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AN APPENDIX TO "CURVATURE AND PROPER HOLOMORPHIC MAPPINGS BETWEEN BOUNDED DOMAINS IN $\mathbf{C}^n$"

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For the implication from Theorem 4 to Theorem 5 in [**1**], we merely considered the boundary convergence case as our proof stood, because of the following fact. This is a folklore following from those old results in [**2**] and [**3**].

Fact. Let D_1 and D_2 be two strongly pseudoconvex bounded domains in $C^n, n \geq 2$. A sequence of proper holomorphic mappings $\{f_i : D_1 \to D_2\}$ can never converge to a *non-proper* holomorphic mapping $f : D_1 \to D_2$ on compact subsets.

Proof. Let's assume that $\{f_i\}$ converges on compact subsets to a non-proper holomorphic map $f : D_1 \to D_2$. Consequently, there is a sequence $\{x_k\}$ in D_1 convergent to $p \in \partial D_1$, such that $\{y_k = f(x_k)\}$ converges to a point $q \in D_2$. By Pincuk's theorem [**2**], all $f_i : D_1 \to D_2$ are finite unbranching covers. We denote by d_i the diameter (with respect to the distance function d_{D_1} induced by Cheng-Yau Einstein Kähler metric on D_1) of the fiber $\{f_i^{-1}(f_i(z_1))\}$ at $z_1 \in D_1$.

Suppose a subsequence of $\{d_i\}$ tends to ∞ as i grows, apparently there is a sequence of covering transformations which will bring z to ∂D_1. Applying [**3**], one concludes $D_1 \cong, B_n$. By Cartan's fixed point theorem ([**1**] Theorem, (i) p. 187), this implies $D_1 \cong B_n \cong D_2$. Thus all f_i are automorphisms. By classical H. Cartan's compactness theorem, f must be a biholomorphism. This contradicts our assumption.

On the contrary, let's assume $\{d_i\}$ are bounded above by a constant $M < +\infty$. By path-lifting property of coverings, triangle inequality of metrics and the assumption of convergence of $\{f_i\}$ to f, it is elementary to show, for sufficiently large $k, d_{D_1}(x_1, x_k) \leq M + d_{D_2}(y_1, q) + \epsilon$, here d_{D_2} = distance function induced by Cheng-Yau Einstein Kähler metric on $D_2, \epsilon =$ any fixed positive number. Since $\lim_{k\to\infty} d_{D_1}(x_1, x_k) = \infty$, we gain a contradiction.

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