105. On the Completion of Algebraic Systems that Statisfy the Conditions T_0 and T_3

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(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 12, 1963)

Continuing the study of the auther's paper [3], we shall give in this paper a generalization of Prof. Nakayama's theorem of uniform algebraic system.

Our generalization is naturally got from the theorems of the paper [3] and the followings.

PROPOSITION. *A completion* $(X^*; \mathfrak{B}^*)$ *of a* T_2 *bow space is a* T_2 *space if and only if for any leg* $\mathfrak{f}$ *in* X *and any point* $x \in X$, *there exist some body* V *of* $\mathfrak{f}$, *and some neighborhood* W *of* x, *such that* $V \frown W = \phi$. (This proposition is an ameliorated one of the proposition mentioned at the end of [3] and can be proved easily.)

A T_3 *algebraic system* G is a T_3 space G which is an algebraic system such that for any composition π defined in G, the function $f(a, b) = a\pi b$ $(a, b \in G)$ is a continuous one from $G \times G$ into G, and that any mappings defined on it as algebraic system, are also continuous. We consider a T_3 algebraic system G, specially, with a bow $\mathfrak{B}$, such that $(G; \mathfrak{B}, \Lambda)$ is a bow space, and let's call it a *bow algebraic system*. Further, if a bow algebraic system is complete as a bow space, then the bow algebraic system is said to be *complete*.

A *completion* $(G^*; \mathfrak{B}^*)$ of a bow algebraic system $(G; \mathfrak{B})$ is a bow algebraic system such that;

(G_1) the T_3 bow space $(G^*; \mathfrak{B}^*)$ is the completion of T_3 bow space $(G; \mathfrak{B})$,

(G_2) $(G; \mathfrak{B})$ is sub-algebraic system of $(G^*; \mathfrak{B}^*)$.

LEMMA. *Assume that a* T_2 *space* $(X; \mathfrak{B})$ *has its completion* $(X^*; \mathfrak{B}^*)$ *which itself is* T_2 *space. Then* $(X^*; \mathfrak{B}^*)$ *is a* T_3 *space, if and only if for any minimal Cauchy filter* $\mathfrak{f}$ *in* X *and for arbitrary* $W \in \mathfrak{f}$, *there exists* $V \in \mathfrak{f}$ *such that for every Cauchy filter of in* X *if* $V \in \mathfrak{g}$ *then* W *belongs to the minimal Cauchy filter contained in* $\mathfrak{g}$.

THEOREM 9. *There exists a completion of bow algebraic system* $(G; \mathfrak{B})$ *if and only if;*

(E_1) $(G; \mathfrak{B})$ *satisfies the conditions* 1, 2, *of auther's paper* [3],

(E_2) *for every Cauchy filter* $\mathfrak{f}$, *in* $(G; \mathfrak{B})$ *and for every composition* π *defined in* $(G; \mathfrak{B})$, *the filter* $\mathfrak{f}\pi\mathfrak{g}$ *is a Cauchy filter,*

(E_3) $(G; \mathfrak{B})$ *satisfies the conditions of lemma and proposition,*

(E_4) *for any mapping* f *defined on it as algebraic system and for any leg* $\mathfrak{f}$ *in* G, $\{f(A) \mid A \in f\}$ *generates a Cauchy filter in* G.

Provided that $\mathfrak{f}\pi\mathfrak{g}$ is the filter generated by $\{A\pi B \mid A\in\mathfrak{f}, B\in\mathfrak{g}\}$, for every filters $\mathfrak{f}$, $\mathfrak{g}$ and for every composition π.

THEOREM 10. *A completion of a bow algebraic system is uniquely determined.*

References

[1] N. Bourbaki: Topology générale, Paris (1940).
[2] S. Mitani: On the completion of topological spaces, Comm. Math. s't Pauli (1961).
[3] ——: A note on the completion theory, Proc. Japan Acad., **39**(5), 270 (1963).
[4] T. Nakayama: Set, Topology and Algebraic System (in Japanese), Tokyo (1943).