CASPER GOFFMAN

By Togo Nishiura

It was in the mid 1950s when I first met Casper at Purdue University. At that time Lamberto Cesari had a large group of mathematicians and students working in differential equations, the calculus of variations and surface area theory. My work with Cesari was on surface area, measures on surfaces and topological aspects related to these topics. Cesari was also known for his work in multiple Fourier series — BV functions were among his specialties. Casper and Cesari had many common mathematical interests. Though Casper and Cesari did not write joint papers it is no surprise that Casper found many collaborators at Purdue. It was not just surface area and BV functions that attracted me to Cas, it was his consummate interest in real functions. He is one of the few people who could actually see real functions. I'm sure many have experienced his descriptions of functions — he would place a dot in the middle of a piece of paper or on a blackboard and proceed to describe, globally or locally, a function that he had in mind with nothing but a dot for the listener to look at.

I have not studied all of Casper's papers, but I do have opinions on some of them. So allow me to tell you of some of my favorites whose influence goes beyond classical real analysis.

The first is one that I had a part in. During my graduate studies at Purdue, research on rings of continuous functions was in its prime and Purdue was a major center of activity — completely regular spaces were in vogue. Casper saw that the analytic notion of approximate continuity could be couched in terms of a completely regular topology determined by the Lebesgue density theorem. Even better, this topology was easily seen to satisfy the countable chain condition, a condition that interested set theoretic topologists. It was not long after these observations were made that J. C. Oxtoby mentioned the density topology at the very end of the 1971 edition of his marvelous book *Measure and Category*. Of course, Oxtoby acknowledged Casper's contribution. (As an aside, let me mention that many topologists attribute the complete regularity of the density topology to Oxtoby and not to Casper; I find this an unfortunate slight.) Connections to Baire spaces were made. Along this line, the famous Blumberg theorem for real-valued functions on $\mathbb{R}$ was investigated for functions defined on other topological spaces. The density topology played a big role in many interesting topological results, some with set theoretic implications. The Casper paper I am referring to is *C. Goffman, C. J. Neugebauer and T. Nishiura*, **Density topology and approximate continuity**, *Duke Math. J.*, **28** (1961)

497-505. A related paper is *C. Goffman and D. Waterman*, **Approximately continuous transformations**, *Proc. Amer. Math. Soc.*, **12** (1961) 116-121.

The second concerns surface area and BV functions. My interests in this area are somewhat at odds with Casper's view. I was interested in continuous parametric transformations and Cas was interested in non-parametric surface area, that is, real-valued functions that are not necessarily continuous. One might say that his research along this line bordered on geometric measure theory — there is an oblique reference to one of Casper's works in Chapter 4: *Homological integration theory* of H. Federer's major 1969 book *Geometric Measure Theory* (see page 343). No one would consider this book as being classical analysis. Casper's approach to the subject followed a more classical vein. Federer must have appreciated Casper's results (*C. Goffman*, **A characterization of linearly continuous functions whose partial derivatives are measures**, *Acta Mat.*, **117** (1967) 165-190) — he placed them in this chapter in the context of locally normal n dimensional currents in $\mathbb{R}^n$. I think another very nice paper on surface area is *C. Goffman and W. P. Ziemer*, **Higher dimensional mappings for which the area formula holds**, *Ann. of Math. (2)*, **92** (1970) 482-488. This paper deals with non-continuous parametric transformations.

Casper wrote on many topics and with many co-authors. He was very generous with his ideas — he loved to talk "mathematics." Even more he loved to talk about mathematicians and rank them as a sport. There wasn't anything he wouldn't try to rank — this amused Ellie, my wife, no end. Some of his rankings were outrageous — it was like sports commentators at play-off time. He even ranked artists — he and Eve collected prints which they would proudly show me when I visited them, wonderful original works of art. We shared a common interest in collecting art — he on a much grander scale than I — with many excursions wandering through art galleries. But he never ranked his students, his co-authors, his friends. He loved to talk about his family — Eve, children and grandchildren. I have digressed, but not too far. This brings me to the dedication to the book *Homeomorphisms in Analysis* (Mathematical Surveys and Monographs, 54, American Mathematical Society, Providence, RI, 1997) which Casper, Dan Waterman and I wrote. The dedication was to Henry Blumberg, Lamberto Cesari and Antoni Zygmund, our mentors who were admired greatly by Casper. These were mathematicians who were leaders in areas he worked in. In his often silly manner, he said to me "isn't it interesting that they have only one given name just like us?" He was full of this sort of fact. The book must be one of his last publications. How the book came to be is an interesting story. During a conference honoring Bill Ziemer, Cas said to me that he had an idea for a book that combined homeomorphisms and real analysis — he described the contents — he thought of Dan as a collaborator. But he was not sure he had the energy to carry it through. I said to him "we'll write it, it's a book that needs to be published." So the project started, he sent me material he thought should be included and he got Dan to write the part on Fourier series. Cas and I put our half of the book into a coherently organized form and Dan brought together his part to tell our story of homeomorphisms in analysis. Working with Cas was very interesting because we had very different ways of looking at the body of research that was to be included — analysis on his part and topology on mine. Cas was happy to see how the book had turned out — I think he was very pleased with and proud of how his works and ideas had influenced mathematics.

Casper, I believe your influence is not over yet.