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AN EMBEDDING OF SEMIPRIME *P.I.*-RINGS

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Let us say an extension R' of a ring R is a *quotient ring* of R if every regular element of R is invertible in R'. In this note we construct a class of quotient rings of semiprime *P.I.*-rings and use this construction to find rapid proofs of several facts about semiprime *P.I.*-rings.

1. Preliminaries. Throughout this paper R will denote a *semiprime P.I.-ring* with unity and center C, i.e., R has no nonzero nilpotent ideals and the standard polynomial

$$S_{2n}(X_1, \cdots, X_{2n}) = \Sigma_\pi(\operatorname{sgn} \pi)X_{\pi(1)} \cdots X_{\pi(2n)},$$

the sum taken over all permutations π of $(1, \cdots, 2n)$, is an identity of R for suitable n (the minimal such n is the *degree* of R). Formanek [5] has constructed a polynomial $g_n(X_1, \cdots, X_{n+1})$ which is central for all semiprime *P.I.*-rings of degree n, and Rowen [11] has used these central polynomials to prove

THEOREM A. *Any nonzero ideal of R intersects C nontrivially.*

Let $S = \{c \in C: cr \neq 0$ for all nonzero r in $R\}$. Define an equivalence relation on $R \times S$ by saying $(r_1, s_1) \sim (r_2, s_2)$ if $r_1s_2 = r_2s_1$, and let rs^{-1} denote the equivalence class of (r, s). Then $R_S = \{rs^{-1}: (r, s) \in R \times S\}$ is a ring when endowed with the (well-defined) operations $r_1s_1^{-1} + r_2s_2^{-1} = (r_1s_2 + r_2s_1)(s_1s_2)^{-1}$, called the *ring of central quotients of R*. The following theorem is a direct consequence of Theorem A (cf., Rowen [11, §2]):

THEOREM B. *If R is a prime P.I.-ring of degree n, then R_S is simple Artinian of dimension n^2 over its center C_S, C_S is the quotient field of C, and R_S satisfies the identities of R.*

Theorem B often enables us to study R by examining R_S. If R is a semiprime *P.I.*-ring of degree n and satisfies the ascending chain condition on annihilators of two-sided ideals, then R_S is the classical semisimple Artinian ring of left and right quotients of R (cf., [12]). Unfortunately, this situation fails for semiprime *P.I.*-rings in general, so one is led to study other extensions of R. The purpose of this paper is to introduce a straightforward type of extension of R and to deduce from it properties of semiprime *P.I.*-rings and their classical quotient rings (if these exist). This paper subsumes Fisher [4]. First we shall derive some easy known properties of R.

For a subset A of R, let $\mathrm{Ann}_R(A)$ denote $\{r \in R \mid Ar = 0\}$. Also we say an ideal A of R is *essential* if for every nonzero ideal B of R, $A \cap B \neq 0$. Since R is semiprime, $A \cap B = 0$ if and only if $AB = 0$. The following lemma is known by Martindale [9].

LEMMA 1. (i) *If E is an essential ideal of C, then ER is an essential ideal of R.*

(ii) *If J is a left ideal of R with $\mathrm{Ann}_R(J) = 0$, then $J \cap C$ is essential in C, so J contains an essential ideal of R.*

Proof. (i) Suppose that $A \cap E = 0$ for some ideal A of R. Then $(A \cap C) \cap E = A \cap (C \cap E) = A \cap E = 0$, implying $A \cap C = 0$. Hence $A = 0$ by Theorem A and thus ER is essential.

(ii) Viewed as a ring (without 1), J is clearly a *P.I.*-ring and can easily be shown to be semiprime. We claim that $J \cap C = \mathrm{cent}\, J$. Indeed $J \cap C \subseteqq \mathrm{cent}\, J$ and if $a \in \mathrm{cent}\, J$, then for all r in R and for all x in J, $(ra - ar)x = rax - a(rx) = rax - r(xa) = rax - rax = 0$. Hence $(ra - ar) \in \mathrm{Ann}_R(J) = 0$ and so $a \in C$.

Now let B be an ideal of C such that $(J \cap C) \cap B = 0$. Then $(J \cap C \cap BR)^2 \subseteqq (J \cap C)BR = B(J \cap C)R \subseteqq (B \cap (J \cap C))R = 0$ and so $(J \cap C \cap BR)^2 = 0$. Since $J \cap C$ has no nonzero nilpotent elements, we have $J \cap C \cap BR = 0$, i.e., $(J \cap C) \cap (J \cap BR) = 0$. But by Theorem A applied to the semiprime ring J (with center $J \cap C$), $J \cap BR = 0$. This implies $RJB = BRJ \subseteqq J \cap BR = 0$, so $B \subseteqq \mathrm{Ann}_R(RJ) = \mathrm{Ann}\, J = 0$. Hence $J \cap C$ is essential in C. The rest of the lemma follows from (i).

2. Definition and elementary properties of $T(R)$.

For the remainder of this paper, we assume that the semiprime *P.I.*-ring R has degree n. This implies that every prime factor ring of R has degree equal to or less than n. The *degree* of a prime ideal P of R is defined as the degree of R/P.

Let $\mathscr{P}$ be a collection (indexed by Λ) of prime ideals P_λ of R such that $\bigcap \{P_\lambda : \lambda \in \Lambda\} = 0$. For each λ in Λ, set $R_\lambda = R/P_\lambda$, let Q_λ equal the simple Artinian ring of central quotients of R_λ, and let Q be the complete direct product $\Pi\{Q_\lambda : \lambda \in \Lambda\}$. There is a natural embedding $R \to \Pi R_\lambda \to Q$ and we shall often view R as a subring of Q under this embedding. Hence R satisfies the identities of Q. On the other hand, any identity f of R is an identity of each R_λ, and is an identity of each Q_λ by Theorem B; hence f is an identity of $Q = \Pi Q_\lambda$. Consequently, R and Q satisfy the same identities.

Clearly Q is *von Neumann regular*, i.e., for any $x \in Q$, there is some y in Q such that $xyx = x$.

As remarked above, each Q_λ has degree $\leqq n$. Let $\Lambda_j = \{\lambda \in \Lambda : Q_\lambda$ has degree $j\}$ and let $\bar{Q}_j = \Pi\{Q_\lambda : \lambda \in \Lambda_j\}$. Then $\bar{Q}_j$ is a semiprimitive ring of degree j with the property that every nonzero homomorphic image of $\bar{Q}_j$ has degree j. This is equivalent to saying, by the Artin [2]-Procesi [10] theorem, that $\bar{Q}_j$ is an Azumaya algebra of rank j. Hence Q is a finite direct sum of the Azumaya algebras $\bar{Q}_j$ of finite rank j.

LEMMA 2. *Any nonzero homomorphic image $\psi(Q)$ of Q is von Neumann regular. Moreover, $\psi(Q)$ is the finite direct sum of the Azumaya algebras $\psi(\bar{Q}_j)$ of finite rank j, and each identity of R is an identity of $\psi(Q)$.*

Proof. Every homomorphic image of a von Neumann ring is von Neumann regular. Also, every homomorphic image of $\psi(\bar{Q}_j)$ is a homomorphic image of $\bar{Q}_j$, thereby having rank j; hence $\psi(\bar{Q}_j)$ is Azumaya of rank j, and clearly $\psi(Q)$ is the direct sum of $\psi(\bar{Q}_j)$ for $j = 1, \cdots, n$. The last assertion is immediate.

For any x in Q, let x_λ denote the component of x in Q_λ and let $W_x = \{\lambda \in \Lambda : x_\lambda \neq 0\}$. Set $V = \{x \in Q : \bigcap \{P_\lambda : \lambda \in W_x\}$ is an essential ideal of $R\}$. Now V is an ideal of Q because, taking x, y in V and q in Q, $W_{x \pm y} \subseteqq W_x \cup W_y$; $W_{qx} \subseteqq W_x$; $W_{xq} \subseteqq W_x$. Let us define $T(R, \mathscr{P}) = Q/V$. From Lemma 2 we have that $T(R, \mathscr{P})$ is a finite direct sum of Azumaya algebras of finite rank and is von Neumann regular.

THEOREM 1. (i) *There is a canonical imbedding $R \to T(R, \mathscr{P})$ given by $R \to Q \to Q/V$.*

(ii) *Half regular elements of R are both left and right invertible in $T(R, \mathscr{P})$.*

(iii) *$T(R, \mathscr{P})$ satisfies precisely the same identities as R.*

Proof. (i) We need show only that $R \cap V = 0$. If $r \in R \cap V$, then $\bigcap \{P_\lambda : \lambda \in W_r\}$ is essential in R and so $\bigcap \{P_\lambda : r \in P_\lambda\} = 0$. Hence $r = 0$.

(ii) Let r in R have right annihilator zero. Then $\mathrm{Ann}_R(Rr) = 0$ and Rr contains an essential ideal E of C by Lemma 1(ii). Let $W'_r = \{\lambda : P_\lambda \not\supseteqq E\}$. Clearly $W'_r \subseteqq W_r$. Moreover, for any λ in W'_r there is an x_λ in Q_λ such that $0 \neq x_\lambda r_\lambda \in \mathrm{cent}\, Q_\lambda$. Since $\mathrm{cent}\, Q_\lambda$ is a field, there is d_λ in $\mathrm{cent}\, Q_\lambda$ such that $d_\lambda x_\lambda r_\lambda = 1_\lambda$. Furthermore, $r_\lambda d_\lambda x_\lambda = 1_\lambda$ because Q_λ is simple Artinian. Define y in Q as follows: $y_\lambda = 0$ for $\lambda \notin W'_r$ and $y_\lambda = d_\lambda x_\lambda$ for $\lambda \in W'_r$. Then $(yr - 1)_\lambda = 0$ and $(ry - 1)_\lambda = 0$ for all λ in W'_r. Thus $\bigcap \{P_\lambda : \lambda \in W_{yr-1}\} \supseteqq \bigcap \{P_\lambda : \lambda \notin W'_r\} \supseteqq$

E. It follows from Lemma 1(i) that $yr - 1 \in V$; likewise $ry - 1 \in V$. Hence, for $\bar{y}$ the image of y in $T(R, \mathscr{P})$, we have $\bar{y}r = 1$ and $r\bar{y} = 1$ in $T(R, \mathscr{P})$.

(iii) $T(R, \mathscr{P})$ satisfies each identity of R by Lemma 2; conversely, by (i), each identity of $T(R, \mathscr{P})$ is an identity of R.

The following theorem of Herstein-Small [8] is a consequence of Theorem 1.

COROLLARY 1. *Half regular elements of R are regular.*

Proof. If r in R is, say, right regular, then for some $y \in T(R, \mathscr{P})$ we have $ry = 1$. Hence r is left regular.

COROLLARY 2. *If R has a classical left ring of quotients R', then R' satisfies the same polynomial identities as R.*

Proof. In view of Theorem 1(ii) the canonical embedding of R into $T(R, \mathscr{P})$ extends to an embedding of R' into $T(R, \mathscr{P})$. Hence R' satisfies the identities of $T(R, \mathscr{P})$ which are precisely the identities of R.

Note that this construction of $T(R, \mathscr{P})$ is related to constructions of Amitsur [1] and Goldie [7]. Also, those versed in logic may wish to regard $T(R, \mathscr{P})$ as the "reduced product" (cf., [6]) of the simple Artinian rings $\{Q_\lambda: \lambda \in \Lambda\}$ by the filter $\{\Lambda - W_x: x \in V\}$.

3. **Definition and structure of $T(R)$.** Now we consider an interesting special case of $T(R, \mathscr{P})$. Index the set of all the prime ideals of R by a set $\bar{\Lambda}$ with $\bar{\Lambda}_i = \{\lambda \in \bar{\Lambda}: P_\lambda$ has degree $i\}$ for $i = 1, \cdots, n$. Set $\bar{N}_i = \bigcap \{P_\lambda: \lambda \in \bar{\Lambda}_i\}$ (if $\bar{\Lambda}_i = \phi$ then $\bar{N}_i = R$), $\Lambda_i = \{\lambda \in \bar{\Lambda}_i: P_\lambda \not\supseteq \bigcap_{j=i+1}^n \bar{N}_j\}$, $\mathscr{P}_i = \{P_\lambda: \lambda \in \Lambda_i\}$, $\mathscr{P} = \mathscr{P}_1 \cup \cdots \cup \mathscr{P}_n$, $\Lambda = \Lambda_1 \cup \cdots \cup \Lambda_n$. Clearly $\bigcap \{P: P \in \mathscr{P}) = \bar{N}_1 \cap \cdots \cap \bar{N}_n = 0$. We define $T(R)$ to be $T(R, \mathscr{P})$. Note that $\Lambda_n = \bar{\Lambda}_n$ and that $\Lambda = \Lambda_n$ if and only if $\bar{N}_n = 0$.

Let $N_i = \bigcap \{P: P \in \mathscr{P}_i\}$ and let $R_i = R/N_i$. Note that $N_n = \bar{N}_n$. Clearly R is a subdirect product of the R_i and this subdirect decomposition is unique with respect to the properties that each of the nonzero subdirect factors has a degree different from each of the other subdirect factors and that for any subdirect factor of degree j, the intersection of its prime ideals of degree j is zero. Our aim is to show how the structure of $T(R)$ is linked to this decomposition. As in Rowen [12], let a polynomial be called *regular* if it is linear in some indeterminant, and let the *central kernel* of a ring be the additive subgroup generated by the values taken (in the center) by regular central polynomials of the ring. The central kernel is an ideal of the center C. If the central kernel is essential in C, we say that R has *essential central kernel*. Let I be the central of R, let $B = N_1 \cap \cdots \cap N_{n-1}$, and let $R'_n = R/B$. It is shown in Rowen [12] that for $\lambda \in \bar{\Lambda}$, $I \not\subseteq P_\lambda$ if and only if $\lambda \in \Lambda_n$.

LEMMA 3. (i) *$(RI + N_n)/N_n$ is an essential ideal of R_n.*

(ii) *$(N_n + B)/B$ is an essential ideal of R'_n.*

(iii) *A semiprime ring R of degree j has essential central kernel if and only if the intersection of its prime ideals of degree j is zero.*

Proof. (i) Suppose that $[(A + N_n)/N_n] \cap [(RI + N_n)/N_n] = 0$ for some ideal A of R. Then $ARI \subseteq N_n \subseteq P_\lambda$ for each $\lambda \in \Lambda_n$. Since $I \not\subseteq P_\lambda$ for $\lambda \in \Lambda_n$, we have $A \subseteq \bigcap \{P_\lambda : \lambda \in \Lambda_n\} = N_n$. So

$$(A + N_n)/N_n = 0 .$$

(ii) Suppose that $[(A + B)/B] \cap [(N_n + B)/B] = 0$ for some ideal A of R. Then $AN_n \subseteq B = N_1 \cap \cdots \cap N_{n-1} \subseteq P_\lambda$ for each $\lambda \in \Lambda - \Lambda_n$. By definition $P_\lambda \not\supseteq N_n$ for $\lambda \in \Lambda - \Lambda_n$, so $A \subseteq \bigcap \{P_\lambda : \lambda \in \Lambda - \Lambda_n\} = B$. So $(A + B)/B = 0$.

(iii) Let $\bar{N}_j$ be the intersection of the prime ideals of degree j. Since every prime ideal of degree $< j$ contains I, we have $I \cap \bar{N}_j = 0$. Since I is essential in C, we have $\bar{N}_j \cap C = 0$, hence $N_j = 0$ by Theorem A. The reverse implication is immediate from (i) and Lemma 1.

Lemma 3(iii) gives us a neater characterization of $R_1, \cdots, R_n$. Namely, the nonzero R_i are uniquely determined if we are to express R as a subdirect product of minimal length of rings with essential central kernel.

LEMMA 4. (i) *Suppose that J is an ideal of R and $N_n \subseteq J$. Then J is essential in R if and only if J/N_n is essential in R_n.*

(ii) *Suppose $B \subseteq J$. Then J is essential in R if and only if J/B is essential in R'_n.*

Proof. (i) ($\Rightarrow$) Suppose that $J/N_n \cap [(A + N_n)/N_n] = 0$ for some ideal A of R. Then $JA \subseteq N_n$ and so $B \cap JA = 0$. Now since $I \subseteq P_\lambda$ for each $\lambda \in \Lambda - \Lambda_n$, we have $RI \subseteq \bigcap \{P_\lambda : \lambda \in \Lambda - \Lambda_n\} \subseteq B$ and $RI \cap JA = 0$, or $IJA = 0$. Hence $(J \cap AI)^2 \subseteq (JAI)^2 = 0$ and $J \cap AI = 0$ since R is semiprime. By hypothesis, we then see $AI = 0$, so $A \subseteq N_n$ by Lemma 3(i). Consequently $(A + N_n)/N_n = 0$.

Conversely suppose that $J \cap A = 0$ for some ideal A of R. Then $JA = 0 \subseteq N_n$, so $A \subseteq N_n$ by hypothesis. Thus $A^2 \subseteq N_nA \subseteq JA = 0$ and so $A = 0$.

(ii) ($\Rightarrow$) Suppose that $J/B \cap [(A + B)/B] = 0$. Then $JA \subseteq B$, or

$JAN_n \subseteqq B \cap N_n = 0$ which implies $AN_n = 0$. Hence $A \subseteqq B$ by Lemma 3(ii) and so $(A + B)/B = 0$. The proof of the converse is analogous to that in (i).

THEOREM 2. $T(R) \cong T(R_1) \oplus \cdots \oplus T(R_n)$.

Proof. We use induction on $n =$ degree of R. The assertion is true for $n = 2$. Since R'_n has degree $\leqq n - 1$, we have by our induction hypothesis that $T(R'_n) \cong T(R_1) \oplus \cdots \oplus T(R_{n-1})$. Let $\bar{Q}_n = \Pi\{Q_\lambda : \lambda \in \Lambda_n\}$, $\bar{Q}'_n = \Pi\{Q_\lambda : \lambda \in \Lambda - \Lambda_n\}$, $V_n = V \cap \bar{Q}_n$, and $V'_n = V_n \cap \bar{Q}'_n$. Clearly $V = V_n \oplus V'_n$ and $T(R) = Q/V \cong \bar{Q}_n \oplus \bar{Q}'_n/V \cong \bar{Q}'_n/V_n \oplus \bar{Q}'_n/V'_n$. But Lemma 4(i) shows $\bar{Q}_n/V_n \cong T(R_n)$ and Lemma 4(ii) shows $\bar{Q}'_n/V'_n \cong T(R'_n)$. Thus $T(R) \cong T(R_n) \oplus T(R'_n) \cong T(R_1) \oplus \cdots \oplus T(R_{n-1}) \oplus T(R_n)$.

Theorem 2 enables us to reduce the study of $T(R)$ to rings with essential central kernel.

THEOREM 3. *Let R be a semiprime P.I.-ring of degree n with essential central kernel. Then $T(R)$ is an Azumaya algebra of rank n^2 and $T(C) \cong$ center $(T(R))$.*

Proof. By Lemma 3(iii), $N_n = 0$. Hence $T(R)$ is a homomorphic image of $\Pi\{Q_\lambda : \lambda \in \Lambda_n\}$. Therefore, $T(R)$ is Azumaya of rank n^2. Write $C_\lambda =$ center Q_λ for $\lambda \in \Lambda$. Since $\Pi\{Q_\lambda : \lambda \in \Lambda_n\}$ is an Azumaya algebra of rank n^2, we have the following fact which we will need later, cent $[(\Pi_{\lambda \in \Lambda_n} Q_\lambda)/(V \cap \Pi_{\lambda \in \Lambda_n} Q_\lambda)] = (\Pi_{\lambda \in \Lambda_n} C_\lambda + V \cap \Pi_{\lambda \in \Lambda_n} Q_\lambda)/(V \cap \Pi_{\lambda \in \Lambda_n} Q_\lambda)$.

We claim that the homomorphism $\varphi: (\Pi_{\lambda \in \Lambda} Q_\lambda)/V \rightarrow (\Pi_{\lambda \in \Lambda_n} Q_\lambda)/(V \cap \Pi_{\lambda \in \Lambda_n} Q_\lambda)$, induced by the projection, $\Pi_{\lambda \in \Lambda} Q_\lambda \rightarrow \Pi_{\lambda \in \Lambda_n} Q_\lambda$, is an isomorphism. Indeed, suppose that $0 \neq x + V$ for x in $\Pi_{\lambda \in \Lambda} Q_\lambda$. Then $\bigcap \{P_\lambda : \lambda \in W_x\}$ is not essential. Since each prime of degree $< n$ contains I and $I \subseteqq \bigcap \{P_\lambda : \lambda \in W_x \cap (\Lambda - \Lambda_n)\}$ is essential, we conclude that $\bigcap \{P_\lambda : \lambda \in W_x \cap \Lambda_n\}$ is not essential and $0 \neq x + (V \cap \Pi_{\lambda \in \Lambda_n} Q_\lambda)$. Consequently φ is an isomorphism.

Now by Rowen [12, Theorem 3] there exists a 1:1 correspondence of $\{P_\lambda : \lambda \in \Lambda_n\}$ and the set of prime ideals of C, not containing I, given by $P_\lambda \rightarrow P_\lambda \cap C$. We claim that $T(C) \cong (\Pi_{\lambda \in \Lambda_n} C_\lambda)/(V \cap \Pi_{\lambda \in \Lambda_n} C_\lambda)$. The proof of this is similar to the one in the preceding paragraph because every prime in C which is not in $\{P_\lambda \cap C : \lambda \in \Lambda_n\}$ contains I which is essential in C.

Finally we have all the requisite pieces to obtain

$$\begin{aligned} T(C) &\cong (\Pi_{\lambda\in\Lambda_n} C_\lambda)/(V \cap \Pi_{\lambda\in\Lambda_n} C_\lambda) \\ &\cong (\Pi_{\lambda\in\Lambda_n} C_\lambda + V \cap \Pi_{\lambda\in\Lambda_n} Q_\lambda)/(V \cap \Pi_{\lambda\in\Lambda_n} Q_\lambda) \\ &\cong \text{cent}\,[(\Pi_{\lambda\in\Lambda_n} Q_\lambda)/(V \cap \Pi_{\lambda\in\Lambda_n} Q_\lambda)] \\ &\cong \text{cent}\,((\Pi_{\lambda\in\Lambda} Q_\lambda)/V) = \text{cent}\,(T(R)) . \end{aligned}$$

REMARK 1. Given $\mathscr{P}$ as in §2, let $\varphi: Q \rightarrow T(R, \mathscr{P})$ be the canonical homomorphism. Then there is a partial order on {ideals A of Q: $\text{Ker}\,\varphi \subseteq A$ and $R \cap A = 0$}. So there exists a maximal such ideal $\bar{A}$. Then $Q/\bar{A} \cong T(R, \mathscr{P})/(\bar{A}/(\text{Ker}\,\varphi)$ is an extension of R which has all the aforementioned properties of $T(R, \mathscr{P})$, and, moreover, any ideal of $Q/\bar{A}$ intersects R (viewed as a subring) nontrivially.

REMARK 2. Suppose that R has an involution (*). Then, for any prime P of degree j, there is a prime P^* of degree j and an isomorphism $R/P \rightarrow R/P^*$ given by $r + P \rightarrow r^* + P^*$. This isomorphism extends to the algebra of central quotients, and one can check that in the definition of $T(R)$, an involution is induced in Q. Moreover, V is stable under this involution, so $T(R)$ inherits an involution which coincides with (*) on R. Hence the embedding $R \rightarrow T(R)$ is actually an embedding in the category of rings with involution.

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