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A COMPLEMENT OF THE ANDO-HIAI INEQUALITY AND NORM INEQUALITIES FOR THE GEOMETRIC MEAN

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ABSTRACT. Let A and B be positive operators on a Hilbert space such that $0 < m \leq A, B \leq M$ for some scalars $0 < m < M$ and put $h = \frac{M}{m}$. If A and B commute, then $A \sharp_\alpha B = A^{1-\alpha}B^\alpha = A \diamondsuit_\alpha B$ for all $0 \leq \alpha \leq 1$, where the α-geomtric mean $A \sharp_\alpha B = A^{\frac{1}{2}} \left(A^{-\frac{1}{2}}BA^{-\frac{1}{2}}\right)^\alpha A^{\frac{1}{2}}$ and the chaotically geometric one $A \diamondsuit_\alpha B = \exp\left((1-\alpha)\log A + \alpha \log B\right)$. In this note, we investigate a complement of the Ando-Hiai inequality: For each $0 \leq \alpha \leq 1$

$$K(h^2,\alpha)^r \|A^r \sharp_\alpha B^r\| \leq \|A \sharp_\alpha B\|^r \leq \|A^r \sharp_\alpha B^r\| \qquad \text{for all } 0 < r < 1,$$

where $K(h,\alpha)$ is a generalized Kantorovich constant. As an application, we prove a norm inequality for the geometric mean and its reverse: For each $\alpha \in [0,1]$

$$K(h^2,\alpha)\|A \diamondsuit_\alpha B\| \leq \|A \sharp_\alpha B\| \leq \|A \diamondsuit_\alpha B\| \leq \|A^{1-\alpha}B^\alpha\|.$$

1. INTRODUCTION.

A (bounded linear) operator A on a Hilbert space H is said to be positive (in symbol: $A \geq 0$) if $(Ax, x) \geq 0$ for all $x \in H$ and strictly positive (in symbol: $A > 0$) if A is positive and invertible. Let A and B be two positive operators on a Hilbert space H. In [8], the α-geometric mean $A \sharp_\alpha B$ for $0 \leq \alpha \leq 1$ is defined by

$$A \sharp_\alpha B = A^{\frac{1}{2}} \left(A^{-\frac{1}{2}}BA^{-\frac{1}{2}}\right)^\alpha A^{\frac{1}{2}}$$

if $A > 0$.

In [4], the chaotically geometric mean $A \diamondsuit_\alpha B$ for $0 \leq \alpha \leq 1$ is defined by

$$A \diamondsuit_\alpha B = \exp\left((1-\alpha)\log A + \alpha \log B\right)$$

if $A, B > 0$.

In the preceding paper [9], we obtained the following estimate for the geometric mean: For each $0 \leq \alpha \leq 1$

$$K(h^2,\alpha)\|A^{1-\alpha}B^\alpha\| \leq \|A \sharp_\alpha B\| \leq \|A^{1-\alpha}B^\alpha\|$$

for positive operators A and B such that $0 < m \leq A, B \leq M$ for some scalars $0 < m < M$ and $h = \frac{M}{m}$, where $K(h,\alpha)$ is a generalized Kantorovich constatnt.

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In this note, we investigate a complement of the Ando-Hiai inequality: For each $0 \leq \alpha \leq 1$

$$K(h^2, \alpha)^r \|A^r \sharp_\alpha B^r\| \leq \|A \sharp_\alpha B\|^r \leq \|A^r \sharp_\alpha B^r\| \qquad \text{for all } 0 < r < 1.$$

As an application, we prove a norm inequality and its reverse on the geometric mean and the chaotically geometric one. In other words, we estimate the sizes of $\|A \sharp_\alpha B\|$ by $\|A \diamondsuit_\alpha B\|$ and $\|A^{1-\alpha} B^\alpha\|$ as mentioned in the abstract. Moreover, we discuss them for the case $\alpha > 1$.

2. Norm inequalities.

First of all, we recall the following Ando-Hiai inequality [1, Theorem 2.1]:

Theorem AH. *If A and B are positive operators, then for each $\alpha \in [0, 1]$*

(1) $$\|A^r \sharp_\alpha B^r\| \leq \|A \sharp_\alpha B\|^r \qquad \text{for all } r \geq 1$$

or equivalently

(2) $$A \sharp_\alpha B \leq I \quad \Longrightarrow \quad A^r \sharp_\alpha B^r \leq I \qquad \text{for all } r \geq 1.$$

Hiai-Petz [7] showed the following result: For selfadjoint operators A, B

(3)
$$\exp((1-\alpha)A + \alpha B) = \lim_{r \to +0} (\exp(rA) \sharp_\alpha \exp(rB))^{\frac{1}{r}} \qquad \text{for all } \alpha \in [0, 1]$$

in the operator norm topology. Incidentally we use also the notation $\natural$ to distinguish it from the operator mean $\sharp$;

$$A \natural_\alpha B = A^{\frac{1}{2}} \left(A^{-\frac{1}{2}} B A^{-\frac{1}{2}}\right)^\alpha A^{\frac{1}{2}} \qquad \text{for all } \alpha \notin [0, 1].$$

Here we point out the fact that the formula (3) holds for $\alpha \notin [0, 1]$ by a similar method to [7]:

(4)
$$\exp((1-\alpha)A + \alpha B) = \lim_{r \to +0} (\exp(rA) \natural_\alpha \exp(rB))^{\frac{1}{r}} \qquad \text{for all } \alpha \notin [0, 1]$$

in the operator norm topology.

We show the following norm inequality for the geometric mean, in which we use the Ando-Hiai inequality:

Theorem 1. *Let A and B be positive invertible operators. Then for each $\alpha \in [0, 1]$*

$$\|A \sharp_\alpha B\| \leq \|A \diamondsuit_\alpha B\| \leq \|A^{1-\alpha} B^\alpha\|.$$

Proof. It follows from (1) of Theorem AH that

$$\|A \sharp_\alpha B\| \le \|A^r \sharp_\alpha B^r\|^{\frac{1}{r}} \qquad \text{for all } 0 < r < 1.$$

On the other hand, the formula (3) implies

$$A \diamondsuit_\alpha B = \lim_{r \to +0} (A^r \sharp_\alpha B^r)^{\frac{1}{r}}$$

in the operator norm topology. Hence we have $\|A \sharp_\alpha B\| \le \|A \diamondsuit_\alpha B\|$.

The following inequality is well known as Segal's inequality [10]:

$$\|\exp(H+K)\| \le \|\exp H \exp K\|$$

for selfadjoint operators H and K. Hence it follows that

$$\begin{aligned} \|A \diamondsuit_\alpha B\| &= \|\exp((1-\alpha)\log A + \alpha \log B)\| \\ &\le \|\exp \log A^{1-\alpha} \exp \log B^{\alpha}\| \\ &= \|A^{1-\alpha} B^{\alpha}\|. \end{aligned}$$

□

Remark 2. *By the proof above, we have*

$$\|A \diamondsuit_\alpha B\| \le \|A^{1-\alpha} B^{\alpha}\| \qquad \textit{for any real number } \alpha \in \mathbb{R}.$$

3. A complement of Ando-Hiai inequality.

We cite Araki's inequality [2, 3] and its reverse [5]:

Theorem B. *If A and B are positive operators such that $0 < m \le A \le M$ for some scalars $0 < m < M$, then*

$$\|BAB\|^p \le \|B^p A^p B^p\| \le K(h,p)\ \|BAB\|^p \qquad \textit{for all } p > 1 \tag{5}$$

or equivalently

$$K(h,p)\ \|BAB\|^p \le \|B^p A^p B^p\| \le \|BAB\|^p \qquad \textit{for all } 0 < p < 1, \tag{6}$$

where $h = \frac{M}{m}$ is a generalized condition number of A in the sense of Turing [12] *and a generalized Kantorovich constant $K(h,p)$ is defined by*

$$K(h,p) = \frac{h^p - h}{(p-1)(h-1)} \left(\frac{p-1}{p} \frac{h^p - 1}{h^p - h} \right)^p \tag{7}$$

for any real numbers $p \in \mathbb{R}$.

We state some properties of $K(h,p)$, cf. [6, Theorem 2.54 and Theorem 2.56].

Lemma 3. *Let $h > 0$ be given. Then a generalized Kantorovich constant $K(h,p)$ has the following properties.*
(i) $K(h,p) = K(h^{-1},p)$ *for all* $p \in \mathbb{R}$.
(ii) $K(h,p) = K(h,1-p)$ *for all* $p \in \mathbb{R}$.
(iii) $K(h,0) = K(h,1) = 0$ *and* $K(1,p) = 1$ *for all* $p \in \mathbb{R}$.
(iv) $K(h^r, \frac{p}{r})^{\frac{1}{p}} = K(h^p, \frac{r}{p})^{-\frac{1}{r}}$ *for* $pr \neq 0$.
(v) $K(h,p) \leq h^{p-1}$ *for all* $p > 1$ *and* $h > 1$.
(vi) $\lim_{r \to 0} K(h^r, \frac{1}{r}) = S(h)$,
where the Specht ratio $S(h)$ ([11]) *is defined by*

$$S(h) = \frac{(h-1)h^{\frac{1}{h-1}}}{e \log h} \quad (h > 0, h \neq 1) \quad \text{and} \quad S(1) = 1. \tag{8}$$

We show the following complement of the Ando-Hiai inequality:

Theorem 4. *Let A and B be positive operators on H such that $0 < m \leq A, B \leq M$ for some scalars $0 < m < M$, $h = \frac{M}{m}$ and $0 \leq \alpha \leq 1$. Then*

$$\|A^r \sharp_\alpha B^r\| \leq K(h^2,\alpha)^{-r} \|A \sharp_\alpha B\|^r \qquad \text{for all } 0 < r < 1 \tag{9}$$

or equivalently

$$A \sharp_\alpha B \leq I \quad \Longrightarrow \quad A^r \sharp_\alpha B^r \leq K(h^2,\alpha)^{-r} \qquad \text{for all } 0 < r < 1. \tag{10}$$

Proof. We firstly show (9). Since a generalized condition number of $A^{-\frac{1}{2}}BA^{-\frac{1}{2}}$ is $h^2 = \left(\frac{M}{m}\right)^2$, it follows that for each $0 \leq \alpha \leq 1$

$$\begin{aligned}
\|A^r \sharp_\alpha B^r\| &= \|A^{\frac{r}{2}} \left(A^{-\frac{r}{2}} B^r A^{-\frac{r}{2}}\right)^\alpha A^{\frac{r}{2}}\| \\
&\leq \|A^{\frac{r}{2\alpha}} A^{-\frac{r}{2}} B^r A^{-\frac{r}{2}} A^{\frac{r}{2\alpha}}\|^\alpha \qquad \text{by } 0 \leq \alpha \leq 1 \text{ and (6) of Theorem B} \\
&= \|A^{\frac{r-r\alpha}{2\alpha}} B^r A^{\frac{r-r\alpha}{2\alpha}}\|^\alpha \\
&\leq \|A^{\frac{1-\alpha}{2\alpha}} B A^{\frac{1-\alpha}{2\alpha}}\|^{r\alpha} \qquad \text{by } 0 < r < 1 \text{ and (6) of Theorem B} \\
&= \|A^{\frac{1}{2\alpha}} A^{-\frac{1}{2}} B A^{-\frac{1}{2}} A^{\frac{1}{2\alpha}}\|^{r\alpha} \\
&\leq \left(K(h^2,\alpha)^{-1} \|A^{\frac{1}{2}} \left(A^{-\frac{1}{2}} B A^{-\frac{1}{2}}\right)^\alpha A^{\frac{1}{2}}\|\right)^r \text{ by } 0 \leq \alpha \leq 1 \text{ and (6) of Theorem B} \\
&= K(h^2,\alpha)^{-r} \|A \sharp_\alpha B\|^r
\end{aligned}$$

for all $0 < r < 1$ and hence we have the desired inequality (9).
(9) $\Longrightarrow$ (10) is obvious.
(10) $\Longrightarrow$ (9): Since $A \sharp_\alpha B \leq \|A \sharp_\alpha B\|$, it follows from the homoginity of the geometric mean that

$$\frac{A}{\|A \sharp_\alpha B\|} \sharp_\alpha \frac{B}{\|A \sharp_\alpha B\|} \leq I.$$

By (10), we have

$$\frac{A^r}{\|A \sharp_\alpha B\|^r} \sharp_\alpha \frac{B^r}{\|A \sharp_\alpha B\|^r} \leq K(h^2,\alpha)^{-r},$$

because a generalized condition number of $A/\|A \sharp_\alpha B\|$, $B/\|A \sharp_\alpha B\|$ is $\frac{M}{\|A \sharp_\alpha B\|} / \frac{m}{\|A \sharp_\alpha B\|} = M/m = h$. Hence we have the desired inequaltiy:

$$\|A^r \sharp_\alpha B^r\| \leq K(h^2,\alpha)^{-r}\|A \sharp_\alpha B\|^r$$

for all $0 < r < 1$. Therefore the proof is complete. □

By Theorem 4, we have the following reverse inequality of the Ando-Hiai one for the case of $r > 1$:

Corollary 5. *Let A and B be positive operators such that $0 < m \leq A, B \leq M$ for some scalars $0 < m < M$, $h = \frac{M}{m}$ and $0 \leq \alpha \leq 1$. Then*

(11) $$K(h^{2r},\alpha)\|A \sharp_\alpha B\|^r \leq \|A^r \sharp_\alpha B^r\| (\leq \|A \sharp_\alpha B\|^r) \qquad \text{for all } r > 1.$$

Proof. For $r > 1$, we have $0 < \frac{1}{r} < 1$ and by (9) of Theorem 4

$$\|A^{\frac{1}{r}} \sharp_\alpha B^{\frac{1}{r}}\| \leq K(h^2,\alpha)^{-\frac{1}{r}}\|A \sharp_\alpha B\|^{\frac{1}{r}}.$$

Replacing A and B by A^r and B^r respectively and a generalized condition number of A^r and B^r is h^r, it follows that

$$\|A \sharp_\alpha B\| \leq K(h^{2r},\alpha)^{-\frac{1}{r}}\|A^r \sharp_\alpha B^r\|^{\frac{1}{r}}$$

and by taking r-th power on both sides we have the desired inequality (11). □

In the remainder of the section, we investigate the Ando-Hiai inequality without the framework of operator mean. The following theorem corresponds to (9) of Theorem 4 in the case of $\alpha > 1$.

Theorem 6. *Let A and B be positive operators such that $0 < m \leq A, B \leq M$ for some scalars $0 < m < M$, $h = \frac{M}{m}$ and $\alpha > 1$. Then*

(12)
$$K(h,r)^\alpha K(h^2,\alpha)^{-r}\|A \natural_\alpha B\|^r \leq \|A^r \natural_\alpha B^r\| \leq K(h^{2r},\alpha)\|A \natural_\alpha B\|^r$$

for all $0 < r < 1$.

Proof. For each $\alpha > 1$, we have

$$\begin{aligned}
||A^r \natural_\alpha B^r|| &= ||A^{\frac{r}{2}} \left(A^{-\frac{r}{2}} B^r A^{-\frac{r}{2}}\right)^\alpha A^{\frac{r}{2}}|| \\
&\leq K(h^{2r},\alpha) ||A^{\frac{r}{2\alpha}} A^{-\frac{r}{2}} B^r A^{-\frac{r}{2}} A^{\frac{r}{2\alpha}}||^\alpha \qquad \text{by } \alpha > 1 \text{ and (5) of Theorem B} \\
&= K(h^{2r},\alpha) ||A^{\frac{r-r\alpha}{2\alpha}} B^r A^{\frac{r-r\alpha}{2\alpha}}||^\alpha \\
&\leq K(h^{2r},\alpha) ||A^{\frac{1-\alpha}{2\alpha}} B A^{\frac{1-\alpha}{2\alpha}}||^{r\alpha} \qquad \text{by } 0 < r < 1 \text{ and (6) of Theorem B} \\
&= K(h^{2r},\alpha) ||A^{\frac{1}{2\alpha}} A^{-\frac{1}{2}} B A^{-\frac{1}{2}} A^{\frac{1}{2\alpha}}||^{r\alpha} \\
&\leq K(h^{2r},\alpha) ||A^{\frac{1}{2}} \left(A^{-\frac{1}{2}} B A^{-\frac{1}{2}}\right)^\alpha A^{\frac{1}{2}}||^r \qquad \text{by } \alpha > 1 \text{ and (5) of Theorem B} \\
&= K(h^{2r},\alpha) ||A \natural_\alpha B||^r
\end{aligned}$$

and hence we have the right-hand side of (12).

Conversely, we have

$$\begin{aligned}
||A^r \natural_\alpha B^r|| &= ||A^{\frac{r}{2}} \left(A^{-\frac{r}{2}} B^r A^{-\frac{r}{2}}\right)^\alpha A^{\frac{r}{2}}|| \\
&\geq ||A^{\frac{r}{2\alpha}} A^{-\frac{r}{2}} B^r A^{-\frac{r}{2}} A^{\frac{r}{2\alpha}}||^\alpha \qquad \text{by } \alpha > 1 \text{ and (5) of Theorem B} \\
&= ||A^{\frac{r-r\alpha}{2\alpha}} B^r A^{\frac{r-r\alpha}{2\alpha}}||^\alpha \\
&\geq \left(K(h,r) ||A^{\frac{1-\alpha}{2\alpha}} B A^{\frac{1-\alpha}{2\alpha}}||^r\right)^\alpha \qquad \text{by } 0 < r < 1 \text{ and (6) of Theorem B} \\
&= K(h,r)^\alpha ||A^{\frac{1-\alpha}{2\alpha}} B A^{\frac{1-\alpha}{2\alpha}}||^{r\alpha} \\
&= K(h,r)^\alpha ||A^{\frac{1}{2\alpha}} A^{-\frac{1}{2}} B A^{-\frac{1}{2}} A^{\frac{1}{2\alpha}}||^{r\alpha} \\
&\geq K(h,r)^\alpha \left(K(h^2,\alpha)^{-1} ||A^{\frac{1}{2}} \left(A^{-\frac{1}{2}} B A^{-\frac{1}{2}}\right)^\alpha A^{\frac{1}{2}}||\right)^r \\
&\qquad\qquad \text{by } \alpha > 1 \text{ and (5) of Theorem B} \\
&= K(h,r)^\alpha K(h^2,\alpha)^{-r} ||A \natural_\alpha B||^r
\end{aligned}$$

and hence we have the left-hand side of (12). □

By Theorem 6, we have the following complement of the Ando-Hiai inequality in the case of $\alpha > 1$:

Theorem 7. *Let A and B be positive operators such that $0 < m \leq A, B \leq M$ for some scalars $0 < m < M$, $h = \frac{M}{m}$ and $\alpha > 1$. Then*

(13) $$||A^r \natural_\alpha B^r|| \leq K(h^{2r},\alpha) ||A \natural_\alpha B||^r \qquad \text{for all } 0 < r < 1$$

or equivalently

(14) $$A \natural_\alpha B \leq I \implies A^r \natural_\alpha B^r \leq K(h^{2r},\alpha) \qquad \text{for all } 0 < r < 1.$$

The following corollary is a complementary inequality for Theorem 6.

Corollary 8. *Let A and B be positive operators such that $0 < m \leq A, B \leq M$ for some scalars $0 < m < M$, $h = \frac{M}{m}$ and $\alpha > 1$. Then*

(15)
$$K(h^2, \alpha)^{-r} \|A \natural_\alpha B\|^r \leq \|A^r \natural_\alpha B^r\| \leq K(h, r)^\alpha K(h^{2r}, \alpha) \|A \natural_\alpha B\|^r$$
for all $r > 1$.

4. Reverse norm inequality for geometric mean.

In this section, we show reverse norm inequalities for the geometric mean by using results obtained in the preceding section.

Theorem 9. *If A and B are positive operators such that $0 < m \leq A, B \leq M$ for some scalars $0 < m < M$ and $h = \frac{M}{m}$, then*

(16)
$$K(h^2, \alpha) \|A \diamondsuit_\alpha B\| \leq \|A \sharp_\alpha B\| \qquad \text{for all } 0 < \alpha < 1.$$

(17)
$$S(h)^{-\alpha} K(h^2, \alpha)^{-1} \|A \natural_\alpha B\| \leq \|A \diamondsuit_\alpha B\| \leq h^{2(\alpha-1)} \|A \natural_\alpha B\| \qquad \text{for all } \alpha > 1,$$
where the Specht ratio $S(h)$ is defined as (8).

Proof. By (9) of Theorem 4, it follows that for each $0 < \alpha < 1$
$$\|A^r \sharp_\alpha B^r\| \leq K(h^2, \alpha)^{-r} \|A \sharp_\alpha B\|^r \qquad \text{for all } 0 < r < 1.$$
By taking $\frac{1}{r}$-th power on both sides, we have
$$\|(A^r \sharp_\alpha B^r)^{\frac{1}{r}}\| \leq K(h^2, \alpha)^{-1} \|A \sharp_\alpha B\|$$
and hence we have the desired inequality (16)
$$\|A \diamondsuit_\alpha B\| \leq K(h^2, \alpha)^{-1} \|A \sharp_\alpha B\|$$
by the formula (3).

Next, it follows from (12) of Theorem 6 that for each $\alpha > 1$
$$\begin{aligned} K(h, r)^{\frac{\alpha}{r}} K(h^2, \alpha)^{-1} \|A \natural_\alpha B\| \leq \|A^r \natural_\alpha B^r\|^{\frac{1}{r}} &\leq K(h^{2r}, \alpha)^{\frac{1}{r}} \|A \natural_\alpha B\| \\ &\leq h^{2(\alpha-1)} \|A \natural_\alpha B\| \qquad \text{for all } 0 < r < 1. \end{aligned}$$
The last inequality follows from (v) of Lemma 3. On the other hand, by (4) we have
$$A \diamondsuit_\alpha B = \lim_{r \to +0} (A^r \natural_\alpha B^r)^{\frac{1}{r}} \qquad \text{for all } \alpha > 1$$
and hence it follows that
$$S(h)^{-\alpha} K(h^2, \alpha)^{-1} \|A \natural_\alpha B\| \leq \|A \diamondsuit_\alpha B\| \leq h^{2(\alpha-1)} \|A \natural_\alpha B\|,$$
since it follows from (vi) of Lemma 3 that $\lim_{r \to 0} K(h, r)^{\frac{\alpha}{r}} = \lim_{r \to 0} K(h^r, \frac{1}{r})^{-\alpha} = S(h)^{-\alpha}$. □

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