PACIFIC JOURNAL OF MATHEMATICS
Vol. 102, No. 1, 1982

MANIFOLDS MODELLED ON THE DIRECT LIMIT OF LINES

RICHARD E. HEISEY

The main theorem of this paper is that topological manifolds modelled on $R^\infty = \varinjlim R^n$ are stable. Combined with previous work this theorem enables us to embed R^∞-manifolds as open subsets of R^∞, classify R^∞-manifolds by homotopy type, and triangulate R^∞-manifolds.

The results established here were announced in the [8].

1. Definitions and results. Let R^n be the cartesian product of n copies of R, where R denotes the reals. Define $i_n: R^n \to R^{n+1}$ by $i_n((x_1, \cdots, x_n)) = (x_1, \cdots, x_n, 0)$. Then $R^\infty = \varinjlim \{R^n; i_n\}$. We regard R^∞ as the set $\{(x_1, x_2, x_3, \cdots,) \mid x_i \in R$, all i, and $x_i \neq 0$ for at most finitely many $i\}$. We identify R^n with $R^n \times \{(0, 0, \cdots, 0)\} \subset R^{n+k}$, $k \geqq 1$, and with $R^n \times \{(0, 0, \cdots)\} \subset R^\infty$. With this identification, a set $\mathscr{O} \subset R^\infty$ is open if and only if $\mathscr{O} \cap R^n$ is open in R^n, $n \geqq 1$. In the terminology of [14], for example, R^∞ is thus the strict inductive limit of $\{R^n\}$. As such it is a locally convex [14, Prop. 1, p. 127], nonmetrizable [14, Prop. 5, p. 129] topological vector space having a natural simplicial structure.

A topological manifold modelled on R^∞, or, more simply, an R^∞-manifold, is a Hausdorff space in which each point has a neighborhood homeomorphic to an open subset of R^∞. By way of example we note that countable direct limits of finite-dimensional manifold are often R^∞-manifolds. Also by [9, Corollary 2], if X is a locally finite polyhedron (more generally, a locally compact, locally finite-dimensional ANR) then $X \times R^\infty$ is an R^∞-manifold. Our main result is Theorem S, below, which asserts that R^∞-manifolds are stable with respect to multiplication by R^∞. We remark that because R^∞ is nonmetrizable and not a countable product (one can show that R^∞ is not homeomorphic to $R^\infty \times R^\infty \times R^\infty \times \cdots$) many of the arguments used in establishing stability of Hilbert space and Hilbert cube manifolds as, for example, in [1] and [16] do not apply here. Our proof uses an inductive argument on finite-dimensional subsets.

By "$\cong$" we denote "is homeomorphic to". We let $I = [0, 1]$. If $\mathscr{U}$ is an open cover of the space Y, two maps $f, g: X\ Y$ are *$\mathscr{U}$-close* if for each $x \in X$ there is a $U \in \mathscr{U}$ such that $\{f(x), g(x)\} \subset U$. A map $f: X \to Y$ is a *near homeomorphism* if for each open cover $\mathscr{U}$ of Y there is a homeomorphism $h: X \to Y$ such that f and h are $\mathscr{U}$-close.

For the remainder of this section let M and N denote paracompact, connected R^∞-manifolds.

THEOREM S (Stability). *The projection map $M \times R^\infty \to M$ is a near-homeomorphism. In particular $M \times R^\infty \cong M$.*

The proof of the stability theorem is given in § 3 of this paper.

In [7] it was shown that $M \times R^\infty$ embeds as an open subset of R^∞. Combined with Theorem S this immediately implies the open embedding theorem for R^∞-manifolds.

THEOREM $\mathscr{O}$ (Open Embedding). *There is an open embedding $f: M \to R^\infty$.*

Using Theorem $\mathscr{O}$, regard M as an open subset of R^∞. Let $\mathscr{C}$ be an open cover of M consisting of convex sets. By Theorem S there is a homeomorphism $h: M \times R^\infty \to M$ which is $\mathscr{C}$-close to the projection. Clearly, then, $H: M \times R^\infty \times I \to M$ defined by $H((m, x, t)) = (1 - t)h((m, x)) + tm$ is a homotopy in M, and the following corollary results.

COROLLARY 1. *There is a homeomorphism $h: M \times R^\infty \to M$ which is homotopic to the projection map.*

Let $f: M \to N$ be a homotopy equivalence. By [7, Theorem II-9] $(f \times \mathrm{id}): M \times R^\infty \to N \times R^\infty$ is homotopic to a homeomorphism g. Let $h_M: M \times R^\infty \to M$ and $h_N: N \times R^\infty \to N$ be homeomorphisms homotopic to the corresponding projection maps. Then $h_N g h_M^{-1}$ is a homeomorphism homotopic to f, and we have proven the following classification theorem.

THEOREM C (Classification by Homotopy Type). *If $f: M \to N$ is a homotopy equivalence, then f is homotopic to a homeomorphism $h: M \to N$.*

Since R^∞-manifolds have the homotopy type of ANR's [7, Theorem II-10], Theorem C has the following corollary.

COROLLARY 2. *If M and N have the same weak homotopy type, then they are homeomorphic.*

In [4] Dobrowolski obtains a special case of Corollary 2; namely, that $R^\infty \cong \varinjlim S^n$, where S^n is the n-sphere. He obtains this result by first showing that compact subsets of $\varinjlim S^n$ are negligible.

Using Theorems $\mathcal{O}$ and C we can now triangulate M. By Theorem $\mathcal{O}$ we may regard M as an open subset of R^∞. Since open subsets of R^∞ are Lindelöf [7, Propositions III. 1 and III. 2] M has the homotopy type of a countable, locally finite, simplicial complex K [13, Theorem 1 and Proposition 2]. By [9, Corollary 2] $|K| \times R^\infty$ is an R^∞-manifold, and clearly, $|K| \times R^\infty$ has the same homotopy type as M. By Theorem C, $M \cong |K| \times R^\infty$. This establishes the triangulation theorem.

THEOREM T (Triangulation). *$M \cong |K| \times R^\infty$, where K is a countable, locally-finite simplicial complex.*

We remark that Theorems S and T answer affirmatively two questions in the Appendix "Open problems of infinite-dimensional topology" in [3].

The author gratefully acknowledges several helpful conversations with Henryk Torunczyk and James West.

§ 2. **Lemmas.** Recall that we identify R^n with $R^n \times \{0, 0, \cdots, 0\} \subset R^{n+k}$ and with $R^n \times \{0, 0, \cdots\} \subset R^\infty$. If $A \subset R^\infty$, let $A^n = A \cap R^n$. Let d_n be the metric induced on R^n by the norm $\|x\| = (\sum x_i^2)^{1/2}$. If $\mathscr{U}$ is an open cover of Y, a homotopy $H: X \times I \to Y$ is *limited* by $\mathscr{U}$ if for every $x \in X$, $H(\{x\} \times I) \subset U$, some $U \in \mathscr{U}$. We abbreviate "finite-dimensional" by f.d. and "piecewise linear" by p.l. If $A \subset X$, by $\bar{A}$ we denote the closure of A in X.

LEMMA 1. *Let A and B be* f.d. *compact metric spaces with $A \subset B$. Let $f: B \to R^n$ be a continuous map such that f/A is an embedding. Then if m is sufficiently large, for every $\varepsilon > 0$ there is an embedding $g_\varepsilon: B \to R^m$ such that $g_\varepsilon/A = f/A$ and $d_m(f, g_\varepsilon) < \varepsilon$.*

Proof. We may assume that $2(\dim B) + 1 \leqq n$ so that there is an embedding $\alpha: B \to R^n$. Let $\beta: R^n \to R^n$ be a continuous extension of $\alpha f^{-1}: f(A) \to R^n$. Define $h: B \to R^n \times R^n$ by $h(b) = (f(b), \alpha(b))$ and $T: R^n \times R^n \to R^n \times R^n$ by $T(x, y) = (x, y - \beta(x))$. Then $g = Th: B \to R^n \times R^n$ is an embedding extending f/A. Choose $r > 0$ such that $g(B) \subset R^n \times \{z \in R^n \mid \|z\| \leqq r\}$. If $e(x, y) = (x, (\varepsilon/r)y)$, then $g_\varepsilon = eg$ is the desired embedding.

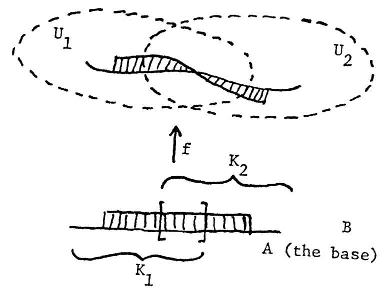
LEMMA 2. *Let X be a* f.d. *locally compact metric space and A and B closed sets in X such that $X = A \cup B$ and B is compact. Let U be an open subset of R^∞, $\mathscr{U}$ an open cover of U. Let $f: X \to U$ be a continuous map such that f/A is a closed embedding. Then there is an embedding $g: X \to U$ such that $g/A = f/A$ and such that*

$H: X \times I \to U$ defined by $H(x, t) = (1 - t)f(x) + tg(x)$ is limited by $\mathscr{U}$.

Proof. If C is a compact subset of U, then $f^{-1}(C)$ is contained in the compact set $(f/A)^{-1}(C) \cup B$. Hence f is proper. Thus, we can choose a relatively compact neighborhood V of the compact set $f^{-1}(f(B))$ in locally compact X.

Let n be such that $f(\bar{V}) \subset U \cap R^n$. By Lemma 1 there is an $m > n$ and an embedding $g_\varepsilon: \bar{V} \to R^m$ with $g_\varepsilon(x) = f(x)$ for $x \in A \cap \bar{V}$ and $d_m(g_\varepsilon, f/\bar{V}) < \varepsilon$, where $\varepsilon > 0$ is chosen smaller than $d_m(f(B), f(A \backslash V) \cap R^m)$ and such that the ε-neighborhood in R^m of any point of $f(\bar{V})$ is contained in a member of $\{W \cap R^m \mid W \in \mathscr{U}\}$. Define $g: X \to U$ by $g(x) = f(x)$ for $x \in A$ and $g(x) = g_\varepsilon(x)$ for $x \in \bar{V}$. Thus, g is one-to-one. Moreover, g is proper, for the same reason for which f is. It follows that g is the desired embedding.

LEMMA 3. *Let A and B be* f.d. *compact metric spaces with $A \subset B$. Let M be a paracompact space such that $M = U_1 \cup U_2$, where U_i, $i = 1, 2$, is an open subset of M homeomorphic to an open subset of R^∞. Let $f: B \to M$ be a continuous map such that f/A is an embedding. Then there is an embedding $f': B \to M$ such that $f'/A = f/A$.*



Let $\{K_1, K_2\}$ be a cover of B by compact sets such that $K_i \subset f^{-1}(U_i)$, $i = 1, 2$. By Lemma 2 there is an embedding $g_1: K_1 \cup [A \cap f^{-1}(U_1)] \to U_1$ such that $g_1(x) = f(x)$ for $x \in A \cap f^{-1}(U_1)$ and such that $f/K_1 \cup [A \cap f^{-1}(U_1)]$ is homotopic to g_1 by a homotopy H fixed on $A \cap f^{-1}(U_1)$ and limited by $\{U_1 \cap U_2, M\backslash f(K_1 \cap K_2)\}$. Note that $H[(K_1 \cap K_2) \times I] \subset U_1 \cap U_2$. Define $H': [(K_1 \cap K_2) \cup (A \cap K_2)] \to U_2$ by $H'(x, t) = H(x, t)$ for $x \in K_1 \cap K_2$, $t \in I$, and $H'(x, t) = f(x)$, $x \in A \cap K_2$, $t \in I$.

By Dugundji's theorem [5, p. 188], R^∞, and, hence, [10, p. 42], U_2 is an absolute neighborhood extensor for the class of metrizable spaces. It follows, as in the proof of [10, Theorem 2.2, p. 117], that U_2 has the homotopy extension property with respect to metric spaces. Since $H_0' = f/A \cap K_2$ extends by f to all of K_2, H' has an extension $\bar{H}: K_2 \times I \to U_2$. Define $g: B \to U_1 \cup U_2$ by $g/K_1 = g_1/K_1$ and $g/K_2 = \bar{H}_1$. Then g extends f/A.

By Lemma 2 there is an embedding $g_2: g^{-1}(U_2) \to U_2$ such that $g_2(x) = g(x)$ for $x \in g^{-1}(U_2) \cap (A \cup K_1)$. Define $f': B \to U_1 \cup U_2$ by $f'(x) = g_2(x)$ if $x \in g^{-1}(U_2)$ and $f'(x) = g(x)$ otherwise. Then $f'/K_1 = g_1/K_1$ and $f'/K_2 = g_2/K_2$ so that f' is continuous. If $f'(x) = f'(y)$, then either both x and y or neither x nor y is in $(f')^{-1}(U_2) = g^{-1}(U_2)$. In the first case $x = y$ since g_2 is one-to-one. In the latter case $x = y$ since g/K_1 is one-to-one. Clearly $f'/A = f/A$. Thus, f' is the desired embedding.

The last lemma is probably known. We include a proof for completeness.

LEMMA 4. *Let X be a finite polyhedron and M a compact* p.l. *manifold with boundary. If $f, g: X \to \operatorname{Int} M$ are homotopic topological embeddings, then for sufficiently large k there is an ambient isotopy H on $M \times [-1, 1]^k$ such that $H_1(f, 0) = (g, 0): X \to M \times [-1, 1]^k$.*

Proof. Let $H: X \times I \to \operatorname{Int} M$ be a homotopy with $H_0 = f$ and $H_1 = g$. Define $\bar{H}: X \times I \to \operatorname{Int}(M \times [-1, 1]^k)$ by $\bar{H}(x, t) = (H(x, t), t/2, 0, 0, \cdots, 0)$. Then $\bar{H}_0 = (f, 0)$ and $\bar{H}_1 = (g, 1/2, 0, \cdots, 0)$. Clearly it is sufficient to show that $(f, 0)$ and $\bar{H}_1$ are ambient isotopic.

Since $\bar{H}/X \times \{0, 1\}$ is an embedding, Theorem 1 of [2] implies that for sufficiently large k, $\bar{H}/X \times \{0, 1\}$ is ε-tame in $\operatorname{Int}(M \times [-1, 1]^k)$. Thus, there is an ambient isotopy K_t: $M \times [-1, 1]^k \to M \times [-1, 1]^k$ such that $K_t(\bar{H}(X \times I)) \subset \operatorname{Int}(M \times [-1, 1]^k)$, $t \in I$, and such that $K_1\bar{H}/X \times \{0, 1\}$ is a p.l. embedding. Using general position [15, 5.4, p. 61] there is, for sufficiently large k, a p.l. embedding $h: X \times I \to \operatorname{Int}(M \times [-1, 1]^k)$ such that $h/X \times \{0, 1\} = K_1\bar{H}/X \times \{0, 1\}$. By [11, Theorem 1.1, p. 426] there is an ambient isotopy $E_t: M \times [-1, 1]^k \to M \times [-1, 1]^k$ such that $E_1 h_0 = h_1$. Then $K_t^{-1}E_tK_t$ is an ambient isotopy on $M \times [-1, 1]^k$ with $K_1^{-1}E_1K_1(f, 0) = K_1^{-1}E_1K_1\bar{H}_0 = K_1^{-1}E_1h_0 = K_1^{-1}h_1 = \bar{H}_1$, as required.

3. Proof of Theorem S. We first prove the following weaker version of the stability theorem.

THEOREM S′. *Let M be a paracompact R^∞-manifold such that $M = U \cup V$, where U and V are homeomorphic to open subsets of R^∞. Then there is a homeomorphism $M \to M \times R^\infty$.*

Proof. We first show that M can be suitably expressed as the direct limit of topological manifolds. Let $\gamma\colon U \to U'$ and $\delta\colon V \to V'$ be homeomorphisms onto open subsets of R^∞. Then $U' = \varinjlim C'_n$ where C'_n is a compact metric subspace of $U' \cap R^n$ and where $C'_n \subset \operatorname{Int}_{n+1} C_{n+1}$. Express $V' = \varinjlim D'_n$ similarly. Let $C_n = \delta^{-1}(C'_n)$ and $D_n = \delta^{-1}(D'_n)$.

Fix $n \geqq 1$. Since $C_n \cup D_n$ is a compact f.d. metric space, there is an embedding $\alpha\colon C_n \cup D_n \to R^k$, some k. Since M is an absolute neighborhood extensor for metric spaces ([5, p. 188] and [10, p. 45]), $\alpha^{-1}\colon \alpha(C_n \cup D_n) \to M$ has a continuous extension β to a compact p.l. submanifold N of R^k containing $\alpha(C_n \cup D_n)$. Let $\pi\colon N \times I \to N$ be the projection. Then $\beta\pi\colon N \times I \to M$ is an embedding on $\alpha(C_n \cup D_n) \times \{0\}$. By Lemma 4 there is an embedding $\beta'\colon N \times I \to M$ such that $\beta'(\alpha(C_n \cup D_n) \times \{0\}) = \beta\pi(\alpha(C_n \cup D_n) \times \{0\}) = C_n \cup D_n$. Let $X = \partial(N \times I)$, a closed p.l. manifold. Let $X_n = \beta'(X)$. Note that $X_n \supset C_n \cup D_n$ and, since $M = \varinjlim (C_n \cup D_n)$, $M = \varinjlim X_n$.

Let $A \subset M$ be a compact subspace. Choose an open cover $\{Y_1, Y_2\}$ of M such that $\bar{Y}_1 \subset U$ and $\bar{Y}_2 \subset V$. Then $A = (A \cap \bar{Y}_1) \cup (A \cap \bar{Y}_2)$. The compactness of $A \cap \bar{Y}_1$ and $A \cap \bar{Y}_2$ implies that for some n, $\gamma(A \cap \bar{Y}_1) \subset C'_n$ and $\delta(A \cap \bar{Y}_2) \subset D'_n$ so that $A \subset C_n \cup D_n$. Thus, every compact subspace of M is contained in some X_n.

Now, let $B_n = [-n, n]^n$, $n \geqq 1$. Then $R^\infty = \varinjlim B_n$. Define $j'_{n,k}\colon X_n \times B_k \to M$ by $j'_{n,k}(x, t) = x$. By Lemma 4 there is an embedding $j_{n,k}\colon X_n \times B_k \to M$ such that $j_{n,k}(x, 0) = x$ for each $x \in X_n$.

Let $j_1 = j_{1,1}$. Choose $n_2 > 1$ such that $j_1(X_1 \times B_1) \subset X_{n_2}$. Consider

$$\begin{array}{ccc} X_1 \times B_1 & \xrightarrow{\alpha_1} & X_{n_2} \times B_{k_2} \\ \big\downarrow{\scriptstyle j_1} & \nearrow{\scriptstyle i_1} & \\ j_1(X_1 \times B_1) & & \end{array}$$

where $k_2 > 1$ is yet to be chosen, $i_1(y) = (y, 0)$ and $\alpha_1(x, t) = (x, (t, 0))$. Since B is contractible $i_1 j_1 \sim \alpha_1$ ("$\sim$" denotes "is homotopic to") with the homotopy taking place in Int $(X_{n_2} \times B_{k_2})$. Choose k_2 so large that, by Lemma 4, there is an ambient isotopy F_2 on $X_{n_2} \times B_{k_2}$ such that $(F_2)_1 \alpha_1 = i_1 j_1$. Let $j_2 = j_{n_2,k_2}\colon X_{n_2} \times B_{k_2} \to j_2(X_{n_2} \times B_{k_2})$. Let $h_1 = j_1$ and $h_2 = j_2(F_2)_1$. Consider

$$\begin{array}{ccc} X_1 \times B_1 & \xrightarrow{\alpha_1} & X_{n_2} \times B_{k_2} \\ \big\downarrow{\scriptstyle h_1} & & \big\downarrow{\scriptstyle h_2} \\ j_1(X_1 \times B_1) & \xrightarrow{\beta_1} & j_2(X_{n_2} \times B_{k_2}) \end{array}$$

where $\beta_1(y) = y$. Since $h_2\alpha_1 = j_2(F_2)_1\alpha_1 = j_2 i_1 j_1 = \beta_1 j_1$, the square commutes. Also, $((y, t), s) \to j_2 F_2((y, t), s)$ defines a homotopy from

j_2 to h_2.

Choose $n_3 > n_2$ such that $j_2(X_{n_2} \times B_{k_2}) \subset X_{n_3}$. Consider

$$\begin{array}{ccc} X_{n_2} \times B_{k_2} & \xrightarrow{\alpha_2} & X_{n_3} \times B_{k_3} \\ \Big\downarrow h_2 & \nearrow i_2 & \\ j_2(X_{n_2} \times B_{k_2}) & & \end{array}$$

where $k_3 > k_2$ is yet to be chosen, $\alpha_2(x, t) = (x, (t, 0))$ and $i_2(y) = (y, 0)$. Since $j_2 \sim h_2$, we obtain homotopies $i_2 h_2 \sim i_2 j_2 \sim \alpha_2$ taking place in $\operatorname{Int}(X_{n_2} \times B_{k_2})$. By Lemma 4 we may choose k_3 so large that there is an ambient isotopy F_3 on $X_{n_3} \times B_{k_3}$ such that $(F_3)_1 \alpha_2 = i_2 h_2$. Let $j_3 = j_{n_3, k_3}$ and $h_3 = j_3 (F_3)_1$.

Continuing, by induction we obtain for every $r \geqq 1$ a commutative diagram

$$\begin{array}{ccc} X_{n_r} \times B_{k_r} & \xrightarrow{\alpha_r} & X_{n_{r+1}} \times B_{k_{r+1}} \\ \Big\downarrow h_r & & \Big\downarrow h_{r+1} \\ j_r(X_{n_r} \times B_{k_r}) & \xrightarrow{\beta_r} & j_{r+1}(X_{n_{r+1}} \times B_{k_{r+1}}) \end{array}$$

where $\alpha_r(x, t) = (x, (t, 0))$, $\beta_r(y) = y$ and h_r is a homeomorphism. Let $D = \underrightarrow{\lim} \{X_{n_r} \times B_{k_r}; \alpha_r\}$ and $E = \underrightarrow{\lim} \{j_r(X_{n_r} \times B_{k_r}); \beta_r\}$. The h_r's induce a homeomorphism $h: D \to E$. As sets clearly $D \equiv M \times R^\infty$ and $E \equiv M$. Since $j_r(X_{n_r} \times B_{n_r}) \supset X_{n_r}$ and $M = \underrightarrow{\lim} X_n$ it follows immediately that $E \cong M$. Also, $M \times R^\infty$ is homeomorphic to an open subset of R^∞ [7, Corollary II-7] and is therefore the direct limit of its compact subsets. If $C \subset M \times R^\infty$ is compact, then $C \subset \pi_1(C) \times \pi_2(C) \subset X_{n_r} \times B_{k_r}$ some r. (Here $\pi_1: M \times R^\infty \to M$ and $\pi_2: M \times R^\infty \to R^\infty$ are the projections.) It follows that $D \cong M \times R^\infty$. Thus, $M \cong M \times R^\infty$, and Theorem S′ is proved.

Theorem S now follows quickly. Let M be a paracompact, connected R^∞-manifold. As shown in [7, Proposition III. 1] every subset of M is paracompact. Say that a paracompact space Z has property P if for every open subset U of Z there is an open embedding $U \to R^\infty$. Then M has property P locally. Let $X = U \cup V \subset M$ where U and V are open in M having property P. By Theorem S′ $X \cong X \times R^\infty$. By [7, Corollary II.7] X has property P. Let $Y = \bigcup_i Y_i$ where Y_i is open in M and has property P, and where $\{Y_i\}$ is discrete. Since M is Lindelöf [7, Proposition III. 1], $\{Y_i\}$ is at most countable, indexed, say, by a subset of the integers. Let $f_i: Y_i \to R^\infty$ be an open embedding. Let $\rho_i: R^\infty \to [(i - 1/3, i + 1/3) \times R \times R \times \cdots] \cap R^\infty$ be a homeomorphism. Then $f: Y \to R^\infty$ defined by $f/Y_i = \rho_i \circ f_i$ is an open embedding, showing that Y has property P. By a theorem of Michael [12, Theorem 3.6] M has property P. That is, there is an open embedding $M \to R^\infty$. By [6, Theorem 1] the projection $\pi: M \times R^\infty \to M$ is then a near homeomorphism. This completes the proof of Theorem S.

REFERENCES

1. R. D. Anderson and R. Schori, *Factors of infinite-dimensional manifolds*, Trans. Amer. Math. Soc. **142** (1969), 315-330.

2. J. L. Bryant, *Taming polyhedra in the trivial range*, Michigan Math. J., **13** (1966), 377-384.

3. T. A. Chapman, *Lectures on Hilbert cube manifolds*, CBMS Number 28, Amer. Math. Soc., 1976.

4. T. Dobrowolski, *Extensions of Bessaga's negligibility technique to certain infinite-dimensional groups*, Bull. Acad. Polon., Ser. Sci. Math. Astronom. Phys., **26** (1978).

5. J. Dugundji, *Topology*, Allyn and Bacon, Boston, 1966.

6. R. E. Heisey, *Factoring open subsets of R^∞ with control*, to appear in Topology Proceedings.

7. ———, *Manifolds modelled on R^∞ or bounded weak-* topologies*, Trans. Amer. Math. Soc., **206** (1975), 295-312.

8. ———, *Stability, classification, open embeddings, and triangulation of R^∞-manifolds*, submitted to the Proceedings of the Conference on Geometric Topology, Warsaw, 1978.

9. D. W. Henderson, *A simplicial complex whose product with any ANR is a simplicial complex*, General Topology and Appl., **3** (1973), 81-83.

10. S. T. Hu, *Theory of Retracts*, Wayne State Univ. Press, Detroit, 1965.

11. J. F. P. Hudson, *Concordance, isotopy, and diffeotopy*, Annals of Math., 2nd Series, **91** (1970), 425-448.

12. E. A. Michael, *Local properties of topological spaces*, Duke Math. J., **21** (1954), 163-172.

13. J. Milnor, *On spaces having the homotopy type of a CW-complex*, Trans. Amer. Math. Soc., **90** (1959), 272-280.

14. A. P. Robertson, and W. Robertson, *Topological Vector Spaces*, second edition, Cambridge Univ. Press, 1973.

15. C. P. Rourke and B. J. Sanderson, *Intro. to Piecewise Linear Topology*, Ergeberisse Math. Grenz. Band **69**, Springer-Verlag, N. Y., 1972.

16. Schori, *Topological stability for infinite-dimensional manifolds*, Compositio Math., **23** (1971), 87-100.

Received June 15, 1979 and in revised form February 9, 1980.

VANDERBILT UNIVERSITY
NASHVILLE, TN 37235