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THE PROBLEM OF EXISTENTIAL IMPORT*
(From George Boole to P. F. Strawson)

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In recent logical theories, one of the most striking features which mark the differences between Aristotelian and modern symbolic logic is the problem of existential import of universal categorical propositions. In the Aristotelian tradition, the subject of a universal proposition is assumed implicitly to be existential in the sense that the class denoted by the subject term has members. On the other hand, in modern symbolic logic the universal propositions are interpreted as non-existential in the sense of not implying the existence of members of the class denoted by the subject term. In the logic of the Aristotelian tradition, the problem of existential import was never raised.[1] The problem has emerged only after the development of mathematical logic. After the publication of George Boole's *The Mathematical Analysis of Logic* in 1847,[2] there has followed a series of discussions on this topic by logicians and mathematicians. The purpose of this paper is to give an expository account of the historical development of the problem in recent logical theories, from George Boole's logical innovation to P. F. Strawson's criticism of symbolic logic.

Although George Boole is the first to outline clearly the program of mathematical logic,[3] he has no intention of instituting any direct comparison between his own treatise and the traditional system of the Aristotelian logic.[4] The sharp contrast between the Boolean algebra of classes and Aristotelian logic in regard to the interpretation of the existential import of propositions is developed gradually through the studies of Franz Brentano, John Venn, Charles Peirce, and other modern logicians.

Brentano's *Psychologie vom empirischen Standpunkt*[5] of 1874 is primarily a work in psychology. But it plays quite an important role in the

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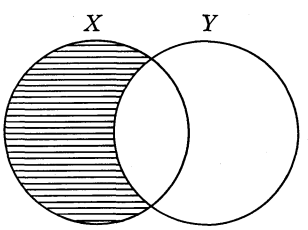
development of the problem of the existential import of propositions. The main feature of his reconstruction of logical doctrine consists in reducing all categorical propositions to what he calls existential propositions, doing away with the distinction between the subject and the predicate of traditional logic.[6] For instance, "Some man is sick" and "No stone is alive" are reduced to "There is a sick man" and "There is not a live stone" correspondingly. "Some man is not learned" is transformed to "There is an unlearned man." "All men are mortal" is replaced by "There is not an immortal man."[7] The most significant point is, that, formerly, a statement was made about a whole class, now Brentano's new formulation makes a statement of the denial of existence.

In John Venn's *Symbolic Logic* of 1881,[8] the problem of existential import has become more explicit. The sharpening of this problem is due to his invention of the diagrammatic method popularly known as the "Venn Diagrams." This method consists in the use of overlapping regions to illustrate relations between classes or the relations of the truth-conditions of propositions.[9] A major feature of his diagrammatic method is that it first represents all possible combinations of classes by distinct areas of the diagram. Next it indicates by marks which combinations are empty and which are not, for the meaning of a given proposition. The traditional A proposition, for example, "*All X is Y*," is to be interpreted to mean that there is no such class of things in existence as "*X that is not-Y*." and is diagrammed as the following:[10]



Here, the shading of an area indicates emptiness of the corresponding class as stipulated in the given proposition. The area which lies within the X circle but outside the Y circle represents the class of "*X that is not-Y*" which is non-existent according to the given proposition "*All X is Y*."

From the above illustration, the proposition "*All X is Y*" is interpreted as the denial of the existence of a certain class, that is, $X\overline{Y}$ or "*X that is not-Y*", rather than as implying the existence of a class of objects of some sort, that is, the class denoted by the subject term X. The diagram for the given proposition shows the emptiness of a certain region rather than asserting the existence or non-emptiness of any area. Being aware of this problem, Venn devotes a whole chapter to the discussion of this topic.[11] He proposes to discuss the problem first in respect to ordinary language and traditional logic, and then to symbolic logic.

Broadly speaking, in ordinary language, as Venn interprets it, "*All X is Y*" does imply directly that there are X's, and consequently indirectly

that there are Y's. For people in general do not talk about what they believe to be non-entities.[12] This interpretation, however, meets difficulty because of exceptions such as assertions about the future or assertions of an ideal. The assertion "Those who pass this examination are lucky men" certainly would be tacitly supplemented by the condition that "if any such there be."[13] The proposition "Perfectly conscientious men think but little of law and rule" does not imply the existence of such men. Turning from ordinary language to ordinary logic, Venn seems to be disappointed in not having found in logical writings in English any examination of this problem.[14] His critical approach to this subject is to inquire what can be elicited from the universally accepted rules of logic.[15] He then tries to examine the case of *Darapti* and concludes that "*all* universal affirmatives postulate the presence, so to say, of actual representatives of their subjects, and consequently of their predicates."[16] Again, if we accept the rule of contra position of propositions, we can derive "*All not-Y is not-X*" from "*All X is Y.*" Also, by the rule of obversion, we may obtain "*All X is not-Y*" from "*No X is Y.*" In both of these cases, "*All not-Y is not-X*" and "*All X is not-Y*" are universal affirmative propositions and they must claim the existence of their subjects and predicates even when they are negative in nature.[17] Venn concludes that this will hinder us in our logical predication, because we ought not to assert anything or deny anything about X or Y when we are not certain either that there are things which are X and Y, or that there are things which are not-X and not-Y.[18]

It is obvious that ordinary language makes no consistent assumption as to the existence of the subject and the predicate of a universal proposition. It is quite difficult to make a generalization as to whether a universal proposition does or does not imply existential import. In traditional logic, the state of the law on this point is full of perplexity.[19] But if we adopt the position of symbolic logic, all the difficulties above mentioned will vanish.[20] The key point is to accept "the simple explanation that *the burden of implication of existence is shifted from the affirmative to the negative form.*"[21] Take the example "*All X is Y.*" Since we have two class terms here, there are four ultimate classes: xy, $x\bar{y}$ (or x not-y), $y\bar{x}$ (or y not-x), and $\bar{x}\bar{y}$ (or not-x not-y). Now, Venn's interpretation of the proposition "*All X is Y*" is that what it does is not to assure us "as to any one of these classes (for instance xy) being occupied, but to assure us one of them (viz. x not-y) being unoccupied." He goes on to say: "Whether there be any x's or y's we cannot tell for certain, but we do feel quite sure that there are no such things existing as 'x which is not y.'"[22] Thus Venn's interpretation of the existential import of universal propositions has revealed a sharp contrast between algebra of logic and logic in the Aristotelian tradition. If the new viewpoint is to be adopted, the traditional square of opposition and rules of syllogism must be subject to revision.

Charles S. Peirce is very sensitive to the needed revision of Aristotelian logic in respect to the square of opposition, as a result of the new interpretation of existential import of universal propositions. In one of his short papers he holds that the distinction between a universal proposition

and a particular proposition is that the former does not, while the latter does, imply the existence of their subjects.[23] In this interpretation, none of the relations shown in the diagram of the square of opposition in traditional logic are preserved except the two pairs of contradictories. Peirce then revises the truth relations among the A, E, I, and O propositions as follows:

> A and E, All S is P, and No S is P, are true together when no S exists, and false together when part only of the S's are P. I and O, some S is P, some S is not P, are true and false together under precisely the opposite conditions.
>
> A and I, Any S is P, Some S is P, are true together when there are S's all of which are P, and are false together when there are S's none of which are P. E and O, No S is P, and Some S is not P, are true and false together under precisely the opposite circumstances . . . .[24]

In the year of 1905, there appeared in *Mind* a short discussion of the problem of existential import. This discussion begins when Hugh MacColl in a very short note challenges the Boolean logicians to explain this problem.[25] As MacColl interprets the Boolean position, the statement "*Every X is A*," if *X* is interpreted to be non-existent, would entail a contradiction "Every non-existence is existent." Bertrand Russell, one of the important founders of modern symbolic logic, formulates a reply in which he points out that it is necessary to make a distinction between two meanings of "existence."[26] The first meaning which occurs in philosophy and in ordinary life is that which can be predicated of a individual. Russell says, "The entities dealt with in mathematics do not exist in this sense." The second sense which is used in symbolic logic is: "To say that *A* exists means that *A* is a class which has at least one member."[27] MacColl's difficulty, as Russell points out, is the confusion between the two senses of existence.

A quarter century later, there appears another discussion on this topic in another famous philosophical journal, *The Monist*. F. S. C. Northrop presents an article which is entitled "An Internal Inconsistency in Aristotelian Logic.[28] In this article, he has listed the following four propositions:

1. All just acts are expedient acts.
4. All inexpedient acts are unjust.
5. Some unjust acts are inexpedient.
15. All unjust acts are expedient acts.[29]

Then he attempts to prove that, according to the accepted rules of Aristotelian logic, propositions 1 and 15 are both consistent and contradictory. Propositions 1 and 15 are consistent "according to those rules of Aristotelian logic which govern the distribution of terms."[30] But by another, equally acceptable set of rules, they can be shown contradictory.

> Proposition 1 implies proposition 4 according to the rule of conversion by contraposition. Proposition 4 implies proposition 5 according to the rule of conversion by limitation. Therefore, proposition 1 implies proposition 5, according to the rules of the syllogism. But proposition 5 contradicts proposition 15. By

> obversion proposition 5 becomes 'Some unjust acts are not expedient.' Since this is the corresponding O form of proposition 15 which is an A, it follows according to the rules governing the opposition of propositions that proposition 5 contradicts proposition 15. But, since 1 implies proposition 5, and 5 contradicts proposition 15, it follows according to the rules of the syllogism that proposition 1 implies the contradiction of proposition 15.[31]

Northrop concludes that since by one set of rules 1 and 5 are consistent and by another equally acceptable set of rules they are contradictory, then Aristotelian logic is a system which contains an internal inconsistency.

But what is the cause of such an inconsistency? According to Northrop, this fallacy is due to the use of the rule of conversion by contraposition along with other rules of immediate inference.[32] Aristotelian logic, however, can remove the contradiction "by admitting within its science the use of the minimum meaning of the universal proposition." He says, "The rule that the universal proposition always implies a particular will no longer hold. Neither will certain 'valid' moods of the syllogism continue to be valid."[33] What Northrop means is that the notion of an empty class should be admitted and the new interpretation of the existential import of the universal proposition should be introduced, if the inconsistency is to be removed.

Northrop's article elicits criticism from A. P. Uchenko. In his article "Aristotelian Logic and the Logic of Classes" he rejects Northrop's argument because it overlooks the fact "that propositions 1 and 15 *ceased to be consistent* when it is found that they are contradictory.[34] Uchenko's point is that "*consistency is relative to the information at hand.*"[35] Therefore, there is nothing inconsistent in Aristotelian logic; there is nothing which would necessitate an appeal to the empty class.[36] Northrop's reply to Uchenko's criticism is a further clarification of his position in regard to the problem of the existential import of propositions.[37] As to Uchenko's point that "consistency is relative to the information at hand," Northrop would think that this is completely irrelevant to formal principles.

A number of logic texts have devoted a chapter or section to the discussion of this topic. Perhaps the most detailed and systematic treatment has been offered by John Neville Keynes in his *Studies and Exercises in Formal Logic*.[38] Keynes has outlined four views for analysis:[39] (1) Every categorical proposition implies the existence both of its subject and predicate terms and their contradictories. (2) Every proposition implies simply the existence of its subject. (3) No proposition necessarily implies the existence either of its subject or of its predicate. (4) Universal propositions do not imply the existence of their subjects but the particular propositions do. Of the above mentioned positions Keynes is in favor of the fourth which is held by John Venn and Charles Peirce. Keyne's conclusion, however, is that the solution of the problem is to some extent a matter of convention and we are guided "partly by the ordinary usage of language, and partly by considerations of logical convenience and suitability."[40]

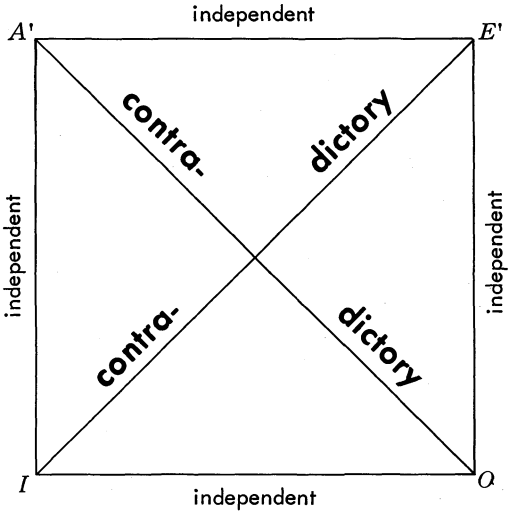
Another text-book writer, W. E. Johnson, in his *Logic*,[41] deals with this problem indirectly. He has constructed five alternative squares of

opposition based on various existential assumptions.[42] To the present writer, his treatment is less systematic than Keynes's and less original than Venn's. His comment, however, on the usage "existential" is instructive. The peculiar use of this term, according to Johnson, has given rise to endless confusion. Consequently, he suggests that this term should be discarded and replaced by some such term as instantial.[43]

When this problem is handed down to Ralph M. Eaton, a great eclectic of logic, the new position held by the modern symbolic logicians is adopted and is applied to the square of opposition. Eaton has revised the traditional square of opposition in the following way:[44]

If anything is S it is P [and Something is S]

If anything is S it is not-P [and Something is S]



Something is S and P

Something is S and not-P

Eaton explains the diagram as follows: The hypothetically interpreted universals are symbolized by A' and E'. The subtracted assumption of instances of the subject is enclosed in square brackets to indicate that it does not get into the meaning of A' and E' correspondingly.[45] "In this scheme," Eaton explains, "A' and I, E' and O do not have the relation of subalternation."[46] That is to say, the truth of the particulars, namely, I and O, does not follow from the truth of the universal propositions, A' and E', correspondingly. Nor does the falsity of the universal propositions follow from the falsity of the particulars.[47] This, indeed, is what Charles Peirce has anticipated.

Those who are symbolic logicians alone and pay no attention to traditional logic, accept universal propositions as non-existential. Those who

are traditional-minded in logic retain the Aristotelian rules. Text-book writers of the last decade, however, still pay considerable attention to this problem. Examples are Bird, Brennan, Cooley, Copi, Leblanc, Lee, Prior, Ruby and Smullyan.[48] The foregoing is sufficient to demonstrate the significance of this problem in contemporary logic.

It is not just a problem in formal logic, however, but also one in scientific explanation. John N. Keynes cites examples from physical science to support the view of the new interpretation.[49] Cohen and Nagel give a very lucid explanation on this point:

> Thus Newton's first law of motion states: All bodies free of impressed forces persevere in their state of rest or of uniform motion in a straight line forever. Will the reader affirm that this proposition asserts the existence of any body which is not under the influence of an impressed force? We need remind him only of the law of gravitation, according to which *all* bodies attract one another. What Newton's first law does assert is the hypothesis that *if* a body were free from impressed forces, it would persevere in its state of rest or in uniform motion in a straight line forever. In the same way, the principle of the lever states what would be the case if the lever were a perfectly rigid body; it does not assert that there is such a body. Indeed, reflection upon the principles of the sciences makes it quite clear that universal propositions in science always function as *hypotheses*, not as statements of fact asserting the existence of individuals which are instances of it.[50]

If Cohen and Nagel are correct, it seems that symbolic logic has superceded the logic in the Aristotelian tradition because it provides a more adequate explanation for scientific method. But the new interpretation of the existential import of propositions has not yet received universal acceptance among logicians and philosophers. On the contrary, it has been challenged severely by an Oxford analyst, P. F. Strawson. In his famous article "On Referring" which is directed primarily to Russell's theory of descriptions, he criticizes the symbolic logicians for their "failure to recognise the special sense in which existential assertions may be implied by the referring use of expressions."[51] He holds that:

> If we interpret the propositions of the schedule as neither positively, nor negatively, nor positively and negatively, existential, but as sentences such that *the questions of whether they are being used to make true or false assertions does not arise except when the existential condition is fulfilled for the subject term*, then all the traditional laws hold good together.[52]

In his *Introduction to Logical Theory*, Stawson has formulated a more detailed and systematic treatment of this problem from the same ordinary language viewpoint.[53] The significance of his criticism of symbolic logicians does not lie in the defense of Aristotelian logic, but in that he points out that the rigid formulations in the logic of classes have led us away from being sensitive to the use of ordinary language.

As Arthur Pap has pointed out, Aristotelian logic was not designed to deal with statements about empty classes. Since counter-factual universal propositions (such as the first law of motion in Newtonian physics) are in-

dispensable to modern science, a more adequate logic is needed. But as far as counterfactuals are concerned, Boolean algebra is equally useless.[54] Pap's insight indicates the significance of, as well as the difficulty involved in, the problem of existential import. As the present author sees it, this problem lies in the gap between logic as pure abstraction and logic as a method applied to existence or human experience. The justification of the applicability of logical forms to existence is still a problem to mathematical logicians. Nevertheless, this is a problem which deserves the immediate attention of scholars in the philosophy of logic. The solution to the problem of existential import presupposes an adequate explanation of the relation between logical forms and existence. This awaits the effort of creative scholars in the field of modern logic.

NOTES

1. Lionel Ruby, *Logic: An Introduction* (2nd ed.; New York: J. B. Lippincott Company, 1960), p. 258.
2. George Boole, *The Mathematical Analysis of Logic* (Cambridge: Macmillan, Barclay, & Macmillan, 1847).
3. I. M. Bocheński, *A History of Formal Logic* (Notre Dame: University of Notre Dame Press, 1961), p. 298.
4. George Boole, *An Investigation of the Laws of Thought, on which are founded the Mathematical Theories of Logic and Probabilities* (New York: Dover Publications; first published, London: Walton and Maberly, 1854), p. 226.
5. Franz Brentano, *Psychologie vom Empirischen Standpunkt* (Leipzig: Duncker & Humbolt, 1874).
6. J. P. N. Land, "Brentano's Logical Innovations," *Mind*, Vol. I, No. 2 (April, 1876), p. 289.
7. *Ibid.*, p. 289.
8. John Venn, *Symbolic Logic* (London: Macmillan and Co., 1881).
9. William and Martha Kneale, *The Development of Logic* (Oxford: The Clarendon Press, 1962), p. 420.
10. The diagram is reproduced as in Venn, *Symbolic Logic*, p. 112.
11. *Ibid.*, Chapter VI, pp. 126-153.
12. *Ibid.*, p. 130.
13. *Ibid.*, p. 130.
14. *Ibid.*, p. 133.
15. *Ibid.*, p. 136.
16. *Ibid.*, p. 137.
17. *Ibid.*, p. 137.
18. *Ibid.*, p. 138.

19. *Ibid.*, p. 144.
20. *Ibid.*, p. 141.
21. *Ibid.*, p. 141.
22. *Ibid.*, p. 141.
23. Charles Sanders Peirce, *Collected Papers*, eds. Charles Hartshorne and Paul Weiss (Cambridge: Harvard University Press, Vols. I-VI, 1931-1935), Vol. II, p. 280.
24. *Ibid.*, Vol. II, p. 283.
25. Hugh MacColl, "Existential Import," *Mind*, New Series, Vol. 14, No. 54 (April, 1905), pp. 295-296.
26. Bertrand Russell, "The Existential Import of Propositions," *Mind*, New Series, Vol. 14, No. 55 (July, 1905), p. 398.
27. *Ibid.*, p. 398.
28. F. S. C. Northrop, "An Internal Inconsistency in Aristotelian Logic," *The Monist*, Vol. 38, No. 2 (April, 1928), pp. 191-210.
29. *Ibid.*, p. 195. The propositions are taken by Northrop from Jevons's *Lessons in Logic*, p. 303.
30. *Ibid.*, p. 204.
31. *Ibid.*, p. 196.
32. *Ibid.*, p. 203.
33. *Ibid.*, p. 204.
34. A. P. Uchenko (Ushenko), "Aristotelian Logic and the Logic of Classes," *The Monist*, Vol. 39, No. 1 (January, 1929), p. 154.
35. *Ibid.*, p. 155.
36. *Ibid.*, p. 155.
37. F. S. C. Northrop, "A Reply Emphasizing the Existential Import of Propositions," *The Monist*, Vol. 39, No. 1 (January, 1929), pp. 157-159.
38. John Neville Keynes, *Studies and Exercises in Formal Logic* (4th ed.; New York: The Macmillan Company, 1906), Chapter VIII, pp. 210-248.
39. *Ibid.*, pp. 219-220.
40. *Ibid.*, p. 235.
41. W. E. Johnson, *Logic* (3 Vols.: New York: Dover Publications, Inc., 1964; first published, Cambridge: Cambridge University Press, 1921).
42. *Ibid.*, Vol. I, p. 138.
43. *Ibid.*, Vol. I, p. 160.
44. This diagram is reproduced as in Ralph M. Eaton, *General Logic: An Introductory Survey* (New York: Charles Scribner's Sons, 1931), p. 224.
45. *Ibid.*, pp. 223-224.

46. *Ibid.*, p. 224.

47. *Ibid.*, p. 224.

48. Otto Bird, *Syllogistic and its Extensions* (Englewood Cliffs: Prentice-Hall, 1964); Joseph G. Brennan, *A Handbook of Logic* (2nd ed.; New York: Harper and Row, 1961); John C. Cooley, *A Primer of Formal Logic* (New York: The Macmillan Company, 1942); Irving M. Copi, *Introduction to Logic* (New York: The Macmillan Company, 1953); Hugues Leblanc, *An Introduction to Deductive Logic* (New York: John Wiley & Sons, Inc., 1954); Harold N. Lee, *Symbolic Logic* (New York: Random House, 1961); A. N. Prior, *Formal Logic* (Oxford: Clarendon Press, 1955); Lionel Ruby, *Logic: An Introduction* (2nd ed.; New York: J. B. Lippincott Company, 1960); Arthur Smullyan, *Fundamentals of Logic* (Englewood Cliffs: Prentice-Hall, 1962).

49. Keynes, *Studies and Exercises in Formal Logic*, pp. 235-236.

50. Morris R. Cohen and Ernest Nagel, *An Introduction to Logic and Scientific Method* (New York: Harcourt, Brace and Company, Inc., 1935), pp. 42-43.

51. P. F. Strawson, "On Referring," *Mind*, New Series, Vol. LIX, No. 235 (July, 1950), p. 343.

52. *Ibid.*, pp. 343-344.

53. P. F. Strawson, *Introduction to Logical Theory* (London: Methuen & Co. Ltd., 1952), Chapter VI.

54. Arthur Pap, *An Introduction to the Philosophy of Science* (New York: The Free Press of Glencoe, 1962), pp. 277-278.

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