90. On AU-property and Countably Compactness

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It is the purpose of this Note to point out that an example due to A. Ramanthan [2] is not countably compact semi-regular space with the AU-property (for the definition, see K. Iséki [1]). Some writers^{*)} have shown that there exist pseudo-compact spaces which are not countably compact. Therefore there exist regular spaces with the AU-property for countable open covering, but not countably compact.

Let a countable space S be defined as follows:

 $S = \{a\} \smile \{b\} \smile \{a_{ij}\} \smile \{b_{ij}\} \smile \{c_i\}.$ $(i, j = 1, 2, \cdots)$

The neighbourhoods of each point of S are defined:

$$egin{aligned} U(a_{ij}) &= \{a_{ij}\}, & U(b_{ij}) &= \{b_{ij}\}, \ U_n(c_i) &= \{c_i\} \smile igcup_{j=n}^\infty (\{a_{ij}\} \smile \{b_{ij}\}), \ U_n(a) &= \{a\} \smile igcup_{j=1}^\infty igcup_{i=n}^\infty a_{ij}, \ U_n(b) &= \{b\} \smile igcup_{j=1}^\infty igcup_{i=n}^\infty b_{ij}, \end{aligned}$$

where $n=1, 2, \cdots$. It is easily seen that every countable open covering has the *AU*-property, and $\bigcup_{i=1}^{\infty} c_i$ is a countable set without cluster point. Therefore, S is the required properties.

References

- K. Iséki: A remark on countably compact normal space, Proc. Japan Acad., 33, 131-133 (1957).
- [2] A. Ramanthan: Maximal Hausdorff spaces, Proc. Indian Acad. Sci., 26, 31-42 (1947).

^{*)} The examples of such spaces have been given by E. Hewitt, J. Novák, S. Mrówka, V. Ptak, W. T. van Est and H. Freudenthal.