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EXAMINATION OF THE AXIOMATIC FOUNDATIONS OF A THEORY OF CHANGE. IV

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*Third Part**

§4

§4. Consistency of the axiomatic system. In order to establish the consistency of our axiom system, it is important to make first the following remarks:

1. The predicate calculus which we have chosen, is consistent. (The proof is given in [13] pp. 93-95.)
2. An expression σ (respectively a set E of expressions) is said to be "satisfiable" if there exists some non-empty domain ω of individuals such that σ (respectively E) is satisfiable in ω.
3. If a predicate calculus is consistent, so is every satisfiable set of expressions.
4. It is then sufficient here to show that there exists a non-empty domain ω of individuals such that the set of our axioms is satisfiable in ω.

The model shall consist of:

I. a) a domain S of individuals for momentaneous subjects. Let R be the following subset of the set of rational numbers:

$$R = \{n \mid n \text{ is a rational number and } 0 \leq n \leq 2\}.$$

Let then

$$S = \{a_i, b_i, c_i\},$$

where $i \in R$ and $\neq a_i, a_j, \neq b_i, b_j, \neq c_i, c_j$, for $i, j \in R$ and $i \neq j$, and

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$\neq a_i, b_j, c_k$, for $i, j, k \in R$, that is, the momentaneous subjects are distinct from one another. We shall interpret our notions in such a way that the domain S will consist of three distinct classes of genidentical momentaneous subjects.

b) a domain Z of individuals for properties:

$$Z = \{\alpha, \beta\}, \text{ where } \alpha \neq \beta.$$

II. an interpretation of our notions as follows:

$\sim$
$a_i \sim a_i$, $b_i \sim b_i$, $c_i \sim c_i$, for each $i \in R$
$a_i \sim b_i$, $a_i \sim c_i$, $b_i \sim c_i$, $b_i \sim a_i$, $c_i \sim a_i$, $c_i \sim b_i$, for each $i \in R$
$\neg a_i \sim a_j$, $\neg a_i \sim b_j$, $\neg a_i \sim c_j$, $\neg b_j \sim a_i$, $\neg b_j \sim c_i$, $\neg c_j \sim a_i$,
$\neg c_i \sim b_j$, $\neg b_i \sim b_j$, $\neg c_i \sim c_j$, for each $i, j \in R$ and $i \neq j$

$<$
$a_i < a_j$, $b_i < b_j$, $c_i < c_j$, for each $i, j \in R$ and "$i < j$"
$\neg a_i < b_j$, $\neg a_i < c_j$, $\neg b_i < c_j$, $\neg b_i < a_j$, $\neg c_i < b_j$, $\neg c_i < a_j$, for each $i, j \in R$
$\neg a_i < a_j$, $\neg b_i < b_j$, $\neg c_i < c_j$, for each $i, j \in R$ and "$j \leq i$"

$\leq$
$a_i \leq a_j$, $b_i \leq b_j$, $c_i \leq c_j$, for each $i, j \in R$ and "$i \leq j$"
$\neg a_i \leq a_j$, $\neg b_i \leq b_j$, $\neg c_i \leq c_j$, for each $i, j \in R$ and "$j < i$"
$\neg a_i \leq b_j$, $\neg a_i \leq c_j$, $\neg b_i \leq c_j$, $\neg b_i \leq a_j$, $\neg c_i \leq b_j$, $\neg c_i \leq a_j$, for each $i, j \in R$

G
$\mathsf{G}a_ia_j$, $\mathsf{G}b_ib_j$, $\mathsf{G}c_ic_j$, for each $i, j \in R$
$\neg\mathsf{G}a_ib_j$, $\neg\mathsf{G}a_ic_j$, $\neg\mathsf{G}b_ic_j$, $\neg\mathsf{G}b_ia_j$, $\neg\mathsf{G}c_ia_j$, $\neg\mathsf{G}c_ib_j$, for each $i, j \in R$

M
$\mathsf{M}a_ib_2\alpha$, for each $i \in R$ and $i \neq 2$
$\neg$ in each other case, i.e., $\neg\mathsf{M}a_ic_i\alpha$ for each $i \in R$, and so on

A
$\neg\mathsf{A}a_i\alpha$, for each $i \in R$
$\neg\mathsf{A}b_i\alpha$, for each $i \in R$ and $i \neq 2$
$\mathsf{A}b_2\alpha$
$\mathsf{A}c_i\alpha$, for each $i \in R$
$\mathsf{A}a_i\beta$, $\mathsf{A}b_i\beta$, $\mathsf{A}c_i\beta$, for each $i \in R$

F
$\neg\mathsf{F}a_i\alpha$, for each $i \in R$
$\mathsf{F}b_i\alpha$, $\mathsf{F}c_i\alpha$, $\mathsf{F}a_i\beta$, $\mathsf{F}b_i\beta$, $\mathsf{F}c_i\beta$, for each $i \in R$

P
$\neg\mathsf{P}a_i\alpha$ for each $i \in R$
$\mathsf{P}b_i\alpha$, for each $i \in R$ and $i \neq 2$
$\neg\mathsf{P}b_2\alpha$, $\neg\mathsf{P}c_i\alpha$, $\neg\mathsf{P}a_i\beta$, $\neg\mathsf{P}b_i\beta$, $\neg\mathsf{P}c_i\beta$, for each $i \in R$

V
$\mathsf{V}b_ib_2\alpha$, for each $i \in R$ and $i \neq 2$
$\neg$ in each other case, i.e., $\neg\mathsf{V}a_ia_j\alpha$, $\neg\mathsf{V}a_ia_j\beta$, for each $i, j \in R$, and so on

B
$\mathsf{B}a_ia_j\alpha$, $\neg\mathsf{B}a_ja_i\alpha$, for each $i, j \in R$, where $j < i$
$\mathsf{B}a_ia_i\alpha$, for each $i \in R$
$\mathsf{B}b_ia_j\alpha$, $\neg\mathsf{B}a_jb_i\alpha$, for each $i, j \in R$
$\mathsf{B}c_ia_j\alpha$, $\neg\mathsf{B}a_jc_i\alpha$, for each $i, j \in R$
$\mathsf{B}b_ib_j\alpha$, $\neg\mathsf{B}b_jb_i\alpha$, for each $i, j \in R$, where $i < j$
$\mathsf{B}b_ib_i\alpha$, for each $i \in R$
$\mathsf{B}b_ic_j\alpha$, $\neg\mathsf{B}c_jb_i\alpha$, for each $i, j \in R$ where $i \neq 2$
$\mathsf{B}c_ic_j\alpha$, for each $i, j \in R$

B $\mathsf{B}a_ia_j\beta$, $\mathsf{B}b_ib_j\beta$, $\mathsf{B}c_ic_j\beta$, $\mathsf{B}a_ib_j\beta$, $\mathsf{B}a_ic_j\beta$, $\mathsf{B}b_ic_j\beta$, $\mathsf{B}b_ia_j\beta$, $\mathsf{B}c_ia_j\beta$, $\mathsf{B}c_ib_j\beta$, for each $i, j \epsilon R$

W
$\mathsf{W}a_ia_j\alpha$, for each $i, j \epsilon R$, where $j < i$
$\mathsf{W}b_ia_j\alpha$, $\mathsf{W}c_ia_j\alpha$, for each $i, j \epsilon R$
$\mathsf{W}b_ib_j\alpha$, for each $i, j \epsilon R$, where $i < j$
$\mathsf{W}b_ic_j\alpha$, for each $i, j \epsilon R$, where $i \neq 2$
$\neg$ in each other case, that is, $\neg\mathsf{W}a_ia_j\alpha$ for each $i, j \epsilon R$, where $i \leq j$, and so on

I
$\mathsf{I}a_ia_i\alpha$, $\mathsf{I}b_ib_i\alpha$, for each $i \epsilon R$
$\mathsf{I}c_ic_j\alpha$ for each $i, j \epsilon R$
$\mathsf{I}b_2c_j\alpha$, $\mathsf{I}c_jb_2\alpha$, for each $j \epsilon R$
$\mathsf{I}a_ia_j\beta$, $\mathsf{I}b_ib_j\beta$, $\mathsf{I}c_ic_j\beta$, $\mathsf{I}a_ib_j\beta$, $\mathsf{I}a_ic_j\beta$, $\mathsf{I}b_ic_j\beta$, $\mathsf{I}b_ia_j\beta$, $\mathsf{I}c_ia_j\beta$, $\mathsf{I}c_ib_j\beta$, for each $i, j \epsilon R$
$\neg$ in each other case, that is, $\neg\mathsf{I}a_ia_j\alpha$, for each $i, j \epsilon R$, where $i \neq j$, and so on

In order to see that the given model is truly a model for our axiomatic system, one has only to verify that the set of our axioms is satisfiable with regard to the non-empty domains S and Z under the given interpretation of our primitive and defined notions.

Let us illustrate this by an example. Suppose we choose the axiom 6.7. Our given interpretation states that the subject to which belong the momentaneous subjects b_i, for each $i \epsilon R$, has undergone a change with regard to the determination α during the time interval determined by b_0 and b_2, and this change came to an end at the point in time belonging to b_2. We must then check that the right part of the implication sign is satisfiable. It is easy to see that this is the case. Under our interpretation, we have:

1) $\mathsf{M}a_ib_2\alpha$ for each $i \epsilon R$ and $i \neq 2$;
2) $a_i \sim b_i$, for each $i \epsilon R$;
3) $b_i \leq b_j$, for each $i, j \epsilon R$ and "$i \leq j$";
4) $b_i < b_j$, for each $i, j \epsilon R$ and "$i < j$".

REFERENCES

References [1]-[8], [9]-[12], and [13] are given at the ends of the first, second, and third parts of this paper respectively. See *Notre Dame Journal of Formal Logic*, vol. IX (1968), pp. 371-384, vol. X (1969), pp. 277-284, and vol. X (1969), pp. 385-409. They are now supplemented by:

[III] Larouche, L., "Examination of the axiomatic foundations of a theory of change. III," in *Notre Dame Journal of Formal Logic*, vol. X (1969), pp. 385-409.

(To be continued)

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