$$A_- = \frac{1}{2}\begin{pmatrix} -i\xi & -(1+i\xi)e^{-i(x+\chi-\eta)} \\ (1+i\xi)e^{i(x+\chi-\eta)} & 2+i\xi \end{pmatrix}.$$

The fact that (20) has two regular singularities and an irregular singularity of rank one implies that the deformation equation $d\Omega - \Omega^2 = 0$ is integrable in terms of a Painlevé transcendent of the fifth kind. In our case, substituting (22) to

$$x\frac{dA_\pm}{dx} = \left[\begin{pmatrix} 0 & e_{12} \\ e_{21} & \pm ix \end{pmatrix}, A_\pm\right] \tag{23}$$

we obtain (7) and

$$x\frac{d\chi}{dx} = 2\xi \cos^2\left(\frac{\eta}{2}\right) + 2i \sin^2\left(\frac{\eta}{2}\right). \tag{24}$$

Finally (18) yields

$$\begin{aligned} \frac{d}{dx}\log\rho(x) &= i(A_+ - A_-)_{22} + \frac{1}{x}e_{12}e_{21} \\ &= \xi + \frac{1}{2x}(1+\xi^2)(1+\cos\eta) - \frac{1}{x}, \end{aligned} \tag{25}$$

which is the results (3) and (8).

Erratum and comment for XI [9].

p. 8, (37) [9] should be corrected as $\sigma[M] = \tau[T_M]^2$.

Theorem 5 has previously been obtained by Widom [10]. He has also shown [11] that the product of half infinite Toeplitz operator (32) [9] of M with that of M^{-1} differs from 1 by a trace-class operator, and that $\sigma[M]$ coincides with its determinant. The authors wish to thank Prof. H. Widom for calling their attention to his results.

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