78. *Axiom Systems of Aristotle Traditional Logic.* IV

By Shôtarô TANAKA

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In this paper, we shall concern with the independence of axiom systems of Aristotle traditional logic. In my paper [2], new axiom systems of the Aristotle traditional logic are given as follows:

1 Aaa,

2 Iaa,

3 any one of AAA_1, AOO_2, OAO_3,

4 any one of AEE_4, EIO_4, IAI_4.

For the notations, see K. Iséki [1].

We shall prove that the above axiom systems are independent.

Let 1, 2, and 3 be values for terms. Let t and f be values for propositions, assuming the following:

$$Ctt=t, \quad Ctf=f, \quad Cft=t, \quad Cff=t,$$
$$Ktt=t, \quad Ktf=f, \quad Kft=f, \quad Kff=f,$$
$$Nt=f, \quad Nf=t,$$

where C, K, and N mean implication, conjunction and negation respectively.

First, we shall prove that *an axiom system* $\langle Aaa, Iaa, AAA_1,$ *any one of* $AEE_4, EIO_4, IAI_4\rangle$ *is independent.*

Proof. Let $Aij=f$, $Iij=t$, then $Eij=f$, $Oij=NAij=t$, for every i, j $(i, j=1, 2, 3)$. Hence we have

$$Aaa=f, \quad Iaa=t,$$
$$AabAcaAcb=AijAkiAkj=fff=ff=t,$$
$$AabEbcEca=AijEjkEki=fff=ff=t,$$
$$EabIbcOca=EijIjkOki=ftt=ft=t,$$
$$IabAbcIca=IijAjkIki=tft=ft=t,$$

where $XabYbcZca$ means XYZ_4, and X, Y, Z denote categorical sentences. Hence Aaa is independent from other axioms.

Let $Iij=f$, $Aij=t$ $(i=j)$, f $(i\neq j)$, then $Eij=NIij=t$, $Oij=NAij$ $=f$ $(i=j)$, t $(i\neq j)$. Hence we have

$$Iaa=f, \quad Aaa=t$$
$$AabEbcEca=AijEjkEki=Aijtt=Aijt=t,$$
$$EabIbcOca=tfOca=fOca=t,$$
$$IabAbcIca=IijAjkIki=fAjkf=ff=t,$$

and $AabAcaAcb=AijAkiAkj$:

(i) $i=j$; $AiiAkiAki=tAkiAki=AkiAki=t$,

(ii) $i\neq j$; $AijAkiAkj=fAkiAkj=fAkj=t$.

Therefore Iaa is independent from other axioms.

Let $Iij=t$, $Aij=f$ $(i=2, j=3)$, t (otherwise), then $Eij=NIij=f$, $Oij=NAij=t$ $(i=2, j=3)$, f (otherwise). Hence we have

$$Aaa=Aii=t, \quad Iaa=Iii=t,$$
$$EIO_4=ftOca=fOca=t,$$
$$AEE_4=Aff=ff=t,$$
$$IAI_4=tAt=At=t.$$

Put $a=1$, $b=3$, $c=2$ in $AabAcaAcb$, then $A13A21A23=ttf=tf=f$. Hence $AabAcaAcb$ is independent from other axioms.

Let $Aij=t$, $Iij=f$ $(i=2, j=1)$, t (otherwise). Put $a=1$, $b=2$, $c=2$ in $IabAbcIca$, then $I12A22I21=ttf=tf=f$. But values of other axioms are t's. Hence $IabAbcIca$ is independent from other axioms.

Similarly let $Aij=t$, $Iij=t$ $(i=j)$, f $(i\neq j)$, then $Eij=NIij=f$ $(i=j)$, t $(i\neq j)$. Then put $a=1$, $b=2$, $c=1$ in AEE_4, $A12E21E11=f$. For the other sentences, we always have t's as values. Hence AEE_4 is independent from others.

Let $Aij=t$, $Iij=t$ $(i=j)$, f $(i\neq j)$, then $Oij=NAij=f$, $Eij=NIij=f$ $(i=j)$, t $(i\neq j)$. Put $a=2$, $b=1$, $c=1$ in EIO_4, then $E21I11O12=f$. The values of other axioms are t's. Hence EIO_4 is independent from others.

Similarly we can prove that *each axiom system* $<Aaa$, Iaa, *any one of* AOO_2, OAO_3, *any one of* AEE_4, EIO_4, $IAI_4>$ *is independent system.*

Therefore the proof is complete.

References

[1] K. Iséki: Axiom systems of Aristotle traditional logic. I. Proc. Japan Acad., **43**, 125–128 (1967).

[2] S. Tanaka: Axiom systems of Aristotle traditional logic. II. Proc. Japan Acad., **43**, 194–197 (1967).