81. *A Table of the Dimensions of the Hilbert Modular Type Cusp Forms for the Hurwitz-Maass Extensions**)

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1. Introduction and the table. For a square-free positive number D, let k be a real quadratic number field $\boldsymbol{Q}(\sqrt{D})$, and $\mathfrak{o}$ the ring of integers in k. There is a unique maximal discrete extension of the Hilbert modular group $G = SL_2(\mathfrak{o})$ in $PL_2^{+}(\boldsymbol{R})^2$, which is called the *Hurwitz-Maass extension* G_m of G. G_m acts properly discontinuously on H^2 (H being a complex upper half plane). We consider the spaces $S(D)_m$ of the cusp forms of weight two with respect to G_m in H^2.

For the ordinary Hilbert modular group G and the extended Hilbert modular group $\hat{G}$, we have already given dimension tables in [5], [6]. This note continues the work of [6]: we tabulate the dimensions of $S(D)_m$ for a square-free D, $1 < D < 1000$, with a computer assistance. In the following table, the number D is given by

(1) $$D = i + j \quad (i = \text{row number}, j = \text{column number}).$$

When the mark '$-$' appears after a figure, $\boldsymbol{Q}(\sqrt{D})$ has a unit of negative norm. The mark '$**$' means that D is not square-free.

2. The method of the computation. For a square-free divisor w of the discriminant d_k of k, we denote by G_w the subgroup of G_m arising from the set of matrices $\begin{pmatrix} a, & b \\ c, & d \end{pmatrix}$ such that $a, b, c, d \in (w)^{1/2}$, $ad - bc = w$, where $(w)^{1/2}$ is an ideal whose square equals (w). Then we have

(2) $$G_m = \bigcup_{w \mid d_k} G_w.$$

Let t be a number of distinct primes dividing d_k. When $t = 1$, G_m coincides with G; when $t = 2$ and k has no units of negative norm, G_m coincides with $\hat{G}$. G_m is also the maximal discrete subgroup of $PL_2^{+}(\boldsymbol{R})^2$ containing $\hat{G}$ ([7]). The elliptic fixed points of G_m was investigated by Hausmann [1]. Note that $[G_m : G] = 2^{t-1}$ and G_m has $h^{+}/2^{t-1}$ inequivalent cusps where h^{+} is the narrow class number of k.

By virtue of [1], [3], we get

Theorem. *The dimension of* $S(D)_m$ *is given by*

(3) $\dim S(D)_m = t_0 + t_1 + t_2 - 1.$

Each term can be written as follows.

(4) $t_0 = (1/2^t)\ \zeta_k(-1)$

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(5) $t_1 = a(D)h(-D) + b(D)h(-3D) + c(D) + \sum_{(w,\tilde{w})} e(D, w)h(-w)h(-\tilde{w})$

(6) $t_2 = \begin{cases} 0 & \textit{if } k \textit{ has a unit of negative norm or } D \textit{ has no primes} \equiv 3 \bmod 4 \\ \sum_{\nu=1}^{h^+/2^{t-1}} \delta(\mathfrak{A}_\nu) & \textit{otherwise} \end{cases}$

where ζ_k is the Dedekind zeta function of k, a set of pairs $(w, \tilde{w})$ runs over all square-free divisors of d_k with $\tilde{w}$ a square-free part of d_k/w, except $w = 1, 4, D, d_k$, $h(-d)$ denotes a class number of $\boldsymbol{Q}(\sqrt{-d})$, $\mathfrak{A}_\nu$ runs over a narrow ideal class in the genus of $\mathfrak{o}$, $\delta(\mathfrak{A}_\nu)$ denotes an invariant of cusp singularity defined in [3]. *$a(D)$, $b(D)$, $c(D)$ and $e(D, w)$ are given in the following tables.*

D	$D \equiv 1 \bmod 4$	$D \equiv 3 \bmod 8$ ($D \neq 3$)	$D \equiv 7 \bmod 8$	$D \equiv 2 \bmod 4$	$D = 3$
$2^{t+2}a(D)$	1	22	4	5	7

D	$D \equiv 1,2 \bmod 3$	$D \equiv 3 \bmod 9$ ($D \neq 3$)	$D \equiv 6 \bmod 9$	$D = 3$	D	$D = 5$	$D \neq 5$
$3 \cdot 2^{t+2}b(D)$	4	34	2	17	$5c(D)$	2	0

D	$D \equiv 1 \bmod 8$				$D \equiv 5 \bmod 8$		
w	$w \equiv 1 \bmod 4$	$w \equiv 3 \bmod 8$ ($w \neq 3$)	$w \equiv 7 \bmod 8$	$w = 3$	$w \equiv 1 \bmod 4$	$w \equiv 3 \bmod 4$ ($w \neq 3$)	$w = 3$
$2^{t+1}e(D, w)$	1	16	4	5	1	4	1

$D \equiv 3 \bmod 4$ ($D \neq 3$)				$D \equiv 2 \bmod 4$				$D = 3$
$w \equiv 3 \bmod 8$ ($w \neq 3$)	$w \equiv 7 \bmod 8$	$w \equiv 0 \bmod 2$	$w = 3$	$w \equiv 1 \bmod 4$	$w \equiv 3 \bmod 8$ ($w \neq 3$)	$w \equiv 7 \bmod 8$	$w = 3$	$w = 2$
10	4	1	3	3	10	4	3	2

Remarks. i) t_0. The method of its calculation was given in [5].

ii) t_1. There are elliptic points of order 2, 3, 4, 5, 6, 12. A point of order 4, 5, 6, or 12 appears only when $D \equiv 2, 3 \bmod 4$, $D = 5$, $D \equiv 0 \bmod 3$ ($D \neq 3$), or $D = 3$, respectively. These contributions are expressed by class numbers of imaginary quadratic fields.

iii) t_2. Cusp contributions are given through continued fraction expansions ([3]).

Table

	0	100	200	300	400	500	600	700	800	900
1	**	4−	5	6	24−	10	50−	39−	**	32−
2	0−	8	33−	45	38	110	61	**	226	108
3	0	12	14	24	44	85	**	91	94	62
5	0−	1	6	9	**	22	**	15	14	39

6	0	15−	29	**	49	53	76	196	101	126
7	0	13	**	56	34	**	140	76	102	269
9	**	4−	6	8	28−	23−	13	43−	67−	**
10	1−	7	10	32	41	29	85−	78	**	75
11	1	7	36	45	42	64	70	**	233	227
13	0−	4−	2	18−	9	**	35−	24	26	47
14	1	8	36	53−	**	124	134	50	124	242−
15	0	8	14	**	46	52	36	53	94	62
17	0−	**	5	13−	15	13	41−	15	37	24
18	**	16	31−	28	46	49	68	188	188−	**
19	2	6	20	32	80	56	161	155	**	288
21	0	**	5	10	21−	35−	**	32	46−	45
22	2	15−	16	31	72	**	142	**	106	273−
23	1	8	34	28	**	119	60	91	232	109
26	3−	**	40−	56	44	126	134−	**	128	242
27	**	16	32	28	47	48	39	188	187	**
29	1−	3	9−	9	4	**	16−	**	52−	78−
30	2	12−	18	17	48	56−	**	109−	102	66
31	2	18	10	67	73	**	166	90	109	**
33	0	2	11−	**	29−	14−	25	42−	**	24
34	4	18	**	64	41	66	171−	164	117	288
35	2	**	21	26	24	62	67	**	124	62
37	1−	5−	6	21−	9	19	**	28	**	90−
38	4	10	21	**	44	123−	68	**	226	113
39	2	21	31	33	92	**	**	212	191	144
41	1−	1	13−	7	**	30−	48−	11	**	53−
42	2	18	**	**	54−	100	75	103	199−	124
43	5	8	**	**	81	61	158	153	104	133
45	**	4−	**	6	14−	19	8	36	**	**
46	4	20	24	71−	84	34	93	179−	**	157
47	3	**	23	57	40	128	125	**	**	230
49	**	6−	7	16−	28−	**	32	19	41	33−
51	3	20	41	**	55	54	44	214	110	132
53	2−	**	5	18−	12	20	33−	32	51−	79−
54	**	13	38	37	104	115−	80	113−	111	**
55	3	11	11	36	22	33	88	86	**	145
57	1	6−	12−	3	32−	27−	**	47−	65−	16
58	7−	18	21	70	83−	**	83	168	61	275
59	7	10	26	57	**	71	152	50	263	122
61	2−	3	**	**	21−	13	37−	59−	13	**
62	6	**	44	59−	26	139	140	90	245	121−
63	**	24	35	**	100	110	38	106	195	**
65	1−	2	8−	9−	8	16−	13	**	45−	28−
66	5	26	22	36	109	116	**	215	215	78
67	8	17	22	70	86	**	86	86	**	288

69	0	**	11−	**	12	41−	20	71−	24	25
70	5	14−	**	42−	46	34	88	48	64	161−
71	5	**	46	34	52	147	72	105	133	259
73	2−	7−	4	19−	15	11	57−	38−	**	31
74	9−	14	53−	36	50	76	153	**	135	259
77	1	4	13−	11	**	43−	34−	17	56−	80−
78	4	29	40	**	103	**	80	215−	195	132
79	8	26	**	81	83	69	94	100	120	161
81	**	7−	15−	10	18−	14	29	25	74−	**
82	11−	13	25	73	89	65	90	85	**	300
83	10	14	51	57	25	71	143	**	262	225
85	2−	4−	2	8	12−	**	21−	31−	14	52−
86	10	14	28	71	**	153−	**	110	274	134−
87	4	16	20	**	106	115	83	208	193	69
89	3−	**	**	18−	17	17	27	20	44	30
90	**	16	26−	21	**	63	45	114	115	**
91	7	23	29	40	101	64	188	90	**	322
93	2	9−	12−	13	15−	38−	**	37	25	45
94	11	29	**	87−	53	**	192	189−	128	163
95	5	8	29	35	**	41	76	55	132	132
97	3−	8−	**	18−	15	18	33−	41−	21	65−
98	**	**	57−	63	56	72	149−	52	266	248
99	**	30	26	22	120	123	88	117	114	**

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