PREFACE

The substance of this booklet was presented in five lectures at the University of Notre Dame, April 12-15, 1948.

The booklet has two purposes. On the one hand it presents certain researches on the methodology of formal systems as explained in the introduction. On the other hand it aims to give a self-contained account of the approach to the logical calculus by means of inferential rules as given by Gentzen in his thesis [35]. These two purposes are not antagonistic; on the contrary they are logically related, and they supplement and mutually influence one another.

In regard to the expository aspect, I share in the opinion that the inferential rules of Gentzen and Jaśkowski form one of the most natural and fruitful approaches to the propositional and predicate calculuses. They have shown themselves to be interesting even to some hardboiled practicians of neighboring fields. A systematic exposition of this approach is, therefore, a desideratum. The exposition attempted here is intended for mature persons. Except in certain portions, easily skipped, no technical knowledge of mathematical logic - or of mathematics either - is presupposed; yet it is assumed that the reader can cope with mathematical arguments of considerable generality and abstractness, including some of the more involved applications of mathematical induction. A rudimentary acquaintance with the meaning, as opposed to the technique, of ordinary logical symbolism, although perhaps not strictly necessary, is nevertheless advantageous. Such an acquaintance may be obtained from Tarski's elementary book [81], especially Chapters I, II, III, IV, and VI; or alternatively from a number of other books, e.g., the following listed in the bibliography: [2,3,4,11,14,29,46,57,87,88,92]. Further suggestions will be found in [91].

In regard to the research aspect, these lectures constitute the publication, with additions, of the paper presented to the American Mathematical Society in September, 1937, for which the abstract is [25]. Various considerations, concerning which my memory is now rather vague, prevented publication of that paper until the war made it necessary to put mathematical logic on the shelf. On receiving the invitation from the University of Notre Dame to deliver these lectures, I decided that the connection of these results with the expository hiatus, mentioned in the preceding paragraph, made them an ideal subject for the purpose. With this connection in mind, and with due regard for what I have learned since, those results have been thoroughly revised. These lectures are the result of that revision.

Those interested in historical questions may wish to know more about the relation of these lectures to the unpublished manuscript of 1937. That manuscript approached the subject strictly from the standpoint of formal deducibility. It began with the natural system, here called the T-system, and introduced the L-system, much as Gentzen did, as an instrument for deriving the theorem analogous to Theorem 8 of Chapter II. The theorem was derived in two ways: first, by applying the results of Gentzen's thesis; and second, by generalizing Gentzen's method of approach so as to include the proof of Gentzen's Hauptsatz as a special case. The first of these methods I found nearly as laborious as the second. Consequently I concluded that a fresh approach to Gentzen's complex of ideas would be the most economical in the long run. This program was carried through for all the logical connectives which are here considered, except that the work in regard to possibility was, and still is, somewhat fragmentary.

The present treatment differs from the earlier one in two principal respects. In the first place, my experience with these inferential methods in other connections (as represented by [17]§8 and [15]) led to the conviction that Gentzen's L-system is really a more profound and, in a sense, a more natural approach than the T-system. Accordingly, the whole structure of the proof has been recast from that standpoint. In the second place, whereas the earlier treatment considered only the intuitionistic systems (here called A, J, and M), the present treatment has been extended to include the classical systems. These extensions were made in the actual writing of this manuscript. Doubtless improvements in these matters will be made in the future. In the appropriate places I have pointed out gaps which remain to be filled.

On account of the dependence of the present approach on the notion of formal system, and also of the relevance of that notion to the general question of formal deducibility, it seemed best to begin these lectures with a general discussion of formal systems as such. This constitutes Chapter I below. This chapter is needed as a general introduction to what follows; but, since the questions discussed there are of some interest in their own right, the chapter goes into somewhat greater detail than is strictly necessary for an introduction. This chapter is mostly a revision of [16], which in turn was a revision and condensation of some parts of [23], (the latter paper, prepared for the International Congress for the Unity of Science in 1939, is still unpublished). However, the notions of U-language and A-language, and some of the notions of grammatics, were written down in the summer of 1947 after reading Carnap's two books [5, 6]; they are published for the first time here and in [20]. (For earlier ideas cf. [21].)

In view of the foregoing origin of these lectures, certain topics related to their subject matter have not been adequately treated. On the one hand, those aspects of Gentzen's thesis which relate to the classical theory exclusively (and are not merely extensions of results for the intuitionistic systems) are not considered here. They are not actually relevant to the main theme. On the other hand, the algebraic approach to the calculus of logic as exemplified in the work of G. Birkhoff, Stone, Tarski, McKinsey and others, is not touched upon. This omission is not due to any lack of appreciation, on my part, of the importance of the algebraic approach. In a really modern introduction to the logical calculus that approach should be combined with the present one. Thus in a recent course in mathematical logic at The Pennsylvania State College I presented the simpler properties of partial order, logical groups (i.e., partially ordered systems with one lattice operation), lattices, and Boolean algebras, including the truth table method as a decision method à la Schröder for Boolean algebra, immediately after an introductory discussion of formal systems; then, on introducing the propositional algebra later by inferential rules, it could be shown that we had to do with lattice systems of such and such kinds. There is much which remains to be done, from the expository standpoint, along such lines. But, from the present point of view, the algebraic approach is a separate subject matter; and I have not attempted to do justice to it, either in the text or in the bibliography.

The bibliography at the end lists all works which I am conscious of having used in the preparation of these lectures. It must be remembered that the war caused a break in my contacts with the literature which has not yet been completely repaired. Consequently, it may happen that an important reference has been omitted. Any such error is regretted; but it is perhaps inevitable, in view of the pressure of other duties, that errors of this and other kinds should occur. The bibliography also contains a selection of elementary works suitable for background. Of the items listed the only ones which I have not examined are [9], [53], [55], [75], and [88].

Acknowledgements are due to the Council on Research of The Pennsylvania State College for assistance in defraying the expense of preparing the manuscript, and to Professor John M. Anderson of the philosophy department, Pennsylvania State College, for help in correcting the manuscript and proofs.

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The Pennsylvania State College
July 16, 1948.