170. Note on Algebras of Strongly Unbounded Representation Type. II

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1. This paper is a continuation of our previous paper¹⁾ on algebras of strongly unbounded representation type. Let A be an algebra over an algebraically closed field k and $g_A(d)$ be the number of inequivalent indecomposable representations of A of degree d where d is a positive integer. Then if A has indecomposable representations of arbitrary high degrees and $g_A(d) = \infty$ for an infinite number of integers d, A is said to be of strongly unbounded representation type.

In his paper [1], James P. Jans proved four sufficient conditions²⁵ for an algebra to be of strongly unbounded representation type and, in our previous paper [3], we added two conditions to them but now in this paper we shall prove another sufficient condition for an algebra to be of strongly unbounded representation type:

(7) The graph $G(A_0)$ associated with a two sided ideal $A_0 \subset N$ is $\begin{cases}
P_{j_4}, P_{k_5} \& P_{j_4}, P_{k_5}, P_{k_5} \& P_{j_3}, P_{j_3}, P_{k_4} \& P_{j_3}, P_{k_4}, P_{k_4} \& P_{j_2}, P_{k_2} \& P_{j_2}, P_{k_2} \& P_{j_2}, P_{k_3} \& P_{j_2}, P_{k_3} \& P_{j_2}, P_{k_4} \& P_{k_5}, P_{k_5} \& P_{k_5} \& P_{k_5}, P_{k_5} \& P_{$

 $P_{k_2}, P_{k_2} \& P_{j_1}, P_{j_1}, P_{k_1} \& P_{j_1}, P_{k_1} \\$

2. First of all we assume that $N^2=0^{4}$ and A is a basic algebra. In order to prove that this condition is sufficient for an algebra to be of strongly unbounded representation type, by the same way as [3] we construct the matrix function R_{cs} , where $c \in k$ and s is an integer,

$$R_{cs}(a) = \begin{bmatrix} X_T(a) & 0 \\ Y(a) & X_B(a) \end{bmatrix}$$
,

as follows:

Let $X_{T}(a)$ be the direct sum of $I_{2s}^{*}X_{j_{4}}(a)$, $I_{6s}^{*}X_{j_{3}}(a)$, $I_{11s}^{*}X_{j_{2}}(a)$ and $I_{5s}^{*}X_{j_{1}}(a)$ and let $X_{B}(a)$ be the direct sum of $I_{4s}^{*}X_{k_{5}}(a)$, $I_{9s}^{*}X_{k_{4}}(a)$, $I_{5s}^{*}X_{k_{3}}(a)$, $I_{8s}^{*}X_{k_{2}}(a)$ and $I_{2s}^{*}X_{k_{1}}(a)$ where $X_{j_{p}}(a)$ and $X_{k_{q}}(a)$ are obtained by the same way as [1] or [3].

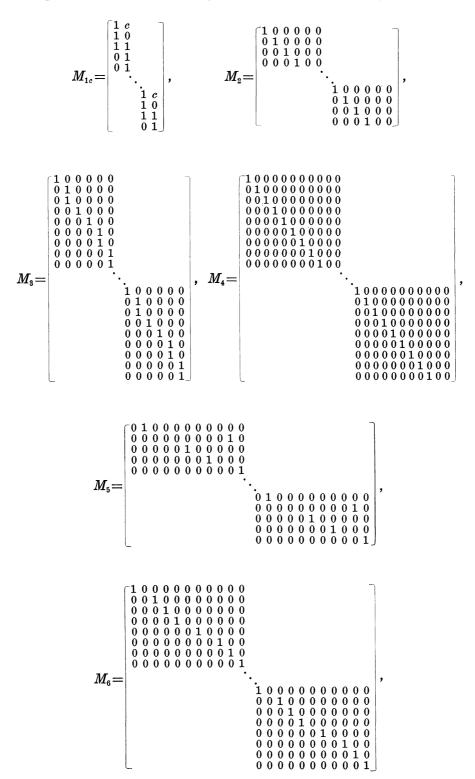
Next we put

¹⁾ T. Yoshii [3].

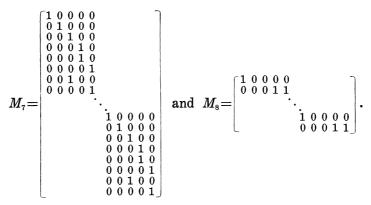
²⁾ James P. Jans [1] or T. Yoshii [3].

³⁾ From now on we use same notations as [1] or [3].

⁴⁾ James P. Jans [1] for the case where $N^2 \neq 0$.



745



Then let Y(a) have $M_{1c} * Y_{54}(a)$ directly below $I_{2s} * X_{j_4}(a)$ and directly to the left of $I_{4s} * X_{k_5}(a)$, $M_2 * Y_{53}(a)$ directly below $I_{6s} * X_{j_3}(a)$ and directly to the left of $I_{4s} * X_{k_5}(a)$, $M_3 * Y_{43}(a)$ directly below $I_{6s} * X_{j_3}(a)$ and directly to the left of $I_{9s} * X_{k_4}(a)$, $M_4 * Y_{42}(a)$ directly below $I_{11s} * X_{j_2}(a)$ and directly to the left of $I_{9s} * X_{k_4}(a)$, $M_5 * Y_{32}(a)$ directly below $I_{11s} * X_{j_2}(a)$ and directly to the left of $I_{5s} * X_{k_3}(a)$, $M_6 * Y_{22}(a)$ directly below $I_{11s} * X_{j_2}(a)$ and directly to the left of $I_{8s} * X_{k_2}(a)$, $M_7 * Y_{21}(a)$ directly below $I_{5s} * X_{j_1}(a)$ and directly to the left of $I_{8s} * X_{k_2}(a)$, $M_8 * Y_{11}(a)$ directly below $I_{5s} * X_{j_1}(a)$ and directly to the left of $I_{2s} * X_{k_1}(a)$. Fill out the rest with zeroes.

Then it is shown by the same way as [1] that R_{cs} is a directly indecomposable representation of A.

Next in order to show that A is of strongly unbounded representation type, we prove that R_{cs} and R_{ds} can not be similar for $c \neq d$.

Now suppose that they were similar. Then there would exist a non-singular matrix P intertwining R_{cs} and R_{ds} . Next let P be divided into submatrices,

$$P = \begin{bmatrix} P_{11} \cdots P_{19} \\ \vdots & \vdots \\ P_{91} \cdots P_{99} \end{bmatrix},$$

corresponding to the divisions of R_{cs} . Then it is clear that $P_{12} \cdots P_{19}$, $P_{21}, P_{23} \cdots P_{29}, P_{31}, P_{32}, P_{34} \cdots P_{39}, P_{41} \cdots P_{43}, P_{45} \cdots P_{49}, P_{56} \cdots P_{59}, P_{67}, P_{68}, P_{69}, P_{78}, P_{79}, P_{89}$ are zero.

Moreover we have

$$P_{55} = \begin{pmatrix} a_{11} & a_{15} \cdots & a_{15} \\ a_{22} & \ddots & a_{33} \\ a_{33} & a_{44} & \ddots \\ a_{51} & a_{55} \\ a_{62} & \ddots & \ddots \\ \ddots & \ddots & \ddots \\ \ddots & \ddots & \ddots \end{pmatrix}$$

and it is impossible from $PR_{cs}(a_{54}) = R_{ds}(a_{54})P$ for $a_{54} \in e_{k_5}Ne_{j_4}$ that P

is non-singular.

Thus we have the following

Theorem. If the graph $G(A_0)$ associated with $A_0 \subset N$ is $\begin{cases}
P_{j_4}, P_{k_5} \& P_{j_4}, P_{k_5}, P_{k_5} \& P_{j_3}, P_{j_3}, P_{k_4} \& P_{j_3}, P_{k_4}, P_{k_4} \& P_{j_2}, P_{j_2}, P_{k_2} \& P_{j_2}, P_{k_3} \& P_{j_2}, P_{k_3} \& P_{j_2}, P_{k_3} \& P_{j_2}, P_{k_3} \& P_{j_2}, P_{k_4} \& P_{j_4}, P_{k_5} \& P_{j_5}, P_{k_5} \& P_{j_5} \&$

Prof. R. Brauer and Prof. R. M. Thrall have conjectured that the class of algebras of unbounded representation type is only that of algebras of strongly unbounded representation type,⁵⁾ but now it is easily shown from the results of [2], [3], and this paper that if $N^2=0$ and k is algebraically closed this conjecture is true.

References

- [1] James P. Jans: On the indecomposable representation of algebras, Dissertation, University of Michigan (1954).
- [2] T. Yoshii: On algebras of bounded representation type, Osaka Math. Jour., 8, No. 1 (1956).
- [3] T. Yoshii: Note on algebras of strongly unbounded representation type, Proc. Japan Acad., 32, No. 6 (1956).