GREEDY ALGORITHM AND COINAGE SYSTEMS

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Given a coinage system with coins of denominations $1 = c_0 < c_1 < \cdots < c_k$, let $f_k(x)$ denote the minimum number of coins needed to give change for the amount x. By the principle of optimality

(1)
$$f_k(x) = \min_{m \ge 0} \{m + f_{k-1}(x - mc_k)\},\$$

where f_{k-1} refers to the corresponding function for the coinage system with denominations c_0, \ldots, c_{k-1} . When the greedy algorithm is applied to the minimization problem, the number of coins used to give change for the amount x is

(2)
$$g_k(x) = [x/c_k] + g_{k-1}(x - [x/c_k]c_k).$$

Our paper gives a partial solution to the problem of characterizing those coinage systems for which $f_k = g_k$. Subsequent to the presentation of our paper, we found that a complete solution of the problem was obtained in [1].

Reference

1. M. J. Magazine, G. L. Nemhauser and L. E. Trotter, Jr., *When the greedy solution solves a class of knapsack problems*. Operations Research (Journal of the Operations Research Society of America) 23 (1975) 207–217.

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