

117. Counter Examples to Wallace's Problem¹⁾

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A. D. Wallace proposed in his paper²⁾ the following problem:

If a compact mob³⁾ has a unique left unit, is this also a right unit?

In this short note we shall show counter examples to the above-mentioned problem without proof. We will write elsewhere⁴⁾ in these connections and about related topics with detailed discussion. Example 1 is given by N. Kimura and Example 2 by T. Tamura.

Example 1. Let S be a set of all pairs (x, y) such that $0 \leq x \leq y \leq 1$. Consider S as a topological space with the usual 2-dimensional plane topology, as well as a multiplicative system with multiplication;

$$(x, y)(x', y') = (xx', xy'),$$

where the multiplication in the parentheses at the right hand side will be understood as usual one.

Then S becomes a compact connected Hausdorff semigroup, and $(1, 1)$ is a unique left unit. Moreover S has no right unit.

Example 2. Let A be a compact connected mob with two-sided unit 1 and two-sided zero 0. Such a mob A really exists, for example, the interval of real numbers from 0 to 1 with the usual topology and multiplication. Let us consider a compact connected Hausdorff space B . If A and B are given, we can construct the union S of A and B such that A has only one 0 in common with B by identifying abstractly one element of B with 0 in A . The product xy in S is defined as the following manner:

$$xy = \begin{cases} x \cdot y & \text{for } x, y \in A, \\ 0 & \text{for } x \in B, y \in A, \\ y & \text{for } x \in S, y \in B, \end{cases}$$

where $x \cdot y$ is the product of x and y in A . Next we shall introduce a topology into S . The neighborhood $N(x)$ of x is defined as the following manner:

$$\begin{aligned} \text{if } 0 \neq x \in A, \quad N(x) &= U(x) && \text{where } U(x) \text{ is a neighborhood of } \\ & && x \text{ in } A, \\ \text{if } 0 \neq x \in B, \quad N(x) &= V(x) && \text{where } V(x) \text{ is a neighborhood of } \\ & && x \text{ in } B, \end{aligned}$$

if $x=0$, $N(0)=U(0) \smile V(0)$ union of a neighborhood of 0 in A and a neighborhood of 0 in B .

Then it is proved that S is a compact connected mob and 1 is a left unit but not a right unit.

References

- 1) Already published in Japanese. See Tannaka's review to Wallace's paper,²⁾ Zentralblatt für Mathematik, **52**, 26 (1955).
- 2) A. D. Wallace: Indecomposable semigroups, Mathematical Journal of Okayama University, **3**, 1-3 (1953).
- 3) A mob is a synonym of a Hausdorff semigroup.
- 4) N. Kimura and T. Tamura: Compact connected mob with a unique left unit (to appear).