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Reduced Group C*-algebras with The Metric Approximation Property by N-positive Maps

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1. Introduction

Choi and Effros [4] and Kirschberg [10] have proved that the nuclearity for a C*-algebra is equivalent to "the complete positive approximation property". Not all C*-algebras have the approximation property. In fact, A.Szankowski [14] has proved, that the algebra of all bounded operators B(H) on an infinite dimensional Hilbert space H, does not have the approximation property. It had been believed that every C*-algebra with the metric approximation property is nuclear. Surprisingly, in 1979, Uffe Haagerup [7] showed an example of a non-nuclear C*-algebra, which has the metric approximation property. Haagerup's example is the reduced group C*-algebra $C_r^*(F_2)$ of the free group F_2 on two generators. Following after [7], M.A.Picardello[12] showed among others that the reduced C^*-algebra $C_r^*(G)$ of amalgamated products $G = \ast_H G_i$, where $\{G_i; i\varepsilon I\}$ is any collection of finite groups and H is a common subgroup, has the metric approximation property, independently of [5]. In this note, modifying the technique of Canniere and Haagerup[2], we shall show that the reduced group C*-algebras generated by the free product of finite groups with one amalgamated subgroup have the metric approximation property by n-positive maps. This improves the result(Theorem 3 of Picardello[12]) in the case of two factors. Positive maps on C*-algebras are very important objects and are investigated by many authors [8],[9],[15]. The contents of this note are the following. In Section 2, we shall give some notations. In Section 3, we shall consider groups acting on trees, in particular we shall study length functions on the free product of finite groups with one amalgamated subgroup. In Section 4, we shall prove that the group C*-algebra $C_r^*(G)$ has the metric approximation property by n-positive maps, if G is a free product of finite groups with one amalgamated subgroup. This is an improvement of our previous result in [5],which is announced in [6].

2. Preliminaries

A Banach space E is said to have the metric approximation property, abbreviated as M.A.P., if there exists a net $(T_\alpha)_\alpha$ of finite rank operators on E such that $\|T_\alpha\| \leq 1$ for all α and $\lim_\alpha \|T_\alpha x - x\| = 0$ for all x in E. Let G be a countable discrete group and let λ be the left regular representation of G. The reduced group C*-algebra $C_r^*(G)$ is the C*-algebra generated by $\{\lambda(g); g \in G\}$. Following after I.M.Chiswell[3], a length function on a group G is a mapping $\Psi: G \longrightarrow \mathbb{R}$ satisfying two axioms proposed by Lyndon [11]:

A2. $\Psi(x) = \Psi(x^{-1})$ for all x in G.

A4. $\tilde{d}(x,y) > \tilde{d}(x,z)$ implies that $\tilde{d}(x,z) = \tilde{d}(y,z)$ for all x,y and z in G, where $\tilde{d}(x,y) = (\Psi(x)+\Psi(y)-\Psi(xy^{-1}))/2$.

Throughout this paper we shall assume that our length functions are integer valued, and <u>normalized</u>, that is, they satisfy the axiom

A1'. $\Psi(1) = 0$.

By Lyndon [11], the following axiom C0 is also considered:

C0. $\tilde{d}(x,y)$ is always an integer.

By a <u>graph</u> X we understand a pair of disjoint sets, V(X), E(X) with V(X) non-empty, together with a mapping $E(X) \longrightarrow V(X) \times V(X)$, $y \longrightarrow (r(y), s(y))$, and a mapping $E(X) \longrightarrow E(X)$, $y \longrightarrow \bar{y}$ satisfying $y \neq \bar{y}$, $y = \bar{\bar{y}}$ and $s(\bar{y}) = r(y)$ for all y in E(X). The elements of V(X) are called <u>vertices</u>, and those of E(X) are called <u>edges</u>. We call a pair of edges $\{ y, \bar{y} \}$ an <u>unoriented</u> edge of X, and an <u>orientation</u> of X is a set consisting of exactly one member of each unoriented edge $\{ y, \bar{y} \}$. For definitions of path, tree, etc. see [13]. For

$P,Q \in V(X)$ we denote the distance from P to Q by $d(P,Q)$. A more general example of a length function satisfying CO is given by I.M.Chiswell [3] as follows: Let G be a group acting on a tree X, and let P_0 be a vertex of X. For g in G, define $\Psi(g)= d(P_0,gP_0)$ to be the distance from P_0 to gP_0. Then $g \longrightarrow \Psi(g)$ is a length function on G satisfying CO. We give some examples about length functions.

<u>Example 1</u>. Let F_n be the free group of n generators and $|g|$ the length of the unique reduced words representing g in F_n in these generators. Then the function $g \longrightarrow |g|$ is a length function.

<u>Example 2</u>. Let $G = A \underset{C}{*} B$ be the free product of two groups A and B with one amalgamated subgroup C(cf.[13]). Then there is a tree X on which G acts as follows: Put

$V(X) = G/A \cup G/B$ (disjoint union),
$E(X) = G/C \cup \overline{(G/C)}$ (disjoint union), $s: G/C \longrightarrow G/A$ and
$r: G/C \longrightarrow G/B$ being induced by the inclusions
$C \longrightarrow A$ and $C \longrightarrow B$.

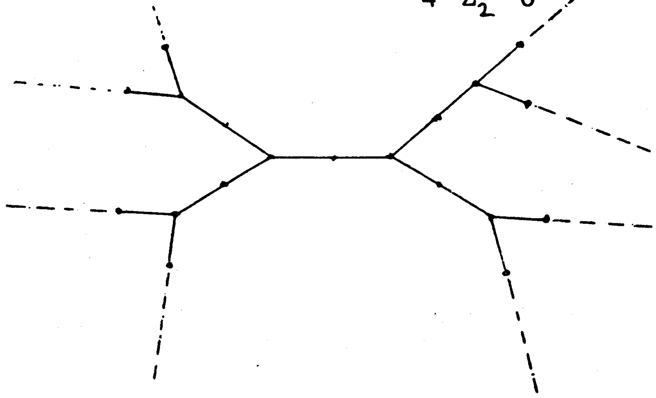
An action of G on the tree X is given by
$g(xA) = (gx)A \in V(X)$, $g(xB) = (gx)B \in V(X)$
and $g(xC) = (gx)C \in E(X)$ for all g,x in G.
Put $P_0 = A \in V(X)$. For g in G, define $\Psi(g) = d(P_0,gP_0)$ to be the distance from P_0 to gP_0. Then Ψ is a length function on

G such that $\Psi(g)$ is even integer for all $g \in G$. Note that edges of X consists of

$$xA \quad \{xC, \overline{xC}\} \quad xB \qquad x \in G$$

If $d(P_0,Q)$ is even (resp. odd) for $Q \in V(X)$, then $Q = gA = gP_0$ (resp. $Q = gB$) for some $g \in G$. The following is the tree for $G = SL(2,Z) = Z_4 \underset{Z_2}{*} Z_6$ defined above:

The tree of $Z_4 \underset{Z_2}{*} Z_6$



3. Length functions

The following lemma 1 is proved by Haagerup [7].

Lemma 1(Haagerup). Let ϕ be a positive definite function on a discrete group G. Then there is a unique, completely positive map $M_\phi : C_r^*(G) \longrightarrow C_r^*(G)$ such that $M_\phi \lambda(s) = \phi(s)\lambda(s)$, and the norm of this map is $\|M_\phi\| = \phi(1)$.

A function Ψ on a group G is negative definite if

$$\Sigma_{i,j=1}^{n}(\Psi(g_i) + \overline{\Psi(g_j)} - \Psi(g_i g_j^{-1}))z_i\overline{z_j} \geq 0$$

for all $g_1,\ldots,g_n \in G$, $z_1,\ldots,z_n \in \mathbb{C}$, or equivalently,

$\Psi(1) \geq 0$, $\Psi(g^{-1}) = \overline{\Psi(g)}$ and $\Sigma_{i,j=1}^{n}\Psi(g_i g_j^{-1})z_i\overline{z_j} \leq 0$ for all $g_1,\ldots,g_n \in G$, $z_1,\ldots,z_n \in \mathbb{C}$ with $\Sigma_{i=1}^{n} z_i = 0$, (cf.[1]).

U.Haagerupp [7] proved that the natural length function $g \longrightarrow |g|$ on the free group is negative definite. This is generalized in [16] as follows:

Lemma 2. <u>Let</u> Ψ <u>be an integer valued length function on a group</u> G <u>satisfying</u> CO. <u>Then</u> Ψ <u>is a negative definite function on</u> G. <u>Moreover, for</u> $\lambda > 0$, <u>the function</u> $s \longrightarrow e^{-\lambda\Psi(s)}$ <u>is a positive definite function on</u> G.

In the following, we assume that a countable group G acts on a tree X. For a vertex P_0 of X, define $\Psi(s) = d(P_0, sP_0)$ for s in G. For s in G and integer $k,\ell \geq 0$, put

$Y(s;k,\ell) = \{(t,u) \in G \times G \; ; \; s = tu, \; \Psi(t) = k$ and $\Psi(u) = \ell\}$

$Z(s;k,\ell) = \{(t,u) \in G \times G \; ; \; s = t^{-1}u, \; \Psi(t) = k$ and $\Psi(u) = \ell\}$.

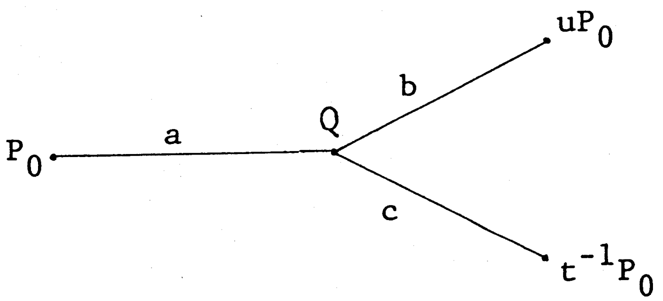
We shall denote the cardinality of a set S by $^{\#}S$ and the set $\{Q \in V(X); d(P,Q) = p\}$ by $S_p(P)$.

Lemma 3. <u>The following statements hold.</u>

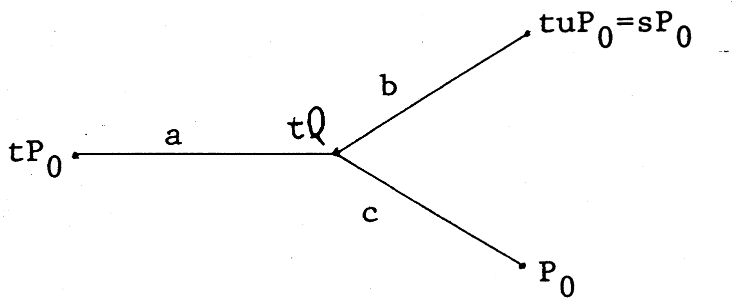
(1) ${}^{\#}Z(s;k,\ell) = {}^{\#}Y(s;k,\ell)$.

(2) <u>If</u> $\Psi(s) = k+\ell-2p$ <u>for</u> $p=0,1,\ldots$, <u>then</u>
${}^{\#}Y(s;k,\ell) \leqq \sup_{Q_1 \in S_p(P_0)} {}^{\#}\{g \in G \; ; \; gQ_1=Q_2$ <u>for some</u> $Q_2 \in S_p(P_0)\}$.

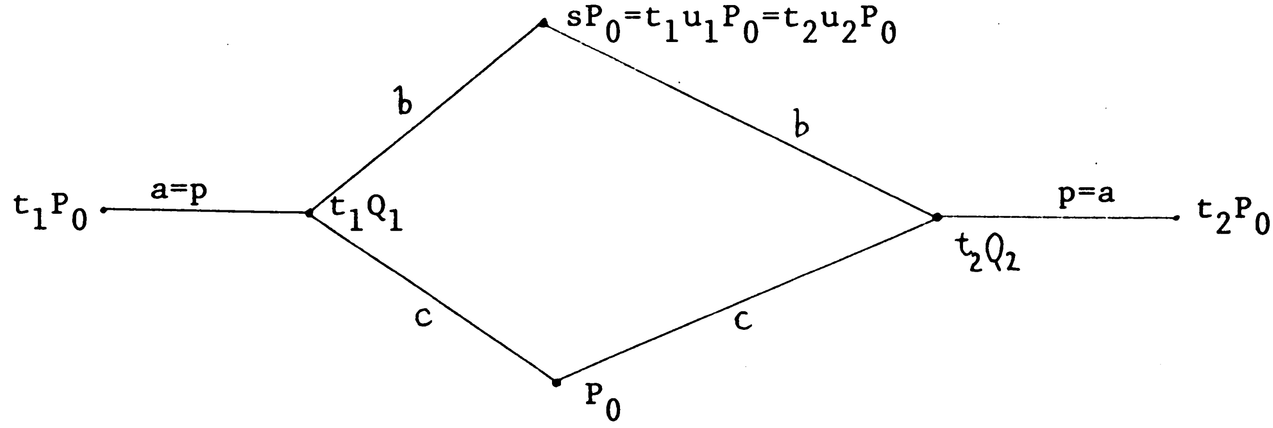
<u>Proof</u>. (1) Since $\Psi(t^{-1}) = \Psi(t)$, ${}^{\#}Y(s;k,\ell) = {}^{\#}Z(s;k,\ell)$.
(2) If $s = tu$ for $s,t,u \in G$, then we have the following diagram(cf.[3]), where the lines represent geodesics, and a,b,c represent lengths:



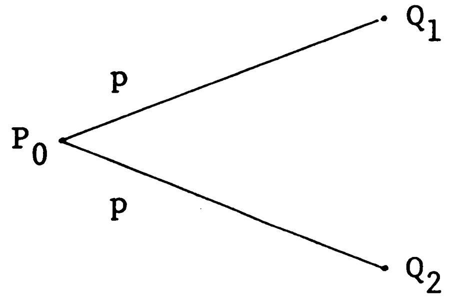
and Q is the point at which the two paths diverge. Translating the diagram by t, we obtain



If $\Psi(t) = k$, $\Psi(u) = \ell$, then $k = a+c$, $\ell = a+b$, $\Psi(s) = b+c$.
Thus $2p = k+\ell-\Psi(s) = (a+c)+(a+b)-(b+c) = 2a$, so that $p = a$.
Suppose that (t_1,u_1) and (t_2,u_2) are in $Y(s;k,\ell)$. Then we have



Since the geodesic from P_0 to sP_0 in the tree X is unique, we have that $t_1Q_1 = t_2Q_2$, so that $t_2^{-1}t_1Q_1 = Q_2$. Remember that



Put $g = t_2^{-1}t_1$. Then $t_1 = t_2g$ and $u_1 = t_1^{-1}s = g^{-1}t_2^{-1}s = g^{-1}u_2$. Since $gQ_1 = Q_2$, $d(P_0,Q_1) = p$ and $d(P_0,Q_2) = p$, we have that

$$\begin{aligned}&\#Y(s;k,\ell)\\ &\leq \sup_{Q_1\in S_p(P_0)}\#\{g \in G;\ gQ_1=Q_2 \text{ for some } Q_2 \in S_p(P_0)\}.\end{aligned}$$ Q.E.D.

In the following we assume that $G = A \underset{C}{*} B$ is the free product of two groups A and B with one amalgamated subgroup C and

X is the associated tree for $G = A \underset{C}{*} B$ as in example 2.

Put $P_0 = A \in V(X)$. Then $\Psi(g) = d(P_0, gP_0)$, $g \in G$, is the length function in example 2.

Lemma 4. <u>If</u> $G = A \underset{C}{*} B$ <u>is the amalgamated free product. Then for</u> $s \in G$, <u>integers</u> $k, \ell \geq 0$ <u>with</u> $\Psi(s) = k+\ell$ <u>or</u> $\Psi(s) = k+\ell-2$,

$$\#Y(s;k,\ell) \leq \#A\#B.$$

<u>Proof</u>. If $\Psi(s) = k+\ell$, then $\#Y(s;k,\ell) = \#\{g \in G;\ gP_0 = P_0\}$ $= \#\{g \in G; gA = A\} = \#A \leq \#A\#B$, by Lemma 3. Let $\Psi(s) = k+\ell-2$. Take $Q_1, Q_2 \in V(X)$ such that $d(P_0,Q_1) = d(P_0,Q_2) = 1$. Then $Q_1 = a_1B$ and $Q_2 = a_2B$ for some $a_1, a_2 \in A$. Moreover we have that

$$\{ g \in G;\ gQ_1 = Q_2 \text{ for some } Q_2 \in S_1(P_0)\}$$

$$= \{g \in G; ga_1B = a_2B \quad \text{for some} \quad a_2 \in A\}$$

$$= \{g \in G;\ a_2^{-1}ga_1 = b \quad \text{for some } a_2 \in A \quad \text{and some } b \in B\}$$

$$= \{g \in G;\ g = a_2ba_1^{-1} \text{ for some } a_2 \in A \quad \text{and some } b \in B\}.$$

Therefore, by Lemma 3, we have that

$$\#Y(s;k,\ell)$$
$$\leq \sup_{Q_1 \in S_1(P_0)} \#\{g \in G;\ gQ_1 = Q_2 \text{ for some } Q_2 \in S_1(P_0)\}.$$

$= \sup_{a_1 \varepsilon A} {}^{\#}\{g \ \varepsilon\ G; g = a_2 b a_1{}^{-1}$ for some $a_2 \ \varepsilon\ A$ and some $b \ \varepsilon\ B\}$

$\leq {}^{\#}A^{\#}B.$ Q.E.D.

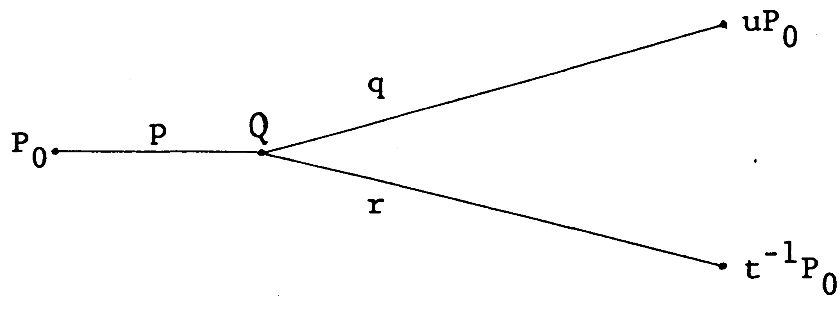
Next we shall show a lemma about a decomposition of an element in G.

Lemma 5. Let $G = A \underset{C}{*} B$ be the amalgamated free product. Suppose that $s = tu$ and $\Psi(s) = \Psi(t) + \Psi(u) - 2p$ for $s, t, u \ \varepsilon\ G$ and an integer $p \geqq 0$.

(1) If p is even, then there exist $t', u', v \ \varepsilon\ G$ such that $t = t'v$, $u = v^{-1}u'$, $\Psi(t') = \Psi(t) - p$, $\Psi(u') = \Psi(u) - p$ and $\Psi(v) = p$.

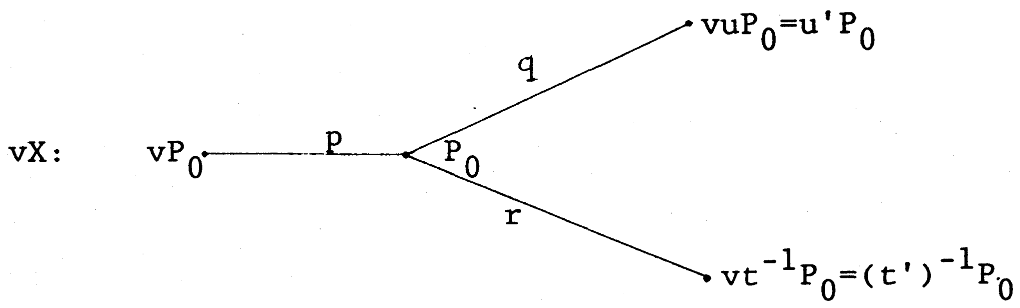
(2) If p is odd, then there exist $t', u', v \ \varepsilon\ G$ such that $t = t'v$, $u = v^{-1}u'$, $\Psi(t') = \Psi(t) - p - 1$, $\Psi(u') = \Psi(u) - p + 1$ and $\Psi(v) = p + 1$.

Proof. We have the following diagram:



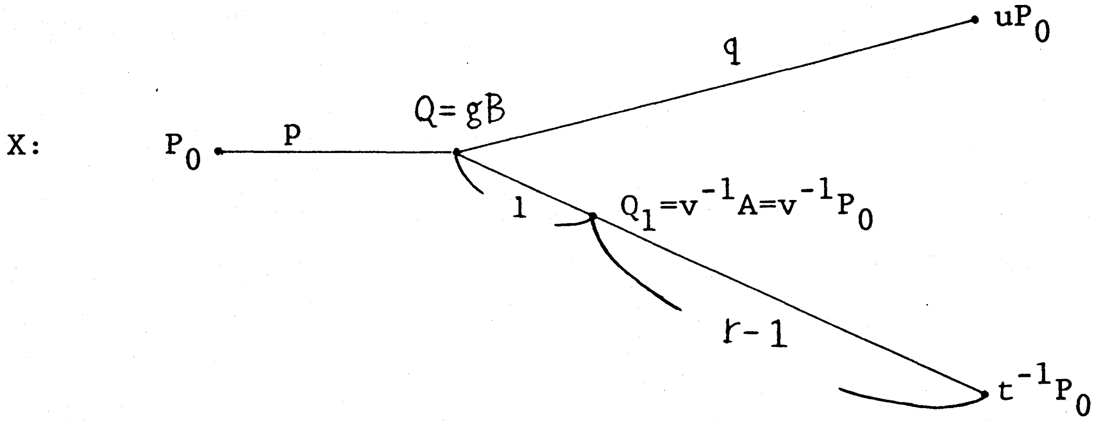
where $P_0 = A \ \varepsilon\ V(X)$, $\Psi(u) = p + q$ and $\Psi(t) = \Psi(t^{-1}) = p + r$.

(1) If p is even, then $Q = v^{-1}A \in V(X)$ for some $v \in G$. Since $P_0 = A$, $Q = v^{-1}A = v^{-1}P_0$, so that $\Psi(v) = d(P_0,vP_0) = d(P_0,v^{-1}P_0) = p$. Put $t' = tv^{-1}$ and $u' = vu$. Translating the tree X by v, we have the following diagram:



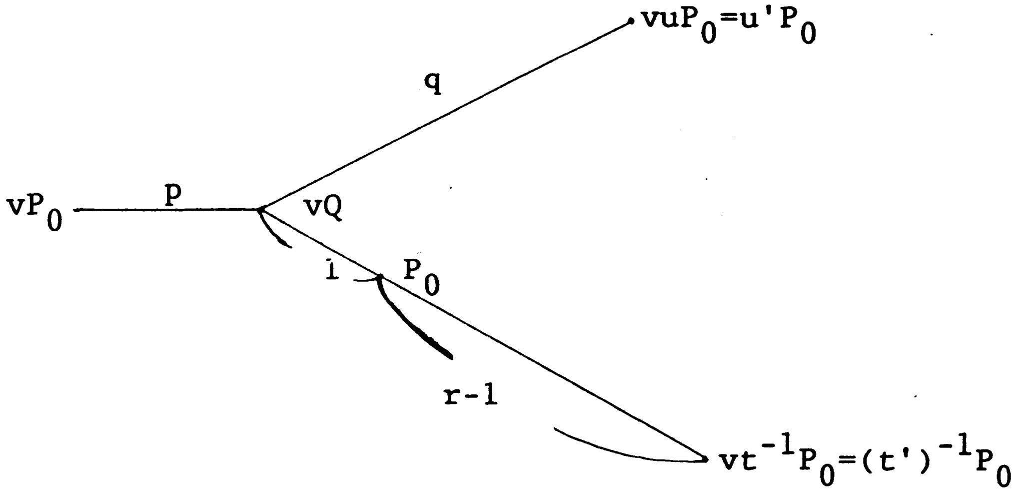
Then $\Psi(t') = d(P_0,t'P_0) = d(P_0,(t')^{-1}P_0) = r = \Psi(t)-p$, and $\Psi(u') = d(P_0,u'P_0) = q = \Psi(u)-p$. Obviously we have that $t = t'v$ and $u = v^{-1}u'$.

(2) If p is odd, then $Q = gB \in V(X)$ for some $g \in G$. Let Q_1 be the vertex in the path from Q to $t^{-1}P_0$ such that $d(Q,Q_1)=1$: Then there exists $v \in G$ such that $Q_1=v^{-1}P_0$. Thus we have the following diagram:



Put $t' = tv^{-1}$ and $v' = vu$. Translating the tree X by v, we have the following diagram vX:

vX:



Then $\Psi(t') = r-1 = \Psi(t)-p-1$, $\Psi(u') = q+1 = \Psi(u)-p+1$ and $\Psi(v) = p+1$. Obviously we have $t = t'v$ and $u = v^{-1}u'$. Q.E.D.

Put $E_n = \{s \in G;\ \Psi(s) = n\}$ and χ_n the characteristic function for E_n. In the following we assume that A and B are finite groups.

Lemma 6. Let $G=A \underset{C}{*} B$ be the free product of finite groups A and B with one amalgamated subgroup C. Then we have the following:

(1) $^{\#}E_n$ is finite for any non-negative integer n. (2) If G is infinite, then for any positive integer k, there exists a positive integer $N \geq k$ such that E_N is not empty.

Proof. (1) If there exist $g_1, g_2 \in G$ such that $g_1 P_0 = g_2 P_0 = Q$ for some $Q \in V(X)$, then $g_2^{-1} g_1 P_0 = P_0$, so that $g_2^{-1} g_1 \in \{g \in G;\ gP_0 = P_0\}$. Therefore $\#\{g \in G;\ gP_0 = Q\} \leqq \#\{g \in G;\ gP_0 = P_0\} = \#A$. If n is odd, then E_n is empty. If n is even, then $n = 2m$ for some non-negative integer m. We have that

$$\#E_n = \#\{g \in G;\ \Psi(g) = n\}$$

$$= \Sigma_{Q \in S_n(P_0)} \#\{g \in G;\ gP_0 = Q\}$$

$$\leqq \Sigma_{Q \in S_n(P_0)} \#A$$

$$= \#A \#S_n(P_0)$$

$$\leqq \#A\{(\#(A/C))^{m-1}(\#(B/C))^{m} + (\#(A/C))^{m}(\#(B/C))^{m})\} < +\infty$$

(2) Suppose that there exists a non-negative integer k such that E_N is empty for all integer $N \geqq k$. Then $G = \bigcup_{i=0}^{k} E_i$. Since $\#E_n$ is finite for any non-negative integer n by (1), G is finite. This contradicts that G is infinite. Q.E.D.

4. Metric approximation property

Proofs of the following lemmas are similar to the ones given by U.Haagerup[7].

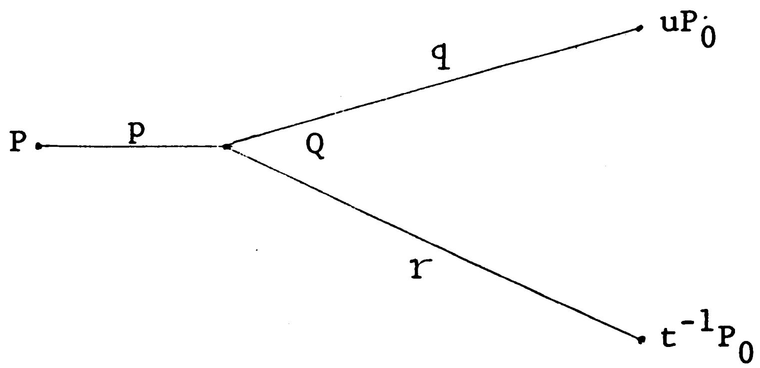
Lemma 7. Let G be a free product of finite groups A and B with one amalgamated subgroup C. Let k, ℓ and m be non-negative integers and f, g be two functions on G with supports in E_k and E_ℓ respectively. Then

$$\|(f * g)\chi_m\|_2 \leqq (\#A\#B)^{3/2}\|f\|_2\|g\|_2$$

, if $|k-\ell| \leqq m \leqq k+\ell$ and $k+\ell-m$ even and $\|(f*g)\chi_m\|_2 = 0$ if not.

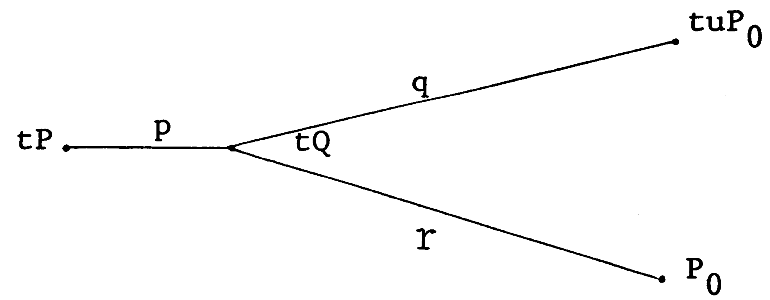
Proof. Take $t, u \in G$ such that $\Psi(t) = k$ and $\Psi(u) = \ell$. Then we have the following diagram.

X:



It follows that $k = \Psi(t) = p+r$ and $\ell = \Psi(u) = p+q$. Translating X by t,

tX:



Then $\Psi(t \cdot u) = q+r = (\ell-p)+(k-p) = \ell+k-2p$, where $0 \leqq p \leqq \min(k, \ell)$.

Therefore $\Psi(tu)$ takes one of the following numbers,
$|k-\ell|$, $|k-\ell|+2$, ... , $k+\ell-2$, $k+\ell$.
On the other hand, we have

$(f*g)(s) = \Sigma_{(t,u)\varepsilon Y(s;k,\ell)} f(t)g(u)$,because $\mathrm{supp}(f) \subseteqq E_k$ and $\mathrm{supp}(g) \subseteqq E_\ell$.

Hence $\|(f*g)\chi_m\|_2 = 0$,when m is not one of these numbers. In the following, according to the property of p, we consider three cases separately. The first case where p is zero, the second one where p is positive even and the third one where p is odd.

Case(1) $p = 0$, that is, $m = k+\ell$.

Then

$$\|(f*g)\chi_m\|_2^2 = \Sigma_{s\varepsilon G}|(f*g)\chi_m(s)|^2$$

$$= \Sigma_{s\varepsilon E_m}|\Sigma_{(t,u)\varepsilon Y(s;k,\ell)} f(t)g(u)|^2$$

$$\leqq \Sigma_{s\varepsilon E_m}|\Sigma_{(t,u)\varepsilon Y(s;k,\ell)} |f(t)||g(u)||^2 ,$$

$$\leqq \Sigma_{s\varepsilon E_m}(\Sigma_{(t,u)\varepsilon Y(s;k,\ell)} |f(t)|^2|g(u)|^2)(\Sigma_{(t,u)\varepsilon Y(s;k,\ell)} 1^2)$$

(by the Schwartz inequality)

$$= \Sigma_{s\varepsilon E_m}(\Sigma_{(t,u)\varepsilon Y(s;k,\ell)} |f(t)|^2|g(u)|^2)\#Y(s;k,\ell)$$

$$\leqq \#A\#B\Sigma_{s\varepsilon E_m}\Sigma_{(t,u)\varepsilon Y(s;k,\ell)} |f(t)|^2|g(u)|^2$$

Since we have that $Y(s;k,\ell) \cap Y(s';k,\ell) = \phi$ for $s \neq s'$,

$$= \#A\#B\Sigma_{(t,u)}|f(t)|^2|g(u)|^2$$

(where $(t,u)\varepsilon G\times G$ such that $\Psi(tu)=m=k+\ell, \Psi(t)=k, \Psi(u)=\ell$)

$$\leq \#A\#B(\Sigma_{t\varepsilon E_k}|f(t)|^2)(\Sigma_{u\varepsilon E_\ell}|g(u)|^2)$$

$$= \#A\#B\|f\|_2^2\|g\|_2^2 .$$

Hence we have

$$\|(f*g)\chi_m\|_2^2 \leq \#A\#B\|f\|_2^2\|g\|_2^2$$

Case (2) p is positive even. Put p=2m. We define functions f' and g' with supports in E_{k-p} and $E_{\ell-p}$ respectively by

$f'(t')=(\Sigma_v|f(t'v)|^2)^{1/2}$ (where $v \varepsilon E_p$ with $t'v \varepsilon E_k$)
, if $\Psi(t')=k-p$

and $f'(t')=0$ otherwise.

$g'(u')=(\Sigma_v|g(v^{-1}u')|^2)^{1/2}$ (where $v \varepsilon E_p$ with $v^{-1}u' \varepsilon E_\ell$)
, if $\Psi(u')=\ell-p$

and $g'(u')=0$ otherwise.

Then we have

$$\|f'\|_2^2 = \Sigma_{t'\varepsilon E_{k-p}} |f'(t')|^2$$

$$= \Sigma_{t'\varepsilon E_{k-p}} (\Sigma_v |f(t'v)|^2) \quad \text{(where } v \ \varepsilon \ E_p \text{ with } t'v \ \varepsilon \ E_k\text{)}$$

$$= \Sigma_{t\varepsilon E_k} \Sigma_{(t',v)\varepsilon Y(t,k-p,p)} |f(t)|^2$$

$$= \Sigma_{t\varepsilon E_k} \#Y(t;k-p,p)|f(t)|^2$$

$$\leq \#A\#B\Sigma_{t\varepsilon E_k} |f(t)|^2$$

$$= \#A\#B\|f\|_2^2.$$

Hence we obtain $\|f'\|_2^2 \leq \#A\#B\|f\|_2^2$.

Similarly we have the following.

$$\|g'\|_2^2 = \Sigma_{u'\varepsilon E_{\ell-p}} |g'(u')|^2$$

$$= \Sigma_{u'\varepsilon E_{\ell-p}} \Sigma_v |g(v^{-1}u')|^2 \quad \text{(where } v \ \varepsilon \ E_p \text{ with } v^{-1}u' \ \varepsilon \ E_\ell\text{)}$$

$$= \Sigma_{u\varepsilon E_\ell} \Sigma_{(v,u')\varepsilon Z(u;p,\ell-p)} |g(u)|^2$$

$$= \Sigma_{u\varepsilon E_\ell} \#Z(u;p,\ell-p)|g(u)|^2$$

$$\leq \#A\#B\Sigma_{u\varepsilon E_\ell} |g(u)|^2 = \#A\#B\|g\|_2^2.$$

Hence we obtain $\|g'\|_2^2 \leq \#A\#B\|g\|_2^2$.

For $s \in G$ such that $\Psi(s) = m = k+\ell-2p$, where p is positive even,

$$|(f*g)(s)| = |\Sigma_{(t,u)\in Y(s;k,\ell)} f(t)g(u)|$$

using lemma 5(1),

$$\leq \Sigma_{(t,u)\in Y(s;k,\ell)} \Sigma_{(t',v,u')} |f(t'v)||g(v^{-1}u')|$$
(where $(t',v,u') \in G\times G\times G$ such that $t=t'v, u=v^{-1}u', \Psi(v)=p$, $\Psi(t')=k-p, \Psi(u')=\ell-p$)

$$= \Sigma_{(t',v,u')} |f(t'v)||g(v^{-1}u')| ,$$
(where $(t',v,u') \in G\times G\times G$ such that $\Psi(v)=p$, $\Psi(t')=k-p$, $\Psi(u')=\ell-p, \Psi(t'v)=k, \Psi(v^{-1}u')=\ell, s=t'u'$)

using the Schwartz inequality with respect to v,

$$\leq \Sigma_{(t',u')\in Y(s;k-p,\ell-p)} (\Sigma_v |f(t'v|^2)^{1/2} (\Sigma_v |g(v^{-1}u')|^2)^{1/2} ,$$
(where $v \in G$ such that $\Psi(v)=p, \Psi(t'v)=k, ,\Psi(v^{-1}u')=\ell$)

$$\leq \Sigma_{(t',u')\in Y(s;k-p,\ell-p)} f'(t')g'(u') = (f'*g')(s)$$

therefore $\quad |(f*g)(s)| \leq (f'*g')(s)$.

Since k, ℓ and m are fixed, p is always even. Therefore

$|f*g|\chi_m \leq (f'*g')\chi_m$. Since $(k-p)+(\ell-p)=m$, it follows from the case (1) that $\|(f'*g')\chi_m\|_2^2 \leq {}^{\#}A\,{}^{\#}B\,\|f'\|_2^2\,\|g'\|_2^2$.

Hence

$$\|(f*g)\chi_m\|_2^2 \leqq \|(f'*g')\chi_m\|_2^2 \leqq \#A\#B\|f'\|_2^2\|g'\|_2^2$$
$$\leqq \#A\#B\#A\#B\|f\|_2^2\#A\#B\|g\|_2^2 = (\#A\#B)^3\|f\|_2^2\|g\|_2^2.$$

At last, we consider the case (3) where p is odd. We define functions f' and g' with supports in E_{k-p-1} and $E_{\ell-p+1}$ represented by

$$f'(t') = (\Sigma_v |f(t'v)|^2)^{1/2} \text{ (where } v \in E_{p+1} \text{ with } t'v \in E_k),$$
$$,\text{if } \Psi(t') = k-p-1$$

and $f'(t') = 0$ otherwise.

$$g'(u') = (\Sigma_v|g(v^{-1}u')|^2)^{1/2} \text{ (where } v \in E_{p+1} \text{ with } v^{-1}u' \in E_\ell)$$
$$,\text{if } \Psi(u') = \ell-p+1,$$

and $g'(u') = 0$ otherwise.

Then we have

$$\|f'\|_2^2 = \Sigma_{t'\in E_{k-p-1}} |f'(t')|^2 ,$$

$$= \Sigma_{t'\in E_{k-p-1}} (\Sigma_v|f(t'v)|^2), \text{(where } v \in E_{p+1} \text{ with } t'v \in E_k)$$

$$= \Sigma_{t\in E_k}\Sigma_{(t',v)\in Y(t;k-p-1,p+1)}|f(t)|^2$$

$= \Sigma_{t\varepsilon E_K} \#Y(t;k-p-1,p+1)|f(t)|^2$

$\leqq \#A\#B\Sigma_{t\varepsilon E_K}|f(t)|^2$, by lemma 4,

$= \#A\#B\|f\|_2^2$,

Hence, we obtain $\|f'\|_2^2 \leqq \#A\#B\|f\|_2^2$.

Similarly

$\|g'\|_2^2 = \Sigma_{u'\varepsilon E_{\ell-p+1}} |g'(u')|^2$

$= \Sigma_{u'\varepsilon E_{\ell-p+1}} (\Sigma_v|g(v^{-1}u')|^2)$ (where $v \varepsilon E_{p+1}$ with $v^{-1}u' \varepsilon E_\ell$)

$= \Sigma_{u\varepsilon E_\ell} \Sigma_{(v,u')\varepsilon Z(u;p+1,\ell-p+1)} |g(v^{-1}u')|^2$

$= \Sigma_{u\varepsilon E_\ell} \#Z(u;p+1,\ell-p+1)|g(u)|^2$

$\leqq \#A\#B\Sigma_{u\varepsilon E_\ell}|g(u)|^2$

$= \#A\#B\|g\|_2^2$.

Hence we obtain $\|g'\|_2^2 \leqq \#A\#B\|g\|_2^2$.

For $s \varepsilon G$ such that $\Psi(s) = m = k+\ell-2p$,where p is odd,

$|(f{\star}g)(s)| = |\Sigma_{(t,u)\varepsilon Y(s;k,\ell)} f(t)g(u)|$

using lemma 5(2),

$\leqq \Sigma_{(t,u)\varepsilon Y(s;k,\ell)} \Sigma_{(t',v,u')} |f(t'v)|\cdot|g(v^{-1}u')|$

(where $(t',v,u') \in G\times G\times G$ such that $t = t'v$, $u = v^{-1}u'$,

$\Psi(v) = p+1$, $\Psi(t') = k-p-1, \Psi(u') = \ell-p+1$),

$= \Sigma_{(t',v,u')} |f(t'v)|\cdot|g(v^{-1}u')|$,

(where $(t',v,u') \varepsilon G\times G\times G$ such that $\Psi(v)=p+1, \Psi(t')=k-p-1$,

$\Psi(u')=\ell-p+1, \Psi(t'v)=k, \Psi(v^{-1}u')=\ell, s=t'u'$),

Using the Schwartz inequality with respect to v,

$\leqq \Sigma_{(t',u')\varepsilon Y(s;k-p-1,\ell-p+1)} (\Sigma_v |f(t'v)|^2)^{1/2} (\Sigma_v |g(v^{-1}u')|^2)^{1/2}$,

(where $v \in G$ such that $\Psi(v) = p+1$, $\Psi(t'v) = k, \Psi(v^{-1}u') = \ell$),

$\leqq \Sigma(t',u')\varepsilon Y(s;k-p-1,\ell-p+1) (\Sigma_v |f(t'v)|^2)^{1/2} (\Sigma_v |g(v^{-1}u')|^2)^{1/2}$,

(where the former $v \in G$ such that $\Psi(v) = p+1, \Psi(t'v) = k$, and the latter $v \in G$ such that $\Psi(v) = p+1$, $\Psi(v^{-1}u') = \ell$),

$= \Sigma_{(t',u')\varepsilon Y(s;k-p-1,\ell-p+1)} f'(t')g'(u') = (f' * g')(s)$.

Since k,ℓ,m are fixed ,p is always odd.

Therefore $|f*g|\chi_m \leqq (f'* g')\chi_m$.

Since $(k-p-1)+(\ell-p+1) = m$, it follows from the case (1) that

$$\|(f'* g')\chi_m\|_2^2 \leqq {}^{\#}A{}^{\#}B \|f'\|_2^2 \|g'\|_2^2.$$

Hence $\|(f*g)\chi_m\|_2^2 \leqq \|(f'*g')\chi_m\|_2^2 \leqq {}^{\#}A^{\#}B\|f'\|_2^2\|g'\|_2^2$

$$\leqq {}^{\#}A^{\#}B^{\#}A^{\#}B\|f\|_2^{2\#}A^{\#}B\|g\|_2^2 = ({}^{\#}A^{\#}B)^3\|f\|_2^2\|g\|_2^2.$$

So, for all of the possible cases about p, we have

$$\|(f*g)\chi_m\|_2^2 \leqq ({}^{\#}A^{\#}B)^3\|f\|_2^2\|g\|_2^2. \qquad \text{Q.E.D.}$$

For a function $f \in \ell^1(G)$ we put as usual $\lambda(f) = \Sigma_{s\varepsilon G} f(s)\lambda(s)$.

Lemma 8. Let f be a function on G with a finite support. Then

$$\|\lambda(f)\| \leqq ({}^{\#}A^{\#}B)^{3/2}\Sigma_{n=0}^{\infty}(n+1)\|f\chi_n\|_2.$$

Proof. Since $f = \Sigma_{n=0}^{\infty} f\chi_n$, it is enough to show that when f has support in E_n, then $\|\lambda(f)\| \leqq ({}^{\#}A^{\#}B)^{3/2}(n+1)\|f\|_2$. Thus let f be a function with support in E_n and let g be an arbitrary ℓ^2-function on G. Put $g_k = g\cdot\chi_k$. Then g_k has support in E_k and $\|g\|_2^2 = \Sigma_{k=0}^{\infty}\|g_k\|_2^2$. Put $h = f*g = \Sigma_{k=0}^{\infty} f*g_k$ and $h_m = h\chi_m$. Note that $h \in \ell^2(G)$ and $\|h\|_2^2 = \Sigma_{m=0}^{\infty}\|h_m\|_2^2$. From lemma 7 we get

$\|(f*g_k)\chi_m\|_2 \leqq ({}^{\#}A^{\#}B)^{3/2}\|f\|_2\|g_k\|_2$ if $|k-n| \leqq m \leqq k+n$ and $k+n-m$ even and $\|(f*g_k)\chi_m\|_2 = 0$ if not.

Therefore,

$$\|h_m\|_2 = \|\Sigma_{k=0}^{\infty}(f*g_k)\chi_m\|_2 \leqq \Sigma_{k=0}^{\infty}\|(f*g_k)\chi_m\|_2$$

$$\leqq (\#A\#B)^{3/2}\|f\|_2\Sigma_{k=|m-n|,m+n-k\text{ even}}^{m+n}\|g_k\|_2.$$

Writing $k = m+n-2\ell$, we get

$$\|h_m\|_2 \leqq (\#A\#B)^{3/2}\|f\|_2\Sigma_{\ell=0}^{\min(m,n)}\|g_{m+n-2\ell}\|_2$$

$$\leqq (\#A\#B)^{3/2}\|f\|_2(\Sigma_{\ell=0}^{\min(m,n)}\|g_{m+n-2\ell}\|_2^2)^{1/2}(\Sigma_{\ell=0}^{\min(m,n)}1)^{1/2}$$

$$\leqq (\#A\#B)^{3/2}(n+1)^{1/2}\|f\|_2(\Sigma_{\ell=0}^{\min(m,n)}\|g_{m+n-2\ell}\|_2^2)^{1/2}$$

Hence

$$\|h\|_2^2 = \Sigma_{m=0}^{\infty}\|h_m\|_2^2$$

$$\leqq (\#A\#B)^3(n+1)\|f\|_2^2\Sigma_{m=0}^{\infty}(\Sigma_{\ell=0}^{\min(m,n)}\|g_{m+n-2\ell}\|_2^2)$$

$$= (\#A\#B)^3(n+1)\|f\|_2^2\Sigma_{\ell=0}^{n}(\Sigma_{m=\ell}^{\infty}\|g_{m+n-2\ell}\|_2^2)$$

$$= (\#A\#B)^3(n+1)\|f\|_2^2\Sigma_{\ell=0}^{\infty}(\Sigma_{k=n-\ell}^{\infty}\|g_k\|_2^2)$$

$$\leqq (\#A\#B)^3(n+1)\|f\|_2^2\Sigma_{\ell=0}^{n}\|g\|_2^2 = (\#A\#B)^3(n+1)^2\|f\|_2^2\|g\|_2^2.$$

This proves that

$$\|f*g\|_2 = \|h\|_2 \leqq (\#A\#B)^{3/2}(n+1)\|f\|_2\|g\|_2 \quad \text{for any } g \in \ell^2(G),$$

i.e. $\|\lambda(f)\| \leqq (\#A\#B)^{3/2}(n+1)\|f\|_2$. Q.E.D.

Lemma 9. Let f be a function on G with finite support. Then

$\|\lambda(f)\| \leqq 2(\#A\#B)^{3/2}(\Sigma_{s\in G}|f(s)|^2(1+\Psi(s))^4)^{1/2}$.

Proof. By lemma 8 and the Cauchy Schwartz inequality, we get that

$$\|\lambda(f)\| \leqq (\#A\#B)^{3/2}\Sigma_{n=0}^{\infty}(n+1)\|f\chi_n\|_2$$

$$= (\#A\#B)^{3/2}\Sigma_{n=0}^{\infty}\{1/(n+1)\}\{(n+1)^2\|f\chi_n\|_2\}$$

$$\leqq (\#A\#B)^{3/2}(\Sigma_{n=0}^{\infty}1/(n+1)^2)^{1/2}\{\Sigma_{n=0}^{\infty}(n+1)^4\|f\chi_n\|_2^2\}^{1/2}$$

$$= (\#A\#B)^{3/2}\sqrt{(\pi^2/6)}\{\Sigma_{s\in G}|f(s)|^2(1+\Psi(s))^4)^{1/2}.$$

Since $\sqrt{(\pi^2/6)} \doteqdot 2$, the assertion follows. Q.E.D.

Let G be a discrete group. We let A(G) denote the Fourier algebra of G (cf.[7]), $A(G) = \{f*\tilde{g};\ f,g \in \ell^2(G)\}$, where $\tilde{g}(s) = \overline{g(s^{-1})}$, $s \in G$. The set A(G) is a Banach algebra with respect to pointwise multiplication and the norm

$$\|\phi\|_{A(G)} = \inf\{\|f\|_2\|g\|_2; \phi = f*\tilde{g}\ ,\ f,g \in \ell^2(G)\}.$$

A complex valued function ϕ on G is called a multiplier of the Fourier algebra A(G) if $\phi\psi \in A(G)$ for all $\psi \in A(G)$. For every multiplier ϕ of A(G) the map $m_\phi : \psi \longrightarrow \phi\psi$, $\psi \in A(G)$. The space of all multipliers of $A(G)$ is denoted by $MA(G)$. Cleary $MA(G)$ is a Banach space with the norm $\|\phi\|_{MA(G)} = \|m_\phi\|$. Recall that a linear map T from a C*-algebra A to a C*-algebra B is called positive if $T(A_+) \subseteq B_+$ and n-positive for some $n \in \mathbb{N}$, if $T \otimes i_n$ is positive. A multiplier ϕ of $A(G)$ is n-positive if the transposition of the map $m_\phi : \psi \longrightarrow \phi\psi$, $\psi \in A(G)$ is an n-positive operator on the von Neumann algebra $R(G) = A(G)^*$.

We need the following proposition 10 due to Canniere and Haagerup[2].

Proposition 10([2]). _Let_ ϕ _be a continuous function on a locally compact group_ G _and let_ $n \in \mathbb{N}$. _The following two conditions are equivalent._

(1) ϕ _is a_ n-_positive multiplier of_ $A(G)$.

(2) _any_ $f_1,\ldots,f_n,g_1,\ldots,g_n \in K(G)$;

$\int_G \phi(s)\Sigma_{i,j=1}^{n}(f_i^* * f_j)(s)(g_i * \tilde{g}_j)(s)ds \geq 0$.

If these two equivalent conditions are fulfilled then

$\|\phi\|_{MA(G)} = \phi(e)$.

Lemma 11. _Let_ $G = A \underset{C}{*} B$ _and_ $n \in \mathbb{N}$. _Let_ ϕ _be a function on_ G. _If_ $\phi(s^{-1}) = \overline{\phi(s)}$ _for_ $s \in G$,

$(\#A\#B)^3|\phi(s)|(1+\Psi(s))^4 \leq (1/n)\phi(e)$ _for_ $s \in G - C$ _and_

$\phi(s) = 0$ _for_ $s \in C - \{e\}$, _then_

ϕ _is an n-positive multiplier on_ $A(G)$.

Proof. By proposition 10, it is sufficient to prove, that any $f_1,\ldots,f_n$, $g_1,\ldots,g_n \in K(G)$:

$$(*) \quad \Sigma_{s\varepsilon G}\phi(s)\Sigma_{i,j=1}^{n}(f_i^*\ast f_j)(s)\Sigma_{i,j=1}^{n}(g_i\ast \tilde{g}_j)(s) \geq 0.$$

Note that since $\phi = \tilde{\phi}$, the sum is always a real number. Assume first, that $g_1,\ldots,g_n$ are orthogonal with respect to the inner product in $\ell^2(G)$. Then $(g_i\ast \tilde{g}_j)(e) = 0$ for $i \neq j$. Hence

$$\Sigma_{i,j=1}^{n}(f_i^*\ast f_j)(e)(g_i\ast \tilde{g}_j)(e)$$

$$= \Sigma_{i=1}^{n}(f_i^*\ast f_i)(e)(g_i\ast \tilde{g}_i)(e)$$

$$= \Sigma_{i=1}^{n}\|f_i\|_2^2\|g_i\|_2^2 .$$

By lemma 8, we have that for any $f \varepsilon K(G)$;

$$\|\lambda(f)\| \leq (\#A\#B)^{3/2}\Sigma_{m=0}^{\infty}(m+1)\|f\chi_m\|_2.$$

Since A(G) may be identified with the predual of the von Neumann algebra R(G), this implies that for any $\phi \varepsilon A(G)$,

$$\|\phi\chi_m\|_2 \leq (\#A\#B)^{3/2}(m+1)\|\phi\|_{A(G)}, \quad m = 0,1,2,\ldots.$$

Obviously $f_i^*\ast f_j$ and $g_i\ast \tilde{g}_j$ are in A(G), and

$$\|f_i^*\ast f_j\|_{A(G)} \leq \|f_i\|_2\|f_j\|_2,$$

$$\|g_i\ast \tilde{g}_j\|_{A(G)} \leq \|g_i\|_2\|g_j\|_2.$$

Hence for $m \in \mathbb{N}$,

$$\Sigma_{s\varepsilon E_m}\phi(s)\Sigma_{i,j=1}^{n}(f_i^*\ast f_j)(s)(g_i\ast\tilde{g}_j)(s)$$

$$\leq \|\phi\chi_m\|_\infty\Sigma_{i,j=1}^{n}\|(f_i^*\ast f_j)\chi_m\|_2\|(g_i\ast\tilde{g}_j)\chi_m\|_2$$

$$\leq \{(^\#A^\#B)^{3/2}\}^2(m+1)^2\|\phi\chi_m\|_\infty\Sigma_{i,j=1}^{n}\|f_i\|_2\|f_j\|_2\|g_i\|_2\|g_j\|_2$$

$$\leq 1/2(m+1)^2(^\#A^\#B)^3\|\phi\chi_m\|_\infty\ \Sigma_{i,j=1}^{n}(\|f_i\|_2^2\|g_i\|_2^2 + \|f_j\|_2^2\|g_j\|_2^2)$$

$$= n(m+1)^2(^\#A^\#B)^3\|\phi\chi_m\|_\infty\ \Sigma_{i=1}^{n}\|f_i\|_2^2\|g_i\|_2^2$$

Hence by the assumption on ϕ and the fact that

$\Sigma_{m=1}^{\infty}(m+1)^{-2} = (\pi^2/6)-1 \leq 1$, we have that

$$|\Sigma_{s\varepsilon G-\{e\}}\phi(s)\Sigma_{i,j=1}^{n}(f_i^*\ast f_j)(s)(g_i\ast\tilde{g}_j)(s)|$$

$$= |\Sigma_{s\varepsilon C-\{e\}}\phi(s)\Sigma_{i,j=1}^{n}(f_i^*\ast f_j)(s)(g_i\ast\tilde{g}_j)(s) + \Sigma_{s\varepsilon G-C}\phi(s)\Sigma_{i,j=1}^{n}(f_i^*\ast f_j)(s)(g_i\ast\tilde{g}_j)(s)|$$

$$= |\Sigma_{s\varepsilon G-C}\phi(s)\Sigma_{i,j=1}^{n}(f_i^*\ast f_j)(s)(g_i\ast\tilde{g}_j)(s)|$$

$$\leq n(^\#A^\#B)^3(\Sigma_{i=1}^{n}\|f_i\|_2^2\|g_i\|_2^2)\Sigma_{m=1}^{\infty}(m+1)^2\|\phi\chi_m\|_\infty$$

$$\leq n(^\#A^\#B)^3(\Sigma_{i=1}^{n}\|f_i\|_2^2\|g_i\|_2^2)\cdot\sup_{m\in\mathbb{N}}\{(m+1)^4\|\phi\chi_m\|_\infty\}(\Sigma_{m=1}^{\infty}(m+1)^{-2})$$

$\leqq (\Sigma_{i=1}^{n} \|f_i\|_2^2 \|g_i\|_2^2)\phi(e)$

$\leqq \phi(e)\Sigma_{i,j=1}^{n}(f_i^* \ast f_j)(e)(g_i \ast \tilde{g}_j)(e)$

This proves (*) in the special case $(g_i|g_j) = 0$, $i \neq j$. The general case is reduced to the special case, $(g_i|g_j) = 0$, $i \neq j$, by the usual trick as shown in [2]. So we get this lemma 11.
Q.E.D.

We are now able to prove our main result.

Theorem 12. <u>Let</u> $G = A \underset{C}{*} B$ <u>be the free product of finite groups</u> A <u>and</u> B <u>with one amalgamated subgroup</u> C. <u>There exists a sequence</u> $(\xi_k)_{k\varepsilon N}$ <u>of functions with finite support, such that</u>

(1) <u>Each</u> ξ_k <u>is an n-positive multiplier of</u> A(G) <u>and</u> $\xi_k(e) = 1$.

(2) $\lim_{k\to\infty}\| \xi_k\phi - \phi\|_{A(G)} = 0$ <u>for any</u> $\phi \in A(G)$.

<u>Proof</u>. The proof is a version of the proof of Canniere and Haagerup([2],Theorem 4.6). Put $\phi_\lambda(s) = e^{-\lambda\Psi(s)}$, $\lambda > 0$.
Then ϕ_λ is positive definite. Thus ϕ_λ is a completely positive multiplier of A(G), and $\|m_{\phi_\lambda}\| = \phi_\lambda(e) = 1$.
For $\lambda > 0$ and $m \in \mathbb{N}$, put

$$\phi_{\lambda,m}^{(n)}(s) = \begin{cases} 1 + n(\#A\#B)^3 \sup_{k>m}(1+k)^4 e^{-\lambda k}, & \text{if } s = e \\ e^{-\lambda\Psi(s)}, & \text{if } 0 \leqq \Psi(s) \leqq m, s \neq e \\ 0 & \text{if } \Psi(s) > m \end{cases}$$

Then $\phi_{\lambda,m}^{(n)}$ is a function on G with finite supports. We shall check that $\phi = \phi_{\lambda,m}^{(n)} - \phi_\lambda$ is an n-positive multiplier of A(G). For s = e,

$$\phi(e) = \phi_{\lambda,m}^{(n)}(e) - \phi_\lambda(e) = 1 + n(\#A\#B)^3 \sup_{k>m}(1+k)^4 e^{-\lambda k} - 1$$
$$= n(\#A\#B)^3 \sup_{k>m}(1+k)^4 e^{-\lambda k}.$$

Hence we have

$$(*) \qquad (\#A\#B)^3 |\phi(s)|(1+\Psi(s))^4 \leq (1/n)\phi(e) \quad \text{if} \quad \Psi(s) > m.$$

For $s \in G - C$, we have $\Psi(s) > 0$. Then, for $0 < \Psi(s) \leq m$, we have $\phi = 0$. For $\Psi(s) > m$, we have the above inequality (*). For $s \in C - \{e\}$, we have $\Psi(s) = 0$. Thus $\phi(s) = 0$. So this ϕ satisfies the assumption of Lemma 11. As a consequence, $\phi_{\lambda,m}$ is an n-positive multiplier on A(G).

For the rest of the proof of Theorem 12, by using proposition 10, we can follow the same line of Cannier and Haagerup[2]. So we omit the rest of the proof . Q.E.D.

Corollary 13. Let G be as Theorem 12 and $n \in \mathbb{N}$. There exists a sequence $(T_k)_{k\in\mathbb{N}}$ of n-positive linear maps on the reduced group C*-algebra $C_r^*(G)$ of G such that

(1) each T_k is of finite rank, and $T_k(I) = I$,

(2) $\lim_{k\to\infty} \| T_k x - x \| = 0$ for any x in $C_r^*(G)$.

Proof. Let (ψ_k) be as in Theorem 12, and put $T_k = M_{\psi_k}$, then each T_k is n-positive. Moreover T_k has finite dimensional range, because $\mathrm{supp}(\psi_k)$ is finite. The statement (2) can be proved as in the proof of Canniere and Haagerup[2;3.11 Corollary]. Q.E.D.

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