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THE LEVI PROBLEM FOR RIEMANN DOMAINS OVER THE BLOW-UP OF $\mathbb{C}^{n+1}$ AT THE ORIGIN

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Abstract

We investigate unbranched Riemann domains $p\colon X \to \tilde{\mathbb{C}}^{n+1}$ over the blow-up of $\mathbb{C}^{n+1}$ at the origin in the case when p is a Stein morphism. We prove that such a domain is Stein if and only if it does not contain an open set $G \subset X$ such that $p|_G$ is injective and $p(G)$ contains a subset of the form $W \setminus A$, where A is the exceptional divisor of $\tilde{\mathbb{C}}^{n+1}$ and W is an open neighborhood of A.

1. Introduction

In 1953 K. Oka [11] gave the solution to the Levi problem for unbranched Riemann domains over $\mathbb{C}^n$ from which follows that an unbranched domain $p\colon X \to \mathbb{C}^n$ is Stein if and only if p is a Stein morphism. As it was shown by Fornaess [6] this result does not remain valid for branched Riemann domains.

Oka's results served as an impulse for a series of research in this area. Through the last few years, various fundamental results concerning the Levi problem were established. In 1960 F. Docquier and H. Grauert [5] proved that if $p\colon Y \to X$ is an unbranched Riemann domain over a Stein manifold X and p is a Stein morphism, then Y is itself Stein. R. Fujita [8] and A. Takeuchi [12] showed that for complex projective spaces there is a similar result as in $\mathbb{C}^n$. T. Ueda [13] investigated the case of Riemann domains over Grassmann manifolds, M. Colţoiu and K. Diederich [1] studied the case of Riemann domains over Stein spaces with isolated singularities. The Levi problem in the blow-up was investigated by M. Colţoiu and C. Joiţa in [2].

In this paper we consider unbranched Riemann domains over the blow-up. We remark that the blow-up of $\mathbb{C}^{n+1}$ in the origin can be regarded as a particular case of a 1-convex manifold. Some important results concerning covering spaces of 1-convex surfaces were established in the recent works [3], [4].

Let us denote the blow-up of $\mathbb{C}^{n+1}$ in the origin by $\tilde{\mathbb{C}}^{n+1}$ and by A the exceptional divisor of $\tilde{\mathbb{C}}^{n+1}$, $A = \mathbb{P}^n$. Let $p\colon X \to \tilde{\mathbb{C}}^{n+1}$ be an unbranched Riemann domain over $\tilde{\mathbb{C}}^{n+1}$.

We shall say that an unbranched Riemann domain $p\colon X \to \tilde{\mathbb{C}}^{n+1}$ *satisfies the condition* (P) if there exist an open set $G \subset X$ and an open neighborhood W of A such

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that $p|_G$ is injective, and $p(G) \supset W \setminus A$.

Our main result is the following.

Theorem 1. *An unbranched Riemann domain* $p\colon X \to \tilde{\mathbb{C}}^{n+1}$, *with* p *Stein morphism, is Stein if and only if it does not satisfy the condition* (P).

2. Preliminaries

An unbranched Riemann domain over $\mathbb{C}^n$ is a pair (Y, p) consisting of a connected Hausdorff space Y together with a locally homeomorphic map $p\colon Y \to \mathbb{C}^n$ (that is, for each point $y \in Y$ and its base point $x := p(y) \in \mathbb{C}^n$ there exist open neighborhoods $U = U(y) \subset Y$ and $V = V(x) \subset \mathbb{C}^n$ such that $p|_U\colon U \to V$ is a homeomorphism). In the following we shall denote the Riemann domain (Y, p) simply by Y. The Riemann domain Y has a unique complex structure such that p is locally biholomorphic.

If we replace in this definition the space $\mathbb{C}^n$ by a complex manifold X, then we get the notion of a Riemann domain over X.

For later use we require the concept of accessible boundary points of a Riemann domain, which was first introduced by H. Grauert and R. Remmert in [9] using the filter theory (Definition 4). We recall here an equivalent definition which was given and studied in [7].

Let us consider the family of all sequences $\{y_k\}_{k=1}^{\infty}$ of points of Y which have the following properties:

i) The sequence $\{y_k\}_{k=1}^{\infty}$ has no cluster point in Y.

ii) The sequence of the images $\{p(y_k)\}_{k=1}^{\infty}$ has a limit $x_0 \in \mathbb{C}^n$.

iii) For every connected open neighborhood $V = V(x_0) \subset \mathbb{C}^n$ there exists a $k_0 \in \mathbb{N}$ such that for any $k, l \geq k_0$ the points y_k and y_l can be joined by a continuous path $\gamma_{k,l}\colon [0, 1] \to Y$, such that $p \circ \gamma_{k,l}([0, 1]) \subset V$, $\gamma_{k,l}(0) = y_k$, $\gamma_{k,l}(1) = y_l$.

Two such sequences $\{y_k\}_{k=1}^{\infty}$ and $\{y'_k\}_{k=1}^{\infty}$ are called equivalent if:

1) $\lim_{k\to\infty} p(y_k) = \lim_{k\to\infty} p(y'_k) = x_0$.

2) For every connected open neighborhood $V = V(x_0)$ there exists a $k_0 \in \mathbb{N}$ such that for any $k, l \geq k_0$ the points y_k and y'_l can be joined by a continuous path $\gamma_{k,l}\colon [0, 1] \to Y$, such that $p \circ \gamma_{k,l}([0, 1]) \subset V$, $\gamma_{k,l}(0) = y_k$, $\gamma_{k,l}(1) = y'_l$.

An accessible boundary point of a Riemann domain $p\colon Y \to \mathbb{C}^n$ is an equivalence class $\sigma_{x_0} = [y_k]$ of such sequences.

Let us denote by $\breve{\partial} Y$ the set of all accessible boundary points of the domain Y and by $\breve{Y} := Y \cup \breve{\partial} Y$.

If $y_0 = \sigma_{x_0}$ is an accessible boundary point, then a neighborhood of y_0 in $\breve{Y}$ is defined as follows:

Take a connected open set $U \subset Y$ such that:

a) U contains almost all points of any sequence $\{y_k\}_{k=1}^{\infty}$ from the equivalence class σ_{x_0}.

b) There exists a connected open neighborhood $V \subset \mathbb{C}^n$ of x_0 such that U is a connected component of $p^{-1}(V)$.

Then add to U all accessible boundary points $z = \sigma_x$ such that almost all points of any sequence from σ_x are contained in U and $x \in \mathbb{C}^n$ is a cluster point of $p(U)$.

We shall denote this neighborhood of $y_0 \in \check{\partial} Y$ by $\check{U}$.

With this neighborhood definition the extended domain $\check{Y}$ becomes a topological space, and $\check{p}\colon \check{Y} \to \mathbb{C}^n$ with

$$\check{p}(y) := \begin{cases} p(y), & \text{if} \quad y \in Y, \\ \lim\limits_{k\to\infty} p(y_k), & \text{if} \quad y = [y_k] \in \check{\partial} Y, \end{cases}$$

is a continuous mapping.

Proposition 1. a) *$\check{Y}$ is a regular topological space.*
b) *For every point $y \in \check{\partial} Y$ there exists a continuous function $\alpha\colon [0,1] \to \check{Y}$ such that $\alpha(1) = y$ and $\alpha(t) \in Y$ for $t \in [0,1)$.*

REMARK 1. Every sequence of points $\{y_k\}_{k=1}^{\infty}$ of Y which satisfies the conditions (ii) and (iii) has a cluster point in $\check{Y}$.

Indeed, if $\{y_k\}_{k=1}^{\infty}$ has a cluster point in Y this statement is trivial. If $\{y_k\}_{k=1}^{\infty}$ has no cluster point in Y, then it defines an equivalence class of such sequences, i.e. an accessible boundary point $y = [y_k] \in \check{\partial} Y$.

The following proposition is Satz 4 in [4].

Proposition 2. *Let T be a locally connected topological space and $S \subset T$ be a nowhere dense subset of T nowhere disconnecting T. Let $p\colon Y \to X$ be a Riemann domain over a complex manifold X and let $\tau\colon T \setminus S \to Y$ be a continuous mapping such that $p \circ \tau$ extends to a continuous mapping on T. Then τ uniquely extends to a continuous mapping $\check{\tau}\colon T \to \check{Y}$.*

DEFINITION 1. A Riemann domain $p\colon Y \to \mathbb{C}^n$ is called pseudoconvex at a boundary point $y \in \check{\partial} Y$, if there exists a neighborhood $\check{U}$ of y such that $\check{U} \cap Y$ is a Stein manifold.

DEFINITION 2. Let $S \subset \mathbb{C}^n$ be an analytic set of positive codimension. A boundary point y of the Riemann domain $p\colon Y \to \mathbb{C}^n$ is called removable along S, if there exists a neighborhood $\check{U}$ of y such that $\check{p}|_{\check{U}}\colon \check{U} \to \mathbb{C}^n$ is injective and $\check{U} \cap \check{\partial} Y$ is contained in $\check{p}^{-1}(S)$.

The next Lemma was proved in [13].

Lemma 1. *Let $S \subset \mathbb{C}^n$ be an analytic set of positive codimension and let $p\colon Y \to \mathbb{C}^n$ be an unbranched Riemann domain over $\mathbb{C}^n$. Assume that Y is pseudoconvex at* *every boundary point $y \in \check{\partial} Y$ with $\check{p}(y) \in \mathbb{C}^n \setminus S$. If there exists no boundary point which is removable along S then Y is Stein.*

Lemma 2. *Let $S \subset \mathbb{C}^n$, $n \geq 2$ be an analytic set that has at least codimension 2, and let $p\colon Y \to \mathbb{C}^n$ be an unbranched Riemann domain over $\mathbb{C}^n \setminus S$. Assume that Y is pseudoconvex at every boundary point y lying over $\mathbb{C}^n \setminus S$. Then Y is not Stein if and only if there exist a connected open subset $U \subset Y$ and a connected open subset $V \subset \mathbb{C}^n$ such that $V \cap S \neq \emptyset$ and $p|_U\colon U \to V \setminus S$ is biholomorphic.*

Proof. Let us consider that Y is not Stein and then, by Lemma 1, there exists a boundary point $y^* \in \check{\partial} Y$ which is removable along S. Let $\check{p}$ be the extension of p to $\check{Y} = Y \cup \check{\partial} Y$. Then there exists an open neighborhood $\check{U}_1$ of y^*, $\check{U}_1 \subset \check{Y}$, such that $\check{p}|_{\check{U}_1}$ is injective and $\check{p}(\check{U}_1 \cap \check{\partial} Y)$ is contained in S. Let $\check{U}$ be another open neighborhood of y^* such that $\overline{\check{U}} \subset \check{U}_1$. There exists such an $\check{U}$ because $\check{Y}$ is regular (see Proposition 1).

Denote by $U = \check{U} \setminus \check{\partial} Y$, and by $x^* = \check{p}(y^*)$, $x^* \in S$. To prove the "only if" statement it suffices to show that there exists an open neighborhood V of x^* such that $V \setminus S \subset p(U)$. Suppose that this is not true. Then for any open neighborhood V of x^* we have that $p(U) \not\supset V \setminus S$. We can choose a sequence of points $\{\xi_k\}_{k=1}^{\infty}$, $\xi_k \in \mathbb{C}^n \setminus (S \cup \check{p}(\check{U}))$, such that it converges to x^*, $\lim_{k \to \infty} \xi_k = x^*$.

Let $\alpha\colon [0, 1] \to \check{U}$ be a continuous path such that $\alpha(1) = y^*$ and $\alpha([0, 1)) \subset U$ (see Proposition 1) and let $\{s_k\}_{k=1}^{\infty}$ be an increasing sequence of positive real numbers, $0 < s_k < 1$, convergent to 1. Denote by $\zeta_k^{(0)} = p(\alpha(s_k))$ and let $\alpha_k\colon [0, 1] \to \mathbb{C}^n$, $k = 1, 2, \ldots$ be a continuous path such that $\alpha_k(0) = \zeta_k^{(0)}$, $\alpha_k(1) = \xi_k$, and $\alpha_k((0, 1]) \subset \mathbb{C}^n \setminus S$. Moreover we may assume that the sequence $\{\alpha_k\}_{k=1}^{\infty}$ converges uniformly to x^* on $[0, 1]$.

We denote by $t_k = \inf\{t \mid t \in [0, 1],\ \alpha_k(t) \in \partial p(U)\}$, and by $x_k = \alpha_k(t_k)$.

Clearly the sequence $\{x_k\}_{k=1}^{\infty}$ also converges to x^*, $x_k \notin S$, and $\alpha_k([0, t_k)) \subset p(U)$, for all k. By Proposition 2 the continuous function $(p|_U)^{-1} \circ \alpha_k\colon [0, t_k) \to Y$ extends to a continuous function $\beta_k\colon [0, t_k] \to \check{Y}$. Let $y_k = \beta_k(t_k)$. Then $p(y_k) = x_k$ and, at the same time, using the path α and the uniform convergence of $\{\alpha_k\}_{k=1}^{\infty}$ to x^* it is easy to see that $\{y_k\}_{k=1}^{\infty}$ satisfies properties ii) and iii). By Remark 1 $\{y_k\}_{k=1}^{\infty}$ has a cluster point $\check{y} \in \check{Y}$. Note that $y_k \in \overline{U} \setminus U$ and, therefore, $\check{y} \in \overline{\check{U}} \setminus \check{U}$. In particular $\check{y} \neq y^*$. At the same time $\check{p}(\check{y}) = x^* = \check{p}(y^*)$ which contradicts the injectivity of $\check{p}$ on $\check{U}_1 \supset \overline{\check{U}}$.

The "if" statement follows easily from Riemann extension theorem. □

3. Proof of Theorem 1

Proof. Let $z_0, z_1, \ldots, z_n$ be the coordinate functions in $\mathbb{C}^{n+1}$, and let denote by $[\xi_0 : \xi_1 : \cdots : \xi_n]$ the homogeneous coordinates in the complex projective space $\mathbb{P}^n$. The blow-up of $\mathbb{C}^{n+1}$ at the origin is the manifold

$$\tilde{\mathbb{C}}^{n+1} := \{(z, \xi) \in \mathbb{C}^{n+1} \times \mathbb{P}^n\colon z_i \xi_j = z_j \xi_i,\ i, j = \overline{0, n}\}.$$

We shall cover $\mathbb{P}^n$ with the sets $U_i = \{\xi \in \mathbb{P}^n : \xi_i \neq 0\}$, $i = 0, 1, \ldots n$. Let us denote by π the projection on the second factor

$$\pi := \mathrm{pr}_2 |_{\tilde{\mathbb{C}}^{n+1}} : \tilde{\mathbb{C}}^{n+1} \to \mathbb{P}^n.$$

Then $\pi^{-1}(\xi) = l(\xi)$ is the complex line determined by ξ. So the blow-up looks like a line bundle over the projective space.

We have the following local trivializations $\psi_i : \pi^{-1}(U_i) \to U_i \times \mathbb{C}$ defined by $\psi_i(z, \xi) := (\xi, z_i)$, $i = 0, 1, \ldots, n$. The mapping ψ_i is biholomorphic and its inverse is

$$\psi_i^{-1}([z], \lambda) = \left(\frac{\lambda}{z_i} \cdot z, [z]\right),$$

where $[z] = [z_0 : z_1 : \cdots : z_n] \in U_i$. Hence, over $U_{ij} = U_i \cap U_j$ we have

$$\psi_i \circ \psi_j^{-1}([z], \lambda) = \psi_i\left(\frac{\lambda}{z_j} \cdot z, [z]\right) = \left([z], \lambda \cdot \frac{z_i}{z_j}\right).$$

Over the blow-up $\tilde{\mathbb{C}}^{n+1}$ we can construct a local trivial fibration with fiber $\mathbb{C}^*$, $F : (\mathbb{C}^{n+1} \setminus \{0\}) \times \mathbb{C} \to \tilde{\mathbb{C}}^{n+1}$.

In [2] was constructed such a fibration F and namely F: $(\mathbb{C}^{n+1} \setminus \{0\}) \times \mathbb{C} \to \mathcal{O}(r)$, where

$$F(z, \lambda) = \psi_k^{-1}\left([z], \frac{\lambda}{z_k^r}\right),$$

$\forall (z, \lambda) \in W_k = \{(z, \lambda) \in (\mathbb{C}^{n+1} \setminus \{0\}) \times \mathbb{C} : z_k \neq 0\}$.

Since one can identify $\tilde{\mathbb{C}}^{n+1}$ with $\mathcal{O}(-1)$, the holomorphic line bundle of degree -1 over $\mathbb{P}^n$, we have $r = -1$ and then for any $(z, \lambda) \in W_k$ we get

$$F(z, \lambda) = \psi_k^{-1}([z], \lambda z_k) = \left(\frac{\lambda z_k}{z_k} \cdot z, [z]\right) = (\lambda \cdot z, [z]).$$

Hence the mapping F can be defined globally by $F(z, \lambda) = (\lambda \cdot z, [z])$.

Then, for every point $(z, [z]) \in \tilde{\mathbb{C}}^{n+1}$ we have

$$F^{-1}(z, [z]) = \left\{\left(\frac{z}{\lambda}, \lambda\right) \;\middle|\; \lambda \in \mathbb{C}^*\right\}.$$

Let us denote by Δ the complex line $\Delta = \{0\} \times \mathbb{C} \subset \mathbb{C}^{n+2} \quad (\{0\} \in \mathbb{C}^{n+1})$.

We construct the fiber product Y of the fibration F and the Riemann domain X, namely

$$Y = \{(w, x) \in (\mathbb{C}^{n+2} \setminus \Delta) \times X \mid F(w) = p(x)\}.$$

We have the following commutative diagram

$$\begin{tikzcd} & Y \arrow[r, "\tilde{F}"] \arrow[d, "\tilde{p}"] & X \arrow[d, "p"] \\ \mathbb{C}^{n+2} \supset & (\mathbb{C}^{n+2} \setminus \Delta) \arrow[r, "F"'] & \tilde{\mathbb{C}}^{n+1}. \end{tikzcd}$$

The mapping $\tilde{F} = \mathrm{pr}_2|_Y \colon Y \to X$, the canonical projection on the second factor, defines a holomorphic principal fibration of fiber $\mathbb{C}^*$.

The mapping $\tilde{p} = \mathrm{pr}_1|_Y \colon Y \to \mathbb{C}^{n+2} \setminus \Delta$, the canonical projection on the first factor, defines an unbranched Riemann domain over $\mathbb{C}^{n+2} \setminus \Delta$.

Since $p \colon X \to \tilde{\mathbb{C}}^{n+1}$ is a Stein morphism, the mapping $\tilde{p} \colon Y \to \mathbb{C}^{n+2} \setminus \Delta$ is also a Stein morphism. As $(\mathbb{C}^{n+2} \setminus \Delta) \subset \mathbb{C}^{n+2}$, consequently we get a Riemann domain $\tilde{p} \colon Y \to \mathbb{C}^{n+2}$ over $\mathbb{C}^{n+2}$. Observe that $\mathbb{C}^{n+2}$ is a Stein variety and $\tilde{p} \colon Y \to (\mathbb{C}^{n+2} \setminus \Delta)$ is a Stein morphism, but it is not known if $\tilde{p} \colon Y \to \mathbb{C}^{n+2}$ is also a Stein morphism since $\mathbb{C}^{n+2}$ contains points from Δ, that is points of the boundary of $(\mathbb{C}^{n+2} \setminus \Delta)$.

By Théorèmes 4 and 5 in [10] of Matsushima and Marimoto, Y is Stein if and only if X is Stein.

Let us suppose that the fiber product Y is not Stein. Then there exists a boundary point $y \in \breve{\partial} Y$ which is removable along Δ.

Then, by Lemma 2, there exist an open neighborhood $\breve{U}$ of y and an open polydisc V_ε of polyradius $\varepsilon > 0$ centered in $x^* = \tilde{p}(y) = (0, \dots, 0, \nu) \in \Delta$ such that $\tilde{p}|_U \colon U \to V_\varepsilon \setminus \Delta$ is biholomorphic, where $U = \breve{U} \setminus \breve{\partial} Y$.

Let us denote by $G = \tilde{F}(U) \setminus p^{-1}(A)$, where A is the exceptional divisor of $\tilde{\mathbb{C}}^{n+1}$. We claim that $p|_G$ is injective.

Let us admit the contrary.

Then there exists an $x \in G$ such that $G \cap p^{-1}(p(x))$ has at least two elements. Let $G \cap p^{-1}(p(x)) = \{x_1, x_2, \dots\}$. Thus

1) $x_i \neq x_j$, $i \neq j$; $i, j = 1, 2, \dots$,
2) $p(x_i) = Q \in \tilde{\mathbb{C}}^{n+1} \setminus A$, for all $i = 1, 2, \dots$.

Let $Q = (q, [q])$, $q = (q_0, q_1, \dots, q_n)$. The preimage of this point is $F^{-1}(Q) = \{(q/\lambda, \lambda) \mid \lambda \in \mathbb{C}^*\}$. Observe that $F^{-1}(Q)$ does not intersect $\Delta = \{0\} \times \mathbb{C}$, and the intersection of $F^{-1}(Q)$ with $V_\varepsilon \setminus \Delta$ is given by $\{|q_j/\lambda| < \varepsilon,\ j = 0, \dots, n,\ \lambda \in \mathbb{C}^*\} \cap \{|\lambda - \nu| < \varepsilon,\ \lambda \in \mathbb{C}^*\}$ and so is open and connected. Let us denote this set by V^*.

Let $D_i = \tilde{F}^{-1}(x_i) \cap (\tilde{p}|_U)^{-1}(V^*)$, $i = 1, 2, \dots$. The sets D_i are open in $\tilde{F}^{-1}(x_i)$, non-empty, and $D_i \subset U$ for all $i = 1, 2, \dots$. Thus $\tilde{p}|_{D_i}$, $i = 1, 2, \dots$ are homeomorphisms and therefore $\tilde{p}(D_i)$ are open in $F^{-1}(Q)$, non-empty and disjoint and

$$V^* = \bigcup_i \tilde{p}(D_i).$$

But this is not possible since V^* is connected.

So $p|_G$ is injective. In addition $F^{-1}(p(G))$ contains a set of the form $V_\varepsilon \setminus \Delta$ and then, by the argument in the proof of Theorem 1 from [2], $p(G)$ contains a set of the form $W \setminus A$, where A is the exceptional set of the blow-up and W is a neighborhood of A. □

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