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PROPOSITIONAL LOGIC IN THE SIXTEENTH AND EARLY SEVENTEENTH CENTURIES

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Until recently, historians of logic have regarded the early modern period with unremitting gloom. Father Boehner, for instance, claimed that at the end of the fifteenth century logic entered upon a period of unchecked regression, during which it became an insignificant preparatory study, diluted with extra-logical elements, and the insights of men like Burleigh into the crucial importance of propositional logic as a foundation for logic as a whole were lost.[1] Nor is this attitude entirely unwarranted, for the new humanism in all its aspects was hostile to such medieval developments as the logic of terms and the logic of consequences. Those who were devoted to a classical style condemned medieval works as unpolished and arid, and tended to subordinate logic to rhetoric; while those who advocated a return to the original works of Aristotle, freed from medieval accretions, naturally discounted any additions to the subject matter of the *Organon*. But it would be a mistake to dismiss the logical work of the period too readily. In the first place, the writings of the medieval logicians were frequently published and widely read. To cite only a few cases, the *Summulae Logicales* of Petrus Hispanus received no fewer than 166 printed editions;[2] Ockham's *Summa Totius Logicae* was well known; the 1639 edition of Duns Scotus included both the *Grammaticae Speculativae* attributed to Thomas of Erfurt and the very interesting *In Universam Logicam Quaestiones of Pseudo-Scotus*;[3] the *Logica* of Paulus Venetus was very popular; and a number of tracts by lesser known men like Magister Martinus and Paulus Pergulensis were printed. Moreover, since logic still played such a preeminent role in education, contemporary scholars were not backward in producing their own textbooks; and numerous rival schools of logic flourished.[4] The purpose of this paper is to make a preliminary survey of some of the wealth of material available from the sixteenth and first half of the seventeenth centuries, in order to ascertain how much of the medieval propositional logic had in fact been retained.[5] It will become clear that the situation was better than has been thought.

The vocabulary and organization of the textbooks under consideration were fairly standard. The discussion of the proposition [*Enuntiatio*, *Propositio*, or, in Ramist texts, *Axioma*] followed sections on the predica-

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ments and predicables or the Ramist equivalent, on arguments. Medieval logicians had called the compound proposition 'hypothetical', but sixteenth and seventeenth century writers more usually referred to *enuntiatio coniuncta or composita,* sometimes with a note to the effect that it is vulgarly or improperly called 'hypothetical'.[6] Melancthon retained the name 'hypothetical', as did one or two others.[7] The Spanish scholastic, Petrus Fonseca, discussed the whole question in some detail, saying that the name 'hypothetical' most properly applies to conditional propositions, but can also be used of disjunctions, because they imply a conditional.[8] A compound proposition was generally said to consist of two (or more) categorical propositions, joined by one (or more) of a list of propositional connectives. The assumption that the truth of these propositions depended upon the truth of the parts, the kind of connective employed, and in certain cases the relationship between the parts usually remained implicit, but the seventeenth century German logician, Joachim Jungius, said explicitly that truth or falsity depended on "the kind of composition involved";[9] while Alsted had written previously that truth or falsity depended "on the disposition of parts".[10]

There was much agreement as to the kinds of compound proposition to be considered. Conditional, conjunctive, and disjunctive propositions were always mentioned. Those logicians in the scholastic tradition, like Campanella, Cardillus, Fonseca, Hunnaeus and John of St. Thomas, included causal and rational propositions, as did some outside the tradition like C. Martini and Jungius, who discussed the causal proposition at length. Only a few, including Fonseca and C. Martini, mentioned the temporal and local propositions which had been discussed by such medieval logicians as Ockham and Burleigh; but both Ramus and Burgersdijck spoke of 'related' propositions which exhibit 'when' and 'where' among other connectives.[11] Ramus and those influenced by him added a new kind of compound proposition, the discretive.

Although compound propositions were rarely called 'hypothetical', the traditional title of 'hypothetical syllogism' was usually retained for the discussion of propositional inference forms. Only a few spoke of *syllogismus compositus* or *coniunctus*.[12] In all cases the categorical syllogism was discussed before the hypothetical, and usually such matters as sorites, example, enthymeme and induction also came first. A few books had, in addition, a section on the rules for valid inference or *bona consequentia*. Melancthon in his *Erotemata Dialectices* included a chapter entitled *De Regulis Consequentiarum* after his discussion of sorites and before his discussion of the hypothetical syllogism. Alsted placed his canons of material consequence in the same position; while the remarks of Caesarius come after his section on the hypothetical syllogism. On the other hand, the three scholastics, Campanella, Fonseca, and Hunnaeus introduced their rules for good consequence before they discussed the syllogism, thus approaching most closely to the later medieval order of priorities.

Now that a brief indication has been given of the topics discussed, I will first consider what was said about each kind of compound proposition;

and then list the various forms of propositional inference which were known to the logicians of the period. The most important compound proposition was, of course, the conditional, sometimes called *enuntiatio connexa*,[13] or, more rarely, *hypothetica*.[14] As will be seen, it can be argued that some logicians were aware of a truth-functional interpretation of the conditional; but their primary concern was with something more closely analogous to strict implication. A number of medieval logicians had given definitions remarkably similar to those of Lewis and Langford in their explicit use of modal terms. Buridan, for instance, wrote that a necessary condition for the truth of a conditional proposition was that it should not be possible for the antecedent to be true and the consequent false;[15] and parallels can be found in such writers as Petrus Hispanus and Ockham.[16] However, while echoes of this modal definition can be found in the discussion of valid inference offered by logicians like Scharfius, who said:

> in bona consequentia antecedens non potest esse sine consequente;[17]

the favored definitions, so far as conditional propositions were concerned, were either in terms of an inner connexion, or in terms of the incompatibility between antecedent and negated consequent.

This latter definition is to be found in Caesarius, Trapezontius, and Campanella; and I will quote the words of Campanella, who uses the standard terminology:

> conditionalis est vera quando oppositum consequentis repugnat antecedenti.[18]

The significance of the definition hinges on the interpretation of 'repugnat'; and it is clear that the word was intended to have a strong meaning. 'Repugnant' propositions are incompatible; their conjunction is logically false. Trapezontius does not offer an explicit definition, but he remarks that "If he is a man, then he is an animal" is a true conditional because it is not *possible* for someone to be a man and yet not to be an animal.[19] An excellent explicit definition of formal repugnance is offered by the fifteenth century writer, Paulus Pergulensis:

> illa dicuntur formaliter repugnare que nec realiter nec conceptibiliter possunt simul stare absque contraditione manifesta.[20]

Hence one may conclude that Caesarius, Trapezontius and Campanella interpreted the conditional connective as being precisely that of strict implication. It is interesting to note that the very same words were used in the definitions of valid inference offered by Hunnaeus and Fonseca;[21] as well as by the earlier writers, Paulus Venetus (d.1428) and Paulus Pergulensis.[22] There is also a very close similarity to the third kind of Stoic implication, tentatively attributed to Chrysippus.[23]

The most common definition of the conditional proposition was in terms of a connexion between antecedent and consequent, and it was emphasized that the truth of either was not relevant. Burgersdijck wrote:

> in horum nexu consistit veritas, non in veritate ipsius antecedentis aut consequentis;[24]

and he goes on to point out that "Si asinus volat, habet pennas", an example which appeared with monotonous regularity in the textbooks of the time,[25] is true, even though both parts are false. He, like Alsted and Hunnaeus,[26] does not make it at all clear what kind of connexion he was dealing with; but more typical are writers like Fonseca, who said of the conditional that:

> eius veritas in sola consequutione consistit,[27]

and Keckermann, who wrote:

> omnis veritas & vis eius est in partium unione & cohaerentia, qua consequens ex antecedente rectê sequatur.[28]

Even more explicit is John of St. Thomas:

> Ad veritatem conditionalis non requiritur quod aliqua pars sit vera, sed sufficit bonitas consequentiae.[29]

It is clear beyond question that the relationship between antecedent and consequent is one of entailment. Unless the consequent can be validly inferred from the antecedent, the conditional proposition is false; or, in other words, to any true conditional there corresponds a valid inference. Thus, in effect, the distinction between a conditional proposition as one kind of compound proposition and entailment as a metalogical relationship between two propositions, whether simple or compound, has been lost. It is not surprising that in many cases the same definition was offered for an *enuntiatio conditionalis* as for a *bona consequentia.*

Some logicians, notably Caesarius, Campanella, and John of St. Thomas, all scholastics, took the further step of saying explicitly, as many medieval logicians had done, that all true conditionals were necessary and all false conditionals impossible.[30] However, there were divergences of opinion among those who mentioned the subject. Fonseca, for instance, rejected the view that all true conditionals were necessary in the light of two counterexamples. Some conditionals, dealing with future contingents, may be judged to be merely probable, he says; while others, such as "If someone is a mother, she loves her son", are usually but not inevitably true.[31] It is unfortunate that he failed to elaborate upon the matter, for so far he has only managed to weaken, if not contradict, his earlier requirement of entailment, without putting anything in its place.

An unusual interpretation of the conditional proposition, or *axioma connexum*, is offered by Conrad Dietericus, one of the most distinguished of the so-called Philippo-Ramist school. He wrote simply that:

> connexum judicatur verum esse, si posito antecedente est consequens;[32]

a definition which is certainly neutral, and could easily be accepted as truth-functional. He went on to distinguish between necessary conditionals, whose parts "necessaria connexione cohaerent"; and contingent conditionals, in which

> hunc fieri potest, ut antecedens esse possit, etiamsi non sit consequens, vel contra.[33]

Clearly Dietericus was aware of the distinction between strict and material implication.

While Dietericus seems to have been alone in his explicit presentation of the distinction, there are a few other logicians whose discussion shows an implicit awareness of material implication. The two following schemata are useful test cases;

$$p \supset q \equiv \sim p \vee q$$

$$p \supset q \equiv \sim (p \cdot \sim q)$$

since neither biconditional holds if '$\supset$' is replaced by '$\rightarrow$'. The inference form, '$p \vee q, \sim p \therefore q$' was standard; and all the writers we are concerned with would also have accepted '$\sim p \vee q, p \therefore q$', which by conditionalization gives us '$(\sim p \vee q) \supset (p \supset q)$'. In the same way, '$\sim (p \,.\, q), p, \therefore \sim q$' was standard; and Fonseca actually cites '$\sim (p \,.\, \sim q), p \therefore q$',[34] which, again by conditionalization, gives us '$\sim (p \,.\, \sim q) \supset (p \supset q)$'. The crucial issue now is whether conditional propositions were ever defined in terms of a disjunction or a negated conjunction to give us the other half of the biconditional; and indeed they were. Paulus Pergulensis wrote:

> A conditionali affirmativa ad disiunctivam affirmativam factam ex contradictorio antecedentis et consequente eiusdem conditionalis est bona consequentia saltem de materia;[35]

and he was echoed by John of St. Thomas over a century later.[36] On the other hand, Fonseca tells us that:

> . . . sunt copulativae, quae inferantur ex conditionalibus: ut si ex hac conditionali, Si Socrates est homo, est animal, colligas hanc copulativam negativam, Non & Socrates est homo & non est animal.[37]

Moreover, both Freigius and Jungius remarked that the inference form, '$\sim (p \,.\, q), \sim p, \therefore q$', owed its force to the equivalence between the negated conjunction and the two conditionals '$p \supset \sim q$' and '$q \supset \sim p$'.[38] Whether these logicians were aware of the significance of what they were doing is another matter. Fonseca certainly was not, for in one place he explicitly denied the validity of the biconditional '$p \supset q \equiv \sim (p \,.\, \sim q)$'.[39] In the context of his definition of the conditional as involving a *consequutio* or relationship of entailment, he was perfectly right; but he had failed to notice the relationship between '$\sim (p \,.\, \sim q), p, \therefore q$' and '$\sim (p \,.\, \sim q) \supset (p \supset q)$'. Had he done so, he would have been forced to distinguish between strict and material implication.

Judging by the examples the textbooks gave both of true conditionals and of valid inferences, it was usually assumed that the connexion between antecedent and consequent, or between premisses and conclusion, was one not only of abstract logical entailment, but of meaning. However, a few logicians followed the commonplace observation that a true proposition implies only true propositions, whereas a false proposition may imply both true and false propositions,[40] with a reference to the paradoxes of modal implication. Melancthon noted that anything follows from an impossible proposition;[41] and Fonseca both that anything follows from an impossible proposition and that anything implies a necessary proposition.[42] A rare example of a discussion of valid inference, including the paradoxes, along the sophisticated lines of the later medieval logicians is afforded us by Augustinus Niphus, a sixteenth century Italian who wrote a commentary on the *Prior Analytics* of Aristotle. It seems worthwhile to present a brief outline of his arguments.[43]

In his attempt to define a valid inference, Niphus takes as his starting point the view of the *Neoterici* [recent writers, who are opposed to the more ancient] that the necessary and sufficient conditions for a valid inference are that it should be impossible for the antecedent to be true and the consequent false.[44] Against this definition, he produces two paradoxical counter-examples. The first, "Omnis propositio est affirmativa, ergo nulla propositio est negativa", he deals with by imposing the condition that the consequent should be "simul formatum cum antecedente". The second, "Deus, est, ergo haec consequentia non valet", he deals with by imposing the additional condition that "consequentis significatio non destruat proprietatem notae illationis." He goes on to distinguish between formal implication, also called *bona per se*, where validity depends on the disposition of terms alone, and material implication, also called *bona per accidens*, where validity depends not only upon the disposition of terms, but upon the terms themselves.[45] Material implication has two subdivisions, material implication *simpliciter*, which becomes formally valid through the addition of a necessary premiss, and material implication *ut nunc*, which becomes formally valid through the addition of a true premiss. Finally he shows firstly that anything follows from an impossible proposition, by proving that "Socrates is and Socrates is not" entails "Man is a horse"; and secondly, that anything follows from a false proposition by proving that "Man is a horse; therefore you are at Rome".[46] In each case his argument follows the same pattern:

$$p \,.\, {\sim} p \,.\!\therefore {\sim} p$$

$$p \,.\, {\sim} p \,.\!\therefore p$$

$$p \,.\!\therefore p \vee q$$

$$(p \vee q) \,.\, {\sim} p \,.\!\therefore q$$

$$\text{Hence: } p \,.\, {\sim} p \,.\!\therefore q$$

However, while it is exciting to read this discussion in a sixteenth century text, one cannot claim that Niphus was original. His arguments on the definition of valid inference are closely parallel to those of Pseudo-Scotus, although one of the three paradoxes discussed by the latter is omitted;[47] the division of *consequentiae* into formal and material was standard; and the proof that '$p \,.\, {\sim} p \,.\!\therefore q$' is found not only in Pseudo-Scotus,[48] but in Ockham, Buridan, and Albert of Saxony, among others.[49]

Before we pass on to other kinds of compound proposition, it is appropriate to say a word about the biconditional. The higher order relationship of logical equivalence was frequently discussed, and Joachim Jungius is noteworthy for the number of examples he gives of *aequipollentia*, but only Crellius gives a clear account of the biconditional as a propositional connective. He does not, it is true, introduce any new vocabulary. Instead, he contrasts the proposition "Si sol ortus est, dies est" with "Si homo est, animal est".[50] The second is a case of *consecutio imperfecta*, for it can be used as a premiss in only two valid inferences, *modus ponens* and *modus tollens*; but the first is a case of *consecutio perfecta*, giving rise not only to *modus ponens* and *modus tollens*, but to the inferences from consequent to antecedent and from negated antecedent to negated consequent.

The four other non-truth-functional connectives to be found in logical textbooks are the casual, rational, temporal and local. I shall not discuss the last two, as very little was said about them and they are not of any interest. Nor shall I go into great detail over the first two. Rational propositions, according to the Roman grammarian Priscian, are those compound propositions which exhibit connectives like *ergo*, *igitur* and *itaque*;[51] and this definition is echoed by those who discuss the matter, like Fonseca and Campanella. They remark that a rational proposition is like a conditional, save for the added requirement that both parts be true.[52] Causal propositions, which exhibit connectives like *quia*, must meet the same conditions as rational propositions, with the further, extra-logical, requirement that a genuine causal relationship must be involved.[53]

We now come to the purely truth-functional connectives, beginning with the copula. There is not much to be said about the truth conditions for the conjunction, since everyone realized that it was true if and only if all the parts were true. Jungius went further, and remarked that a conjunction remained true, no matter what the order of parts.[54] He made the same statement about the disjunction,[55] thus making explicit the two laws of commutativity which, according to E. A. Moody, were never more than implicit in medieval logic.[56] More interesting is the general attitude to the use of the conjunction in inference. A number of logicians, mainly scholastics, recognized the inference from a conjunction to one of its parts;[57] but it was regarded very dubiously. Melancthon in an early work felt that one should argue '$p \,.\, q,\ p,\ \therefore q$', but said he wasn't entirely sure about the merits of this.[58] Caesarius reported that one can argue vulgarly that '$p \,.\, q \therefore p$' or that '$\sim p \therefore \sim(p \,.\, q)$, but added that this was not useful.[59] Fonseca thought that '$p \,.\, q \therefore p$' was an enthymeme, and should be expanded into '$(p \,.\, q) \supset p,\ p \,.\, q,\ \therefore p$'.[60] Horstius concluded that nothing could be done with '$p \,.\, q$' as a premiss, since one had already said all one wanted to say;[61] and C. Martini preferred to argue that if p and q are true then either is true, p and q are true, therefore p is true and q is true.[62] Much more generally accepted was the inference from a negated conjunction and the assertion of one of its parts to the negation of the other; though both Willichius and Trapezontius mistakenly argued that '$\sim(p.\sim q),\ q \therefore \sim p$'.[63]

The most interesting of the truth-functional connectives to be discussed is the disjunction. Unlike the medieval logicians, who had usually accepted the weak disjunction,[64] most logicians of the period in question preferred the strong disjunction, by which a disjunctive proposition is true if and only if one part alone is true. The reason usually given for this preference was that the parts of a disjunction should be opposed to one another, or incompatible. A disjunction should, that is, be *inter dissentanea*[65] or *inter pugnantia*.[66] C. Martini added that a true disjunction must list all the possibilities. It is no use saying that "Either Peter is white or Peter is black", because he may also be red, blue, or green.[67] Some authors discussed both weak and strong disjunction, remarking that the *veteres*, including Cicero and Boethius, preferred the latter, but the *recentiores* or *iuniores* the former.[68] Fonseca, whom C. Martini quotes with approval on this point, felt that the ancients were closer to the true nature of a disjunction, but that the more recent writers were closer to common speech.[69] Hunnaeus quoted Virgil to show that the weak disjunction was acceptable;[70] but he used only the strong disjunction. A few writers, including Campanella, John of St. Thomas, and Melancthon, gave only the weak disjunction;[71] and Jungius is conspicuous for the fact that he distinguishes with great care between the two kinds of disjunction, and seems to accept them on an equal footing. Like Priscian, he calls the strong disjunction *disjunctiva* and the weak disjunction *subdisjunctiva*;[72] and he points out that it is often very difficult to tell with which one is dealing.[73]

The fact that most people accepted the strong disjunction had an effect on the kinds of inference forms and equivalences which were accepted. De Morgan's laws, for instance, depend upon the weak disjunction, and only Campanella and Fonseca of the writers I have studied include either or both of them.[74] Nor was the inference form '$p \therefore p \vee q$' much used, although Campanella, John of St. Thomas and Niphus all refer to it.[75] That strong disjunctive propositions are equivalent to the four conditionals '$p \supset \sim q$', '$\sim p \supset q$', '$q \supset \sim p$', and '$\sim q \supset p$' was spotted by Alsted, Burgersdijck, Fonseca and Jungius, who all used the traditional example "Aut dies est aut nox".[76] Jungius also gave the two conditionals to which a weak disjunctive proposition is equivalent.[77] The standard inference forms given in every text are either '$p \vee q, \sim p, \therefore q$' or '$p \not\equiv q, \sim p, \therefore q, p, \therefore \sim q$', or both, in the case of Jungius.

A further propositional connective is introduced by Jungius under the name of '*posterior subdisjunctiva*.'[78] He describes it as a disjunction whose parts cannot both be true, but yet can both be false; and this is clearly equivalent to non-conjunction or: '$p|q =_{def} \sim(p \,.\, q)$.' He can, moreover, be credited with a knowledge of non-disjunction, or '$p{\downarrow}q =_{def} \sim(p \vee q)$', through his use of '*neque—neque*'. This form of words also appears in Burgersdijck; and Erastus lists the following inference forms:[79]

$(p \not\equiv q) \not\equiv r, p \therefore$ neque q neque r

$(p \not\equiv q) \not\equiv r, p,$ Non igitur vel q vel p.

The last kind of compound proposition to be discussed is the discretive proposition, which exhibits such connectives, as *tamen*, *quamquam*, *quamvis* etc. The standard example was "Non formosus erat, sed erat facundus Ulysses".[80] The parts of a discretive proposition are said to be opposed, but the distinction is one of reason alone,[81] so that, unlike the strong disjunction, all parts can, and indeed must, be true. The truth conditions were laid down most clearly by Jungius, who said that a discretive proposition was true in so far as it included both an affirmative conjunction and a negated conditional.[82] This latter requirement makes it plain that the proposition cannot be given a completely truth-functional interpretation, for an affirmative conjunction implies an affirmative material conditional. According to Scharfius, these propositions were discussed only by Ramus and those influenced by him;[83] but obviously the accounts of discretive propositions echo Priscian's section on *adversativae*;[84] and Burgersdijck uses this word rather than *discretivae*.

Finally, I will list some of the rules of inference to be found in the textbooks of the period. The five Stoic 'indemonstrables' are frequently found, although Burana's attribution of them to Chrysippus is rare.[85] They are:

1. $p \supset q,\ p,\ \therefore\ q$
2. $p \supset q,\ \sim q,\ \therefore\ \sim p$
3. $p \not\equiv q,\ p,\ \therefore\ \sim q$
4. $p \not\equiv q,\ \sim p,\ \therefore\ q$
5. $\sim(p\ .\ q),\ p,\ \therefore\ \sim q$

Hunnaeus, Sturm, C. Martini and Ramus in his *Dialectique* give these five rules alone; some, like Alsted, Burgersdijck, Dietericus and Polanus omit rule 5. Others add extra rules. Caesarius, Trapezontius and Willichius, for instance, have '$\sim(p \supset q),\ p,\ \therefore\ \sim q$'; while elaborations on 3 and 4 are to be found in Burgersdijck, Dietericus, Erastus and Jungius. Jungius also includes such equivalences as:

$$(p \mid q) \mid r \equiv p \supset (q \mid r)$$

and

$$(p \vee q) \supset r \equiv (p \supset r)\ .\ (q \supset r);$$ [86]

the latter being particularly worthy of note.

Those who discuss the rules for valid inference give us some or all of the following set:[87]

1. Ex vero non nisi verum, verum autem tum ex vero, tum ex falso colligitur.

2. Ex falso & falsum & verum, falsum autem non nisi ex falso concluditur.

3. Ex necessario non nisi necessarium, necessarium autem ex quolibet. . . .

4. Ex contingenti nunquam colligitur impossible, sed vel necessarium, vel contingens: contingens autem nunquam ex necessario, sed vel ex contingenti, vel impossibili concluditur.

5. Ex impossibili sequitur quodlibet.... impossible autem non nisi ex impossible colligitur.

6. Quicquid stat cum antecedente stat cum consequente:

$$p \supset q, (p . r), \therefore (q . r)$$

7. Quicquid repugnat consequenti, repugnat antecedenti:

$$p \supset q, \sim(q . r) \quad \therefore \sim(p . r)$$

8. In bona consequentia, ex opposito contradictorio consequentis infertur contradictorium antecedentis:

$$p \supset q, \sim q, \therefore \sim p$$

9. Ex quocunque sequitur antecedens, sequitur consequens:

$$p \supset q, \; r \supset p, \therefore r \supset q$$

10. Quicquid sequitur ex consequente, sequitur ex antecedente:

$$p \supset q, q \supset r, \therefore p \supset r$$

It is indeed true that the logicians of the sixteenth and early seventeenth centuries failed to appreciate the fundamental importance which the logicians of the later middle ages had attributed to propositional logic; and a number of the texts I have been concerned with even give instructions for the reduction of hypothetical syllogisms to categorical syllogisms.[88] On the other hand, the amount of propositional logic retained was by no means negligible, and some authors, such as Fonseca and Jungius, included a great deal. No startling advances were made, but there were innovations in detail, like Jungius's discussion of the *posterior subdisjunctiva*, or the linking of the conditional with a negated conjunction. One may therefore conclude that, while the period is not one of great excitement for the historian of logic, it merits considerably more attention than it has been granted in the past.

NOTES

1. See P. Boehner, "Bemerkungen zur Geschichte der De Morgannsche Gesetze in der Scholastik," *Archiv für Philosophie*, 4 (1951), p. 145.

2. See J. P. Mullally, *The Summulae Logicales of Peter of Spain* (Notre Dame, Indiana, 1945), p. lxxviii.

3. In Joannes Duns Scotus, *Opera Omnia*, edited by L. Wadding (Lugduni, 1639), Vol. I.

4. For a comprehensive account of the various schools of logic, see Dr. Wilhelm Risse, *Die Logik der Neuzeit. I. Band 1500-1640*, (Stuttgart-Bad Cannstatt, 1964).

5. I have limited myself to material in the British Museum and the Cambridge University Library for the purposes of this introductory survey.

6. Cf. Thomas Campanella, *Philosophiae Rationalis Partes quinque. 2. Dialectica* (Parisiis, 1638), p. 334; Augustinus Hunnaeus, *Dialectica seu generalia logices praecepta omnia* (Antverpiae, 1585), p. 147; and Amandus Polanus, *Logicae libri duo* (Basileae, 1599), p. 147.

7. Philippus Melancthon, *Erotemata Dialectices*, (---, 1540?), p. 96. Cf. Johannes Caesarius, *Dialectica* (Coloniae, 1559), Tract. IV [No pagination]; and Cornelius Martini, *Commentatiorum logicorum adversus Ramistas* (Helmstadii, 1623), p. 204.

8. Petrus Fonseca, *Institutionum Dialecticarum libri octo* (Conimbricae, 1590), Vol. I, p. 173. Cf. Abelard's discussion of the same point in his *Dialectica*, edited by de Rijk (Assen, 1956), p. 488.

9. Joachim Jungius, *Logica Hamburgensis*, edited by R. W. Meyer (Hamburg, 1957), p. 98. "[Enuntiatio conjuncta] . . . secundum illam compositionis speciem, veritatis et falsitatis est particeps".

10. J. H. Alsted, *Logicae Systema Harmonium* (Herbonae Nassoviorum, 1614), p. 321. "Compositi axiomatis veritas & necessitas pendet specialiter ex partium dispositione".

11. Petrus Ramus, *Dialecticae libri duo* (Parisiis, 1560), p. 126; and Franco Burgersdijck, *Institutionum Logicarum libri duo*, (Lugduni Batavorum, 1634), pp. 166-167.

12. E.g., Fonseca, *op. cit.*, vol. II, p. 100, refers to "syllogismus coniunctus"; and Polanus, *op. cit.*, p. 165, refers to "syllogismus compositus".

13. E.g., Jungius, *op. cit.*, p. 98; Alsted, *loc. cit.*, uses the word "connexivus".

14. See Burgersdijck, *op. cit.*, p. 165; and Fortunatus Crellius, *Isagoge Logica* (Neustadii, 1590), p. 117.

15. Buridan, *Tractatus Summularum*, quoted by Philotheus Boehner in *Collected Articles on Ockham*, edited by E. M. Buytaert (St. Bonaventure, N. Y., Louvain, Paderborn, 1958), p. 332, n. 8. "Ad veritatem conditionalis requiritur quod antecedens non possit esse verum sine consequente".

16. Petrus Hispanus, *Summulae Logicales*, edited by I. M. Bocheński (Turin, 1947), p. 8. "Ad veritatem conditionalis exigitur quod antecedens non possit esse verum sine consequens". Cf. William Ockham, *Tractatus minor logicae* [attributed] quoted by Boehner, *Collected Articles*, pp. 325-326.

17. Johannes Scharfius, *Institutiones Logicae* (Wittenbergae, 1632), p. 490. Cf. John of St. Thomas, quoted by Alberto Moreno in "Logica Propositional en Juan de Sante Tomás", *Notre Dame Journal of Formal Logic*, 4, (1963), p. 132, n. 62. Cf. also Ockham and Pseudo-Scotus.

18. Campanella, *op. cit.*, p. 333. Cf. Caesarius, *loc. cit.*; and Georgius Trapezontius, *Dialectica brevis* (Coloniae, 1526) [No pagination] "De propositione conditionali".

19. Trapezontius, *loc. cit.* ". . . non enim potest homo esse, cum animal non sit".

20. Paulus Pergulensis, *Compendium perclarum ad introductionem iuvenum in facultate logice* (Venetiis, 1486), Tract IV [No pagination].

21. Fonseca, *op. cit.*, Vol. II, p. 6; and Hunnaeus, *op. cit.*, p. 231.

22. Paulus Venetus, *Logica* (Venetiis, 1559), p. 27v^{o}; and Paulus Pergulensis, *loc. cit.* The latter repeats the words of his definition of the conditional in Tract. I.

23. "And those who introduce the notion of connexion say that a conditional is sound when the contradictory of its consequent is incompatible with its antecedent". Sextus Empiricus, quoted by W. & M. Kneale in *The Development of Logic* (Oxford, 1962), p. 129.

24. Burgersdijck, *op. cit.*, p. 165.

25. It is also found in Burleigh, *De Puritate Artis Logicae Tractatus Longior*, edited by Philotheus Boehner (St. Bonaventure, N. Y., Louvain, Paderborn, 1955), p. 79; and the Stoic, Philo, had "If the earth flies, then the earth has wings". See Kneale, *op. cit.*, p. 130.

26. Alsted, *op. cit.*, p. 321; and Hunnaeus, *op. cit.*, p. 176.

27. Fonseca, *op. cit.*, vol. I, p. 176. Cf. vol. II, p. 1. "Consequentia . . . sive consequutio". The word "consequutio" (or "consecutio") used to express the relationship of entailment is also found in Jacobus Martini, quoted by Risse, *op. cit.*, p. 504, n. 353; Crellius, *op. cit.*, p. 239; and Gasparus Cardillus Villalpandeus, *Summa Summae Summularum* (Madrid, 1615), p. 151 v^{o}. Cf. Abelard, *op. cit.*, p. 271.

28. Bartholomaeus Keckermann, *Systema Logicae* (Hanoviae, 1606), p. 365. Cf. Cardillus, *op. cit.*, p. 151; Polanus, *op. cit.*, p. 33; and Jungius, *op. cit.*, p. 99.

29. Quoted by Ivo Thomas in "Material Implication in John of St. Thomas", *Dominican Studies*, 3 (1950), p. 180.

30. Caesarius, *op. cit.*, Campanella, *op. cit.*, p. 333; John of St. Thomas, quoted by Moreno, *op. cit.*, p. 131, n. 60.

31. Fonseca, *op. cit.*, vol. I, p. 177.

32. Conrad Dietericus, *Institutiones Dialecticae* (Giessae Hassorum, 1655), p. 242.

33. Dietericus, *op. cit.*, p. 243.

34. Fonseca, *op. cit.*, vol. II, p. 103.

35. Paulus Pergulensis, *loc. cit.*

36. Quoted by Ivo Thomas, *loc. cit.*

37. Fonseca, *op. cit.*, vol. I, p. 173.

38. J. T. Freigius, *Logica, ad usum rudiorum in epitomen redacta* (---, 1590) [No pagination] "De syllogismo composito"; Jungius, *op. cit.*, p. 157.

39. Fonseca, *op. cit.*, vol. I, pp. 201-202.

40. See Alsted, *op. cit.*, p. 384; Caesarius, *op. cit.*, Tract. VI; Cardillus, *op. cit.*, p. 15; Crellius, *op. cit.*, p. 233; Dietericus, *op. cit.*, p. 266; Fonseca, *op. cit.*, vol. II, pp. 15-16; and Hunnaeus, *op. cit.*, p. 235.

41. Melancthon, *op. cit.*, p. 174.

42. Fonseca, *op. cit.*, vol. II, pp. 16-19.

43. Augustinus Niphus, *Super libros priorum Aristotelis* (Venetiis, 1554), pp. 11-12.

44. Niphus, *op. cit.*, p. 11v^{o}, col. I. ". . . ad bonitatem consequentiae requiritur & sufficit, quod impossibile est antecedens esse verum, & consequens falsum".

45. *Loc. cit.*, col. II. "... per materialem vero intelligunt, quando servata simili dispositione terminorum, non in omnibus terminis est bona".

46. Niphus, *op. cit.*, p. 12, col. I.

47. Pseudo-Scotus, *op. cit.*, pp. 286-287. Niphus omits the discussion of "Nulla chimaera est hircocerus; igitur homo est asinus", whose significance seems rather obscure.

48. Pseudo-Scotus, *op. cit.*, p. 288, pp. 333-334.

49. See I. M. Bocheński, *A History of Formal Logic*, translated and edited by Ivo Thomas (Notre Dame, Indiana, 1961), pp. 108-205. See also, e.g., *Parvorum Logicalium* pp. 287-287v^{o} in *Petri Hispani Summulae Logicales cum Versorii ... expositione* (Venetiis, 1568) [Bryn Mawr College Library].

50. Crellius, *op. cit.*, p. 240.

51. Priscian, *Institutionum Grammaticarum libri XVIII*, in *Grammatici Latini*, edited by H. Kiel (Leipsig, 1855) Vol. III, p. 100.

52. Fonseca, *op. cit.*, vol. I, p. 178; and Campanella, *op. cit.*, p. 333.

53. Fonseca, *loc. cit.*, Cf. Hunnaeus, *op. cit.*, p 177; and Jungius, *op. cit.*, p. 103.

54. Jungius, *op. cit.*, p. 102. "Eadem enim manet eius sive veritas sive falsitas, quocunque ordine membra proponantur . . ."

55. Jungius, *op. cit.*, p. 104.

56. E. A. Moody, *Truth and Consequence in Medieval Logic* (Amsterdam, 1953), p. 86.

57. Campanella, *op. cit.*, p. 379; Cardillus, *op. cit.*, p 156v^{o}; Crellius, *op. cit.*, p. 242; Melancthon, *op. cit.*, p. 177; Trapezontius, *op. cit.*

58. Melancthon, *Compendaria Dialectices ratio* (Coloniae, 1522), Bk. IV [No pagination].

59. Caesarius, *op. cit.*, Tract. VI.

60. Fonseca, *op. cit.*, Vol. II, p. 103.

61. Gregorius Horstius, *Institutionum Logicarum libri duo* (Witebergae, 1608), p. 318.

62. C. Martini, *op. cit.*, pp. 366-367.

63. Jodocus Willichius, *Erotematum Dialecticae libri tres* (Argentorati, 1540), p. 191. Cf. Trapezontius, *op. cit.*

64. For instance, William of Shyreswood accepts both, Petrus Hispanus is hesitant, and Ockham, Burleigh and Albert of Saxony accept the weak disjunction.

65. E.g., Polanus, *op. cit.*, p. 151; Dietericus, *op. cit.*, p. 246. Cf. Burgersdijck, *op. cit.*, p. 166.

66. Cardillus, *op. cit.*, pp. 160-161. Cf. Caesarius, *op. cit.*, Tract. IV.

67. C. Martini, *op. cit.*, p. 263.

68. E.g., Caesarius, *loc. cit.*

69. Fonseca, *op. cit.*, vol. I, pp. 182-183. C. Martini, *loc. cit.*

70. Hunnaeus, *op. cit.*, p. 176.

71. Campanella, *op. cit.*, p. 380; Melancthon, *Dialectices libri IIII* (Lugduni, 1634), p. 132; John of St. Thomas, quoted by J. J. Doyle, "John of St. Thomas and Mathematical Logic", in *The New Scholasticism* 27 (1953), p. 6.

72. Priscian, *op. cit.*, pp. 97-98. Jungius, *op. cit.*, pp. 104-105.

73. Jungius, *op. cit.*, p. 105.

74. Campanella, *op. cit.*, p. 380. '$\sim p \vee \sim q \equiv \sim(p \cdot q)$'. Fonseca, *op. cit.*, vol. I, p. 203, gives:

$$\begin{aligned} \sim(p \cdot q) &\equiv \sim p \vee \sim q \\ \sim(\sim p \cdot \sim q) &\equiv p \vee q \\ \sim(p \vee q) &\equiv \sim p \cdot \sim q \\ \sim(\sim p \vee \sim q) &\equiv p \cdot q \end{aligned}$$

75. Campanella, *op. cit.*, p. 380; John of St. Thomas, quoted by Doyle, *op. cit.*, p. 7.

76. Alsted, *op. cit.*, p. 358; Burgersdijck, *op. cit.*, p. 303; Fonseca, *op. cit.*, vol. I, p. 173; Jungius, *op. cit.*, p. 104.

77. Jungius, *op. cit.*, p. 105.

78. Jungius, *loc. cit.*

79. Thomas Erastus, *Ratio Formandorum Syllogismorum* (Basileae, 1565), pp. 52-53.

80. E.g., Dietericus, *op. cit.*, p. 244; Burgersdijck, *op. cit.*, p. 166; Scharfius, *op. cit.*, p. 419.

81. Dietericus, *loc. cit.*

82. Jungius, *op. cit.*, p. 110.

83. Scharfius, *op. cit.*, p. 418. Cf. Ramus, *La Dialectique en deux livres* (Paris, 1576), p. 43vo.

84. Priscian, *op. cit.*, p. 99. Cf. Burgersdijck, *loc. cit.*

85. J. F. Burana, *Aristotelis Priora Resolutoria* (Parisiis, 1589), p. 119.

86. Jungius, *op. cit.*, p. 102. "Copulata ex duabus connexis constare potest, ut Rom. 14.8. *Sive vivimus, sive morimur, Domini sumus*, hoc est, *Et, si vivimus Domini Sumus, et, si morimur, Domini sumus*."

87. The quotations are from Fonseca, *op. cit.*, Vol. II, Bk. VI, Ch. 5, pp. 15-22. The chapter is entitled "*Regulae Generales Consequentiarum*". For other references to (1)-(5) see above, notes 40-43. (6) is found in Alsted, *op. cit.*, p. 385; (7) in Alsted, *loc. cit.*, and John of St. Thomas, quoted by Moreno, *op. cit.*, p. 133, n. 73; (8) in Caesarius, *op. cit.*, Tract. VI, Hunnaeus, *op. cit.*, p. 230, and John of St. Thomas, *loc. cit.*, n. 75; (9) in Alsted, *loc. cit.*, Burgersdijck, *op. cit.*, p. 298, Campanella, *op. cit.*, p. 374, and Ramus, *Dialecticae*, p. 179; and (10) in Caesarius, *loc. cit.*, Campanella, *loc. cit.*, Dietericus, *op. cit.*, p. 275, Erastus, *op. cit.*, pp. 52-53, John of St. Thomas, *loc. cit.*, n. 72, Jungius, *op. cit.*, p. 225, and Melancthon, *Erotemata Dialectices*, p. 163.

88. E.g., Dietericus, *op. cit.*, p. 312; Crellius, *op. cit.*, pp. 243-246; and Jungius, *op. cit.*, *passim*.

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