
- q = first countable
- strict q = first countable

(See diagram (2) in regard to the above results.) (14)

The following are equivalent:
(a) Fréchet,
(b) property K,
(c) hereditarily a k-space,
(d) sequential + accessibility,
(e) sequential + Z,
(f) sequential + accumulation complete,
(g) countable tightness + accumulation complete,
(h) countable tightness + hereditarily k_0',
(i) countable tightness + k_0' + accessibility (if regular),
(j) k' + property H,
(k) k + accessibility. (15)

The following hold:

(a) If X is a space which is sequential but not Fréchet, then X contains a subspace which, with the sequential closure topology (sequentially closed sets are closed), is homeomorphic to example 25 of Section 4.

(b) If X is a space which is Fréchet but not countably bi-sequential, then X contains a subspace which is homeomorphic to example 9 of Section 4 (the sequential fan). (16)

The following are equivalent:

(a) first countable,

(b) for each point x there is a sequence $\{U_i\}$ of neighborhoods of x such that whenever a point $x_i \in U_i$ for each i, the sequence $\{x_i\}$ converges to x,

(c) E_0 + point countable type,

(d) $E_1 + q$ (or + strict q),

(e) accumulation complete + weak first axiom of countability,

(f) accessibility + weak first axiom of countability,

(g) Fréchet + weak first axiom of countability. (17)

If a space is Fréchet + countably bi-quasi-k + regular, then it is countably bi-sequential. Thus a compact Fréchet space is countably bi-sequential. If a space is compact + perfectly normal, then it is first countable. (18)

A space which is countable + regular + q is a separable metrizable space. (19)

For a space X, the following relations hold:

(a) $\Psi X \leqq \chi X \leqq wX$.

(b) $\Psi X \leqq |X|$.

(c) $\chi X \leqq 2^{\delta X}$, if X is a regular space.

(d) If K is a compact subset of X, $\chi(K, X) \leqq m$, and $\chi(x, K) \leqq m$ for $x \in X$, then $\chi(x, X) \leqq m$.

(e) $\chi X = \Psi X$, if X is of point countable type, or if there exists a cover $\{K_\alpha\}$ of compact sets with $\chi(K_\alpha, X) \leqq \Psi X$. Compare (13).

(f) $|X| \leqq 2^{LX \cdot \chi X}$. Thus, if X is compact + first countable, then $|X| \leqq c$.

(In the above, $|X|$ is the cardinality of X, χX is the character of X, δX is the density of X (i.e., the least cardinality of a dense subset of X), LX is the Lindelof degree of X (i.e., the least cardinal number m such that every open cover of X has a subcover of cardinality m or less), ΨX is the pseudo-character of X, and wX is the weight of X (i.e., the least cardinality of an open base for X). See Comfort [99] for an excellent survey of cardinality concepts.) (20)

For a completely regular space X, $\chi(x, X) = \chi(x, \beta X)$ for every x in X; $\chi(x, \beta X)$ is uncountable for every point x of $\beta X - X$. (21)

	quotient	pseudo-open	countably bi-quotient	bi-quotient
quotient	----	accessi-bility	strong accessi-bility	
pseudo-open	----	----		
compact-covering	k	k'	strongly k'	locally compact
km-covering	sequential	Frechet	countably bi-sequent.	loc. compact + loc.metriz.
sequence-covering	sequential	Frechet	countably bi-sequent.	
sequentially quotient	sequential	Frechet	countably bi-sequent.	
countable covering	countable tightness	countable tightness		

Characterizations of spaces X by the property that every $\cdots$ (left column) $\cdots$ mapping onto X is a $\cdots$ (top row) $\cdots$ mapping. (22)

	quotient	pseudo-open	countably bi-quotient	bi-quotient	almost-open
convergent sequences	sequential	Frechet	countably bi-sequent.		
compact metric spaces	sequential	Frechet	countably bi-sequent.	loc. compact + loc. metriz.	loc. compact + loc. metriz.
metric spaces	sequential	Frechet	countably bi-sequent.	bi-sequent.	
first countable spaces	sequential	Frechet	countably bi-sequent.	bi-sequent.	
compact spaces	k	k'	strongly k'	locally compact	locally compact
paracompact H-spaces	k	singly bi-k	countably bi-k	bi-k	
countable spaces	countable tightness	countable tightness			
spaces of character m	m-sequent.	m-Frechet			

Characterizations of spaces as images of a discrete union of · · · (left column) · · · under a · · · (top row) · · · mapping. (23)

Credits and proofs for the above results.

Diagram (2) is taken from Michael [**267**]; most of diagram (4) is from Aull [**39**]; the fact that a space which is $k + S_4$ belongs to $\mathfrak{N}$, in diagram (4), is similar to a result of Isiwata [**182**, Theorem 1.3] which the reader may verify by modifying Isiwata's proof. (Isiwata has more on this last subject. See also Scarborough [**345**, Corollary 5], Nadler [**288**], Hanai [**152**, Theorem 3], and Noble [**302, 303, 307**]. A partial converse of Hanai [**152**, Theorem 4] may be improved somewhat to say: If X is a T_1 non-discrete space having countable tightness and Y is a space such that the projection of $X \times Y$ onto X is closed, then Y is countably compact. A related result is also in [**345**, Corollary 5, and **288**].)

Result (5) is easily proved and may be left to the reader. Statement (6) is due to Paul Meyer.

The Fréchet and countably bi-sequential cases of (7) and (8) are proved by Michael in [**267**]. Noting that for countable spaces: $k_0 =$ weakly-k and $k_0{}' = k'$, the proofs of the other cases are trivial.

Result (9) is due to Kannan [**187**]. The second statement of (10) is a well-known result of Arhangel'skii [**23**] and independently, of Mary Ellen Rudin (see [**399**] and [**244**]), while the first statement of (10) is an immediate corollary.

It is easily proved that a regular + accessibility + $k_0{}'$-space is accumulation complete. The other parts of (11) and (12) follow immediately from the mapping characterizations of the spaces given in charts (22) and (23).

The results of (13) and (14) with the E_0-space and E_1-space assumptions have been discussed by several researchers; see Michael [**267** Section 7] in particular. The writer showed in [**351** or **353**] that the E_0 space assumption could be replaced by the assumption of a KM-space (i.e., a space in which every compact subspace is metrizable). Here we show a better result, that KFC-space suffices. (Notice that every E_0-space is a KFC-space by (20e).) The cases of a $k_0{}'$-space, k-space, and space of point countable type easily follow directly from the definitions and (20d). The three "bi-k" cases may be proved directly or in the manner of the following: If X is a space which is bi-k and KFC then, by results given in Michael [**267**, Theorem 3.E.3, etc.], the space X is a bi-quotient image of a paracompact M-space Z which is a subspace of $X \times M$ for some metric space M. Then Z is a KFC-space because if K is compact in Z, then its projection $\pi_1(K)$ into X is compact in X. Since X was assumed to be KFC, $\pi_1(K) \times M$ is first countable, so that the subspace K is also first countable. But a paracompact M-space is known to be of point countable type [**267**], so that Z which is KFC and of point countable type is first countable by what we have mentioned above. The space X being a bi-quotient image of the first countable space Z is then bi-sequential [**267**]. The two remaining cases of (13) are now trivial. Notice that the results of (14) expect for the q-space case may be generalized in a like manner. In particular the E_1-space assumption may be replaced by the assumption that every countably compact subspace is first countable.

The equivalence of Fréchet to (b) and (c) of (15) is (10) above. The equivalence to (d) is a repeat of (11), (e) is due to Aull [**39**], (f) is due to Howes and Chandler [**175**], (g) is a repeat from (8), (h) follows from (10), while (i) follows from (11). The last two conditions are due to Shirley [**346**] and Aull [**44**]. Result (a) of (16) is due to Franklin (see example 25 of Section 4), while (b) is a variation of some yet unpublished work of P. Harley and K. van Doren dealing with metrizability. We will prove this result here. Thus, let X be a space which is Hausdorff, Fréchet, but not countably bi-sequential. We wish to show that X contains a subspace homeomorphic to example 9. Since X is not countably bi-sequential, there exists a decreasing sequence $\{A_n\}$ of sets having a common accumulation point x_0 and such that no sequence $\{x_n\}$ with $x_n \in A_n$ for all n, can converge to x_0. Let $\{x_n{}^1\}$ be a sequence of distinct points of A_1 converging to x_0. There exists an $i_2 \in N$ such that $i_2 > 1$ and no subsequence of $\{x_n{}^1\}$, which is contained in A_{i_2}, can converge to x_0. Let $\{x_n{}^2\}$ be a sequence of distinct points of A_{i_2} converging to x_0. Thus $\{x_n{}^2\}$ may be assumed to have no points in common with the sequence $\{x_n{}^1\}$. There exists an $i_3 \in N$ such that $i_3 > i_2$ and no subsequence of $\{x_n{}^1\}$ and $\{x_n{}^2\}$, which is contained in A_{i_3}, can converge to x_0. Continuing this procedure, we obtain $x_n{}^j$, and we may set $X' = \{x_0, x_n{}^j \mid j, n \in N\}$. Then for each j, there are at most finitely many points $x_n{}^j$ which are not isolated in X'. For the proof of this, suppose there is a $j \in N$ for which there are infinitely many $x_n{}^j$ not isolated in X'. Denote these by D. Then each $x_n{}^j$ of D is an accumulation point of X'. Since X' is Fréchet, there is a sequence S_{jn} in X' which converges to $x_n{}^j$. Clearly S_{jn} has only finitely many points in common with each sequence $\{x_m{}^k \mid m \in N\}$. We may assume that S_{jn} was chosen to be contained in $\{x_m{}^k \mid k \geqq n, m \in N\}$. Let S be the union of all these S_{jn} (for $x_n{}^j \in D$), with $\{x_0\} \cup D$ removed from S. Then x_0 is an accumulation point of S. Since S is Fréchet, there is a sequence in S which converges to x_0. But then this sequence has only finitely many points in common with each sequence $\{x_m{}^k \mid m \in N\}$. Let S' be a subsequence having at most one point in common with each $\{x_m{}^k \mid m \in N\}$. Let S'' be S' with a repetition of terms, if necessary, so that S'' has an element y_n of each A_n with $S'' = \{y_n \mid n \in N\}$.

Then $y_n \to x_0$ which contradicts the original assumption. Thus, for each j there are at most finitely many points x_n^j not isolated in X'. Let X'' consist of the isolated points of X' union $\{x_0\}$. Then X'' has x_0 as its only non isolated point. To show that X' is example 9, let G be an arbitrary open neighborhood of x_0 in the topology of example 9. Then $X'' - G$ is of the form $\bigcup \{D_j \mid j \in N\}$, where each $D_j \subset \{x_n^j \mid n \in N\}$. By Theorem 5.1 of Olson [**312**], since each D_j is closed, their union is closed in X, and so also closed in X''. Thus G is open in X''.

A countable regular space is easily seen to be an E_1-space, thus (19) follows from (17d). Results (d) and (e) of (20) are due to Coban [**91**, **90**, p. 143], while (f) is due to Arhangel'skii [**26**] (see also [**28**, **99**, **322**, **340**]). The results of (21) may be found in Wilansky [**406**, p. 151 #209 and p. 195 #112].

Chart (22) may be found in [**352**, **356**, and **399**]. In chart (23) almost all the spaces X characterized have the property that the discrete union referred to consists of the discrete union of all subspaces of X of the stated type. The "sequential" entries are due to Franklin [**118**], the "Fréchet" entries to Arhangel'skii [**13**], the "countably bi-sequential" entries to Siwiec [**352**]. The "bi-quotient" column is due in whole or in part to Michael [**267**] and Morita [**274**]. In the "compact" row the first entry is due to Morita [**274**], the second to Arhangel'skii [**15**], and the third to Siwiec [**352**]. The "paracompact M-space" row is due to Michael [**267**], the following row to Rishel [**331**] and Kannan [**187**]. The chart appearing here is very similar to a chart of Kannan [**187**, p. 164] and a chart of Michael [**267**] to which the reader might also refer. Charts (22) and (23) are taken from [**353**].

4. **The Examples.** In this section we present a number of examples of Hausdorff, non-first countable spaces gathered from the literature. The properties which the following examples satisfy are given in a chart which follows. Some good references for additional examples are Franklin [**118** and **120**], Franklin and Rajagopalan [**126**], Michael [**267**], and Steen and Seebach [**361**].

1. The one-point compactification, R^*, of R, where R is the set of real numbers with the discrete topology [**361**, #24]. By Michael [**267**, Example 10.15] this space is bi-sequential. It is in fact a perfect image of a first countable space (the space given in Bourbaki [**72**, Exercise 2.13d]).

2. Let Z be the unit square with lexicographic order and the order topology [**361**, #48], let $Y = Z \times Z$, K the diagonal in Y, and $X = Y/K$. (The space Y/K is the quotient image of Y under the mapping

f which identifies K to a point. In other words, $Y/K = (Y - K)_r \cup \{p\}$ where p is a point not in $Y - K$, $f: Y \rightarrow Y/K$ is defined by $f(x) = x$ for all x in $Y - K$ while $f(x) = p$ for x in K, and Y/K has the topology obtained by requiring f to be a quotient mapping.) Then X is a perfect image of the space Y (a compact first countable space), and X is not first countable because K is not a G_δ-set in Y. [**267**, Example 10.4].

3. The ordinal space $[0, \omega_1)$, where ω_1 is the first uncountable ordinal, with the set of countable limit ordinals identified to a point (see example 2 above for "identified to a point"). This is Y of example 3.3 in [**259**].

4. The space X/S described in remark 3.3 of Borges [**69**].

5. The space X/K where K is the closed unit interval in the x-axis and X is the example of remark (2) on page 105 of Heath [**160**].

6. The space X/K where K is the closed unit interval in the x-axis and X is the space of example 12.1 of Michael [**260**], [**267**, Example 10.3].

7. The plane with the topology generated by sets of the form U where U is an open interval of a line through the origin such that the interval does not contain the origin, or U is the union of the collection of open intervals each of which contains the origin and such that each line through the origin contains one of these open intervals. Example C of Bing [**60**, or 361 #141]. Compare example 13.

8. The one-point compactification of a discrete space whose cardinality is a measurable cardinal. Michael [**267**, Example 10.13].

9. A countable space X of Arens [**9**, the space X on p. 233] commonly called the "sequential fan". Let $X_n = \{0, 1, 1/2, \cdots\}$ with the usual topology for each $n \in N$. Consider the discrete union of these X_n, and let X be the quotient obtained by identifying the zeros to a point. See also [**15**, page 25 Example 2.3, **126**, page 311, 252]. This is a subspace of example 16.

10. The second example on page 475 of Duda [**104**].

11. The space of rational numbers with the set of integers identified to a point [**118**, example 1.11].

12. The space of real numbers with the set of integers identified to a point.

13. The plane with the topology generated by sets of the form U where U is an open interval of a line through the origin such that the interval does not contain the origin, or U is an open sphere centered at the origin with a finite number of open radii removed. Anderson [**6**, example 3]. Compare example 7.

14. The plane with the x-axis identified to a point [**199**, example 3.R].

15. The first example on page 475 of Duda [**104**].

16. The discrete union of a countable number of copies of the closed unit interval, with the point zero in each of the copies identified to one point. This space contains example 9 as a subspace.

17. Okuyama's image space in section 4 of [**311**].

18. The countable, zero-dimensional, homogeneous space of Franklin and Rajagopalan [**126**, example 3.1].

19. The zero-dimensional topological group of Franklin and Rajagopalan [**126**, example 2.1].

20. The discrete union of $2^{\aleph_0}$ copies of example 1, with the point at infinity of each copy identified to one point [**265**, p. 17].

21. The one-point compactification of the space Ψ of Isbell. Franklin [**120**, example 7.1].

22. The real numbers provided with the topology generated by the union of the usual topology and all sets of the form $U \cup \{0\}$, where U is a usual open neighborhood of the sequence $\{1/n\}$. Franklin [**118**, example 1.8].

23. Let X be the real line, let $f(x) = x$ if x is not a positive integer, and let $f(x) = -1/x$ otherwise. The image space, with the quotient topology, is the example of T'ong [**381**, example 1, or **18**, example 2.2].

24. Ceder's example 9.3 in [**83**].

25. This is a countable space due to Arens [**9**, Y on page 233] or see Franklin [**120**, example 5.1]. Franklin has shown that every space which is sequential but not Fréchet contains a subspace which, with the sequential closure topology, is homeomorphic to this example. (In the sequential closure topology every sequentially closed set is closed.) The space $X = (N \times N) \cup N \cup \{0\}$ with each point of $N \times N$ an isolated point. A basis of neighborhoods of $n \in N$ consists of all sets of the form $\{n\} \cup \{(m, n) \mid m \geqq m_0\}$. And U is a neighborhood of 0 iff $0 \in U$ and U is a neighborhood of all but finitely many $n \in N$. Example 38 is the subspace obtained by deleting N from X.

26. The radial topology of the plane: a set U is open iff for each point p of U, U contains a line segment through p in each direction. The space is separable, but not Lindelöf and not regular. [**410**]. Compare example 28.

27. The product space of the space of example 12 with a closed unit interval. Franklin [**120**, example 7.4].

28. The Archimedean topology of the plane: a set is open iff its intersection with each horizontal line and each vertical line is a set open in the usual topology of the line [**15**, page 30 and page 58; also **256**]. Compare example 26.

29. The space X_1 of example 3.2 on page 35 of [**15**].

30. The countable, zero-dimensional space S_ω of Arhangel'skii and Franklin [**34**], which has no point of first countability.

31. An example of Kofner [**209**, example 2, or **210**, example (9.2)].

32. A countable example of Kofner [**209**, example 3]. The product of this space with itself is not a k-space.

33. Example 4 of Kofner [**209**].

34. An example of Arhangel'skii [**18**, p. 149, example 5.1, or equivalently, **20**, p. 1260].

35. The product space of the real line with example 11. Michael [**260**, example 12.6].

36. The product space of the space of example 16 and a closed unit interval. Michael [**267**, example 10.2].

37. A countable, connected, nowhere first countable example of Kannan [**189**, Theorem 3].

38. The countable pseudo-finite (every compact subset is finite) example of Arens. [**9** the space Z on page 234, or **199**, Problem 2.E]. See example 25.

39. The subspace $N \cup \{p\}$ of βN where p is any point of $\beta N - N$. This space is pseudo-finite, not a subspace of any sequential space [**117**], and is not of class $\mathfrak{N}$ [**212**]. See: [**136**, Problem 6.E.5, or **260**, example 12.5].

40. The product space of the space of rational numbers with the space of example 11. Franklin [**118**, example 1.11, or **260**, example 12.6].

41. A countable space of Arhangel'skii identified as example L in [**15**, p. 15, example 2.2].

42. A countable, zero-dimensional example of Appert [**361**, #98].

43. A countable space of Arhangel'skii identified as example π in [**15**, p. 57, example 3.5]. No point of the space is a κ-point (in the sense of Alexandroff and Urysohn), in fact the space is pseudo-finite.

44. The space is βN retopologized with a topology generated by the union of the usual topology for βN and the family $\{N \cup \{p\} \mid p \in \beta N - N\}$. Franklin [**122**, Theorem A]. Example 55 is βN with its usual topology.

45. Example 1.2 of Franklin and Rajagopalan [**126**].

46. An example of Heath [**162** or **163**].

47. The product space of the space of rational numbers with the space of example 16 [**267**, Example 10.1].

48. An example of Aull [**37**, example 1]. This is not an E_1-space though it is an E_0-space.

49. The subspace of $[0, \omega_1]$, where ω_1 is the first uncountable ordinal, obtained by removing the set of countable limit ordinals [**165**, example H on p. 85]. This example of a pseudo-finite space is very similar to the following example.

50. The set of real numbers with an additional point "at infinity". A set is closed if it contains the point at infinity or if the set is countable. This example of Gál [**39**, example 5 or **352**, Problem 2.8b] and the preceding example are very similar.

51. The set of ordinal numbers less than or equal to the first uncountable ordinal with the usual order topology [**361**, #43].

52. The Tychonoff plank in the version with the corner point (ω_1, ω_0) included [**361**, #86]. Compare example 60.

53. The product of $2^{\aleph_0}$ copies of the closed unit interval [**361**, #105, **197**, **198**]. Priestley shows that this space is badly non-Fréchet since it has a countable dense subset that contains no nontrivial convergent sequences [**323**, **324**].

54. The product of $2^{\aleph_0}$ copies of a discrete two point space.

55. The Stone-Cech compactification βN of the countable discrete space N [**361**, #111, or **136**, example 6.10]. This example is not an H_1 space [**39**] and does not belong to class $\mathfrak{R}$ [**182**, example 2.3]. For the character of this space see [**149**]. Example 44 is a modification of this example.

56. The Stone-Cech compactification βQ of the space of rational numbers [**136**, example 6.10]. βQ does not belong to $\mathfrak{R}$ [**182**, example 2.3].

57. The Stone-Cech compactification βR of the space of real numbers [**136**, example 6.10]. βR does not belong to $\mathfrak{R}$ by [**182**, example 2.3].

58. Example 1.1 of Franklin and Rajagopalan [**126**].

59. The Stone-Cech compactification of example 50. This is the space Y of [**267**, Example 10.5].

60. The Tychonoff plank in the version with the corner point (ω_1, ω_0) deleted [**136**, example 8.20, or **257**, #87]. Compare example 52.

61. The space X in section 4 of Morita [**276**].

62. The space Y in section 4 of Morita [**276**].

63. The product space of $[0, \omega_1)$ with $[0, \omega_1]$. [**199**, Problem 4.E, or **165**, Example 2.3].

64. Example X of Suzuki [**372**] with $M_\alpha = [0, \omega_1]$ and cardinality of $A = \aleph_1$.

65. The real line with an open base consisting of the usual open intervals with at most a countable set of points deleted. This is a pseudo-finite example of Alexandroff and Urysohn [**2**, example 2].

66. Example 10.5 of Michael [**267**].

67. The space $X \times Y$, where $X = \{0, 1, 1/2, \cdots\}$ with the usual topology and Y is example 9 (the sequential fan). Bagley and Weddington [**47**, Theorem 1].

68. The example $P \cup N$ of Novak [**361**, #112, or **136**, example 9.15].

69. The product space of $2^{\aleph_0}$ copies of the real line.

70. The modification of Bing's example G by Michael in [**258**, example 2].

71. Isiwata's modification ([**183**], or the space X of [**267**] Example 10.7) of Novak's example, $P \cup N$, number **68** above.

72. Isiwata's modification of Novak's product space (the space $X \times Y$ of [**267**] Example 10.7).

In the chart which follows, the second column refers to the separation axioms which the examples satisfy — note that all examples are Hausdorf — with "P" meaning paracompact Hausdorff, "K" meaning compact Hausdorff, and"RC" meaning regular and countable. An "H" means that the example has the stated property hereditarily, "+" means the example has the property, but some subspace does not have the property, "⊣" means the example has the property, and "—" means the example does not have the property. Blanks in the chart indicate cases which have not been answered. The letter "c" denotes the cardinality of the continuum. However, the continuum hypothesis is at times assumed, so that in some cases "c" actually denotes that the cardinality, m, satisfies $\aleph_0 < m \leqq c$.

		bi-sequential	count. bi-sequen	Frechet	sequential	det. by count	weak 1st count	strong access	accessibility	strongly k'	k'	k'_0	accum. complete	pt. count. type	bi-k	count. bi-k	singly bi-k	k	strict q	quasi k	weakly k	k_0	character	pseudo character
1	K	H	H	H	H	H	−	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	c	c
2	K	H	H	H	H	H	−	H	H	H	H	H	H	⊣	H	H	H	H	⊣	H	H	H	c	c
3	K	H	H	H	H	H	−	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	$\aleph_1$	$\aleph_1$
4	P	H	H	H	H	H	−	H	H	H	H	H	H	−	H	H	H	H	−	H	H	H	c	$\aleph_0$
5	P	H	H	H	H	H	−	H	H	H	H	H	H	−	H	H	H	H	−	H	H	H	c	$\aleph_0$
6	P	H	H	H	H	H	−	H	H	H	H	H	H	−	H	H	H	H	−	H	H	H	c	$\aleph_0$
7	P	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	2^c	$\aleph_0$
8	K	−	H	H	H	H	−	H	H	H	H	H	H	H	H	H	H	H	H	H	H	H	meas	
9	RC	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	c	$\aleph_0$
10	P	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	c	$\aleph_0$
11	RC	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	c	$\aleph_0$
12	P	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	c	$\aleph_0$
13	3.5	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	c	$\aleph_0$
14	P	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	c	$\aleph_0$
15	P	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	c	$\aleph_0$
16	P	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	c	$\aleph_0$
17	P	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	c	$\aleph_0$
18	RC	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	c	$\aleph_0$
19	3.5			H	H	H	−		H		H	H	H				H	H		H	H	H		$\aleph_1$
20	P	−	−	H	H	H	−	−	H	−	H	H	H	−	−	−	H	H	−	H	H	H	2^c	c
21	K	−	−	−	+	H	−	−	−	+	+	+	−	+	+	+	+	+	⊣	⊣	+	+	c	c
22	P	−	−	−	+	H	+	−	−	−	−	−	−	−	−	−	−	+	−	+	+	+	c	$\aleph_0$
23	P	−	−	−	+	H	+	−	−	−	−	−	−	−	−	−	−	+	−	+	+	+	c	$\aleph_0$
24	P	−	−	−	+	H	+	−	−	−	−	−	−	−	−	−	−	+	−	+	+	+	c	$\aleph_0$

		bi-sequential	count. bi-sequen	Frechet	sequential	det. by count	weak 1st count	strong access	accessibility	strongly k'	k'	k_0	accum. complete	pt. count. type	bi-k	count. bi-k	singly bi-k	k	strict q	quasi k	weakly k	k_0	character	pseudo character
25	RC	—	—	—	+	H	+	—	—	—	—	—	—	—	—	—	—	+	—	+	+	+	c	$\aleph_0$
26	2	—	—	—	+	H	+	—	—	—	—	—	—	—	—	—	—	+	—	+	+	+		$\aleph_0$
27	P	—	—	—	+	H	—	—	—	—	—	—	—	—	—	—	—	+	—	+	+	+	c	$\aleph_0$
28	2	—	—	—	+	H	+	—	—	—	—	—	—	—	—	—	—	+	—	+	+	+		$\aleph_0$
29	2	—	—	—	+	H		—	—	—	—	—	—	—	—	—	—	+	—	⊣	⊣	⊣		$\aleph_0$
30	RC	—	—	—	+	H		—	—	—	—	—	—	—	—	—	—	+	—	+	+	+	c	$\aleph_0$
31	3.5	—	—	—	+	H	+	—	—	—	—	—	—	—	—	—	—	+	—	⊣	+	+	c	$\aleph'_0$
32	RC	—	—	—	+	H	+	—	—	—	—	—	—	—	—	—	—	+	—	+	+	+	c	$\aleph_0$
33	2	—	—	—	+	H	+	—	—	—	—	—	—	—	—	—	—	+	—	+	+	+		$\aleph_0$
34	2	—	—	—	+	H	+	—	—	—	—	—	—	—	—	—	—	+	—	+	+	+		$\aleph_0$
35	P	—	—	—	+	H	+	—	—	—	—	—	—	—	—	—	—	+	—	+	+	+	c	$\aleph_0$
36	P	—	—	—	+	H		—	—	—	—		—	—	—	—	—	+	—	+	+	+		
37	2				⊣	H								—				⊣		⊣	⊣	⊣	c	$\aleph_0$
38	RC	—	—	—	—	H	—	H	H	—	—	—	—	—	—	—	—	—	—	—	—	—	c	$\aleph_0$
39	RC	—	—	—	—	H	—	H	H	—	—	—	—	—	—	—	—	—	—	—	—	—	c	$\aleph_0$
40	RC	—	—	—	—	H	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	c	$\aleph_0$
41	RC	—	—	—	—	H	—	H	H	—	—	—	—	—	—	—	—	—	—	—	—	—	c	$\aleph_0$
42	RC	—	—	—	—	H	—	—	H	—	—	—	—	—	—	—	—	—	—	—	—	—	c	$\aleph_0$
43	2	—	—	—	—	H	—		H	—	—	—	—	—	—	—	—	—	—	—	—	—	c	$\aleph_0$
44	2	—	—	—	—	H	—			—	—	—	—	—	—	—	—	—	—	—	—	—	c	$\aleph_0$
45	2	—	—	—	—	H	—						—					+		⊣	+	+		
46	RC	—	—			H								—	—	—			—				c	$\aleph_0$
47	P	—	—	—	—	H	—			—	—	—	—	—	—	—	—	—	—					
48	2	—	—	—	—	—	—	H	H	—	—	H	H	—	—	—	—	—		—	⊣	⊣		$\aleph_0$

		bi-sequential	count. bi-sequen	Frechet	sequential	det. by count	weak 1st count	strong access	accessibility	strongly k'	k'	k'_0	accum. complete	pt. count. type	bi-k	count. bi-k	singly bi-k	k	strict q	quasi k	weakly k	k_0	character	pseudo character
49	P	—	—	—	—	—	—	—	H	—	—	H	H	—	—	—	—	—	—	—	—	H	$\aleph_1$	$\aleph_1$
50	P	—	—	—	—	—	—	—	H	—	—	H	H	—	—	—	—	—	—	—	—	H	c	c
51	K	—	—	—	—	—	—	—	—	+	+	+	—	+	+	+	+	+	⊣	⊣	⊣	⊣	$\aleph_1$	$\aleph_1$
52	K	—	—	—	—	—	—	—	—	+	+	+	—	+	+	+	+	+	⊣	⊣	⊣	⊣	$\aleph_1$	$\aleph_1$
53	K	—	—	—	—	—	—	—	—	+	+	+	—	+	+	+	+	+	⊣	⊣	⊣	⊣	c	c
54	K	—	—	—	—	—	—	—	—	+	+	+	—	+	+	+	+	+	⊣	⊣	⊣	⊣	c	c
55	K	—	—	—	—	—	—	—	—	+	+	+	—	+	+	+	+	+	⊣	⊣	+	⊣	c	c
56	K	—	—	—	—	—	—	—	—	+	+	+	—	+	+	+	+	+	+	⊣	⊣	⊣	c	c
57	K	—	—	—	—	—	—	—	—	+	+	+	—	+	+	+	+	+	+	⊣	⊣	⊣	c	c
58	K	—	—	—	—	—	—	—	—	+	+	+	—	+	+	+	+	+	⊣	⊣	⊣	⊣	$\aleph_1$	$\aleph_1$
59	K									⊣	⊣	⊣		+	⊣	⊣	⊣	⊣	⊣	⊣	⊣	⊣		
60	3.5	—	—	—	—	—	—	—	—	+	+	+	—	+	+	+	+	+	⊣	⊣	⊣	⊣	$\aleph_1$	$\aleph_1$
61	3.5									⊣	⊣	⊣		⊣	⊣	⊣	⊣	⊣	⊣	⊣	⊣	⊣	$\aleph_1$	$\aleph_1$
62	3.5									⊣	⊣	⊣		⊣	⊣	⊣	⊣	⊣	⊣	⊣	⊣	⊣	$\aleph_1$	$\aleph_1$
63	3.5	—	—	—	—	—	—	—	—	+	+	+	—	+	+	+	+	+	⊣	⊣	⊣	⊣	$\aleph_1$	$\aleph_1$
64	P	—	—	—	—	—	—	—	—	—	⊣	+		—	—	—	+	+	—	+	⊣	⊣	$2^{\aleph_1}$	$\aleph_1$
65	2	—	—	—	—	—	—	—	—	—	—	H	H	—	—	—	—	—	—	—	—	H	c	$\aleph_0$
66	P	—	—	—	—	—	—	—	—	—	—			+	+	+	+	+	⊣	⊣	⊣	⊣		
67	3.5	—	—	—			—	—	—	—	—							+		⊣	⊣	⊣		
68	3.5	—	—	—	—	—	—			—	—	—	—	—	—	—	—	—	⊣	⊣	—	—	c	c
69	3.5	—	—	—	—	—	—			—	—			—	—	—	—	—	—				c	c
70	4	—	—	—	—		—			—	—			—	—	—	—	—			—			
71	3.5	—	—	—	—		—			—	—			—	—	—	—	—	⊣					
72	3.5	—	—	—	—		—			—	—			—	—	—	—	—	—					

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