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REDUCTIO-AD-ABSURDUM: A FAMILY FEUD BETWEEN COPI AND SCHERER

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Scherer's verdict, in his article, "The form of reductio-ad-absurdum" (*Mind*, April 1971), that Copi's account of reductio[1] is confusing, is based on the contention that Copi's form of reductio fails to manifest the essential basis upon which a reductio is conceived to rest, and that it is given a form which is less than intuitive and, in fact, is both epistemologically and formally impossible.[2] Scherer develops an alternative formulation* which, he claims, is an adequate manifestation of reductio, free from Copi's epistemological errors and formal fallacies. It would be too ambitious on my part to claim that I have understood Copi perfectly, but, it is my plea, that his formulation of reductio may lend itself to an interpretation whereby his formulation appears more true to the spirit and form of reductio than that of Scherer.

Both Scherer and Copi agree that the contradiction is central to their formulations of reductio, i.e., both accept $r.-r$ as a valid derivation from $-q$, the negation of the original conclusion and p, the original premiss, which means that they regard $r.-r$ as being meaningful. Scherer's use of $-(r.-r)$ as a premiss in his formulation means that he considers its negation $r.-r$ well formed and thus, meaningful. That the contradiction is false, also receives emphasis in both the formulations. Then, why this family feud?

Scherer's grievance seems to be that Copi's form fails to manifest the essential basis of reductio, the logical falsehood of the contradiction,

*For the sake of avoiding confusion, I have adopted Scherer's symbolic notations in all the formulations.

1. Irving M. Copi, *Symbolic Logic*, Macmillan Company, New York, Second Edition (1965), pp. 61-66.

2. Donald Scherer, "The form of reductio-ad-absurdum," *Mind*, Vol. LXXX, No. 318 (1971), p. 247.

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just because his form at no point involves the denial of the contradiction, the line, -(*r*.-*r*), simply does not occur.[3] For Scherer, the logical falsehood of the contradiction, as expressed by the denial of *r*.-*r*, i.e., in the form of -(*r*.-*r*), is indispensable to any formulation of reductio.

But, should one always deny the contradiction to assert its falsehood? Usually, to assert the falsity of a proposition, e.g., *p*, one has to deny it, i.e., -*p* has to occur. But, the falsehood of the contradiction seems to lend itself also to another type of expression. The logical make-up of the contradiction makes it impossible that it be true, but, should this impossibility be expressed only by an explicit denial? Instead of accommodating -(*r*.-*r*) cannot it be accepted that "a contradiction can never be true," by recognising that *r*.-*r* is always false?

1	2	3
r	-*r*	.
T	F	F
F	T	F

Acceptance of "F" in column 3 is the recognition that the contradiction is tautologously false. Scherer is aware of this type of recognition of the logical falsehood of *r*.-*r* when he accepts that an adequate demonstration could be developed on the basis of the tautological character of (*r*.-*r*) ⊃ *q*, because this is a function of the fact that its antecedent, *r*.-*r*, is always false.[4] Yet, his criticism is based on the assumption that *r*.-*r* has to be denied, that -(*r*.-*r*) has to occur. This is understandable. Scherer, using the conditional proof, derives -*q* ⊃ (*r*.-*r*), which means that the supposition of -*q* leads him to the predicament of having to accept the truth of the contradiction, *r*.-*r*. Hence, to escape from this, he resorts to the usual method of asserting a proposition false by denying it. On the other hand, he is unable to detect the occurrence of -(*r*.-*r*) in Copi's formulation and hence concludes that Copi accepts the contradiction. But, Copi's account contains two formulations of reductio, presented so much together, that they appear as one. The first one ends abruptly with line 13, *r*.-*r*, and after that, instead of completing it, he introduces a comment, "Here line 13 is an explicit contradiction, so the *demonstration is complete*, for the validity of the original argument follows by the *rule of Indirect Proof*."[5] This rule, mentioned by him when he cites Euclid's use of reductio, stipulates that if the assumption of what one wants to prove leads to a contradiction or 'reduces to an absurdity,' then *that assumption is false* and so its negation is true.[6] When this rule is applied, the falsehood of the assumption, which

3. *Ibid.*, p. 249.

4. *Ibid.*, p. 251.

5. Copi, *op. cit.*, p. 61.

6. *Ibid.*, p. 61

leads to the contradiction, is derived through the denial of the contradiction, i.e., through $-(r.-r)$. Hence he denies the contradiction and on this basis concludes that the assumption, $-q$, which leads to the contradiction, is false. Then, through the use of Double Negation, he establishes q.

Copi could avoid confusion if he continued his formulation to derive q, instead of commenting on the manner of reaching such a derivation. The completion of his formulation (as found on page 61), using his comment and his citation of Euclid's use of reductio, would be somewhat like this.

→4 $-q$
5 .
6 .
7 .
8 .
9 .
10 .
11 r
12 $-r$
13 $r.-r$

14 $-q \supset (r.-r)$ . . . . C.P.
15 $-(r.-r)$
16 $--q$ . . . . M.T.
17 q . . . . D.N.

This is in substance Scherer's formulation. In this formulation $-(r.-r)$ occurs very much. But Copi develops an alternative formulation which deviates from lines 11 and 12 but not from line 13. Lines 14 and 15 have to be viewed, not as a continuation from line 13, but as a deviation from lines 11 and 12, presenting an alternative formulation.

Copi's First Formulation
4 $-q$
5 .
6 .
7 .
8 .
9 .
10 .
11 r ⟶
12 $-r$
13 $r.-r$
14 His comment after line 13 and his citation of Euclid
15 $\therefore q$

Copi's Alternative Formulation
⟶ r
$-r$
$r.-r$
r

$r \vee q$ 11, Add.
$-r$ 12
$\therefore q$ D.S.

The alternative formulation is conceived as a method of proof which proceeds through the contradiction to the conclusion of the original argument.[7] Here Copi does not find the need for -(*r*.-*r*), for he recognises the tautological falsehood of *r*.-*r*. But, it is Scherer's contention that Copi supposes the truth of *r* and -*r* conjunctively and separately and hence, he cannot recognize the tautological falsehood of *r*.-*r*. Scherer's representation stands in need of correction. Perhaps, his misrepresentation is because he views Copi's formulation in lines 14 and 15 as a continuation from line 13 and not as an alternative formulation.

Copi's form[8]			Scherer's representation[9]		
11	*r*		1	*r*.-*r*	
12	-*r*		2	*r*	Simp.
13	*r*.-*r*	Conj.	3	-*r*.*r*	Comm.
14	*r* v *q*	11, Add.	4	-*r*	Simp.
15	*q*	12, 14, D.S.	5	*r* v *q*	Add.
			6	*q*	D.S.

Scherer's representation gives one the impression,

(a) that Copi not only accepts *r*.-*r* as a valid derivation, but also makes it the basic premiss to derive *q*. As Scherer puts it, "steps 1 to 6 are a purported proof that 6, *q*, is the valid consequence of 1, *r*.-*r*,"[10]

and,

(b) that Copi uses the rule of Simplification to derive *r* and -*r* from *r*.-*r*, the use of which is permissible, if the truth of the conjunction, *r*.-*r*, is accepted.

A glance at Copi's formulation makes it clear that he neither makes *r*.-*r* the basic premiss nor derives *r* and -*r* from it through simplification. He accepts *r*, -*r* and from them derives the conjunction, *r*.-*r*. If Scherer concludes that this acceptance of this derivation is supposition of its truth, then, he too is guilty of the same error for he accepts *r*.-*r* as a valid derivation. According to his representation, Copi does not stop with the acceptance of *r*.-*r* as a valid derivation, but goes further to make use of *r*.-*r* as a premiss to derive *r* and -*r*, through simplification. Simplification is possible only if the truth of *r*.-*r* is accepted, but Copi does not use simplification. He side-steps the contradiction (step 13) and makes use of *r* and -*r*, accepting them as derivations obtained earlier than step 13 (steps 11 and 12), and not as conjuncts of *r*.-*r*. He finds himself having to

7. *Ibid.*, p. 62.

8. *Ibid.*, pp. 61 and 62.

9. Scherer, *op. cit.*, p. 249.

10. *Ibid.*, p. 249.

accept the truth of $r.-r$ through conjunction. He avoids the acceptance of $r.-r$ as true by resorting to the use of the elementary valid arguments of Addition and Disjunctive Syllogism. In this inferential leap, does he accept the truth of r and $-r$ separately, as alleged by Scherer?

Scherer's contention is, that for the derivation of $r \vee q$ by Addition, r is supposed true and for the derivation of q from $r \vee q$, by the rule of Disjunctive Syllogism, $-r$ is supposed true. This piece of inference has to be viewed without dissecting it into artificially isolated temporal units but as a series of interdependent steps. No doubt, both r and $-r$ are employed to derive q, but the assertion of $-r$, which Copi accepts from step 12, negates the truth of r and enables him to derive q. If he supposes the truth of r, then he cannot accept the negation of it by $-r$ and this non-acceptance will not permit him to derive q from $r \vee q$. It is the employment of $-r$ as true and r as false which enables him to derive q and not the truth of both. He will not object to a formulation in which the truth of r and the falsity of $-r$ is asserted:

11	r	
12	$-r$	
13	$r.-r$	11, 12, Conj.
14	$-r \vee q$	12, Add.
15	q	11, 14, D.S.

To him, it does not matter which of the conjuncts is true, but that both should not be supposed true in the same piece of inference. The negation of r by $-r$ or $-r$ by r could be made explicit by expanding his formulation through the use of the rules of Commutation and Double Negation and the method of definitional substitution. Then, we have the following formulations:

I			II		
1	r	11	1	r	11
2	$-r$	12	2	$-r$	12
3	$r \vee q$	11, Add.	3	$-r \vee q$	12, Add.
4	$q \vee r$	Comm.	4	$q \vee -r$	Comm.
5	$-q \supset r$	Mat. Impl.	5	$-q \supset -r$	Mat. Impl.
6	$-r$	12	6	r	11
7	$-(-q)$	M.T.	7	$-(-q)$	M.T.
8	q	D.N.	8	q	D.N.

In formulation I, $-r$ is supposed true and the truth of $-r$ negates the truth of r enabling, by Modus tollens, $--q$. In formulation II, r is supposed true and the truth of r negates the truth of $-r$ enabling, by Modus tollens, $--q$. His introduction of the rule of Addition has been one of the causes for misunderstanding his formulation. In fact, even without the use of the rule of Addition, $-q \supset r$ may be derived through the use of conditional proof. His formulation would then be like this:

```
Step ┌─►4   -q
     │  5   .
     │  6   .
     │  7   .
     │  8   .
     │  9   .
     │ 10   .
     │ 11   r
     └──────────
            -q ⊃ r     4, 11, C.P.
       12   -r
            ─────────────────
            - -q       M.T.
       ∴      q        D.N.
```

That $-q$, the negation of the original conclusion, implies r is accepted by Scherer too, but this acceptance of r as a consequent of the implication does not mean that it is supposed true.

These paraphrased formulations are all meant to make it clear that he does not suppose the truth of both r and $-r$ to derive q. Instead he introduces Addition, even at the risk of being misunderstood, because he decided to derive q from the *falsity of the contradiction itself.*[11] In the above formulations, Modus tollens is adopted

$$\begin{array}{ccc} (1) & & (2) \\ \begin{array}{l} -q \supset r \\ \underline{\quad -r \quad} \\ \therefore\ --q \end{array} & \text{or} & \begin{array}{l} -q \supset -r \\ \underline{\quad r \quad} \\ \therefore\ --q \end{array} \end{array}$$

and when this is adopted, the assertion of $-r$ as true and r as false or vice versa will render us only $--q$ and not q. To derive q, the rule of Double Negation has to be further employed. But the form adopted by Copi,

$$\begin{array}{l} r \vee q \\ \underline{-r} \\ q \end{array} \quad \text{or} \quad \begin{array}{l} -r \vee q \\ \underline{\quad r \quad} \\ q \end{array},$$

has to be taken as strictly a case of Modus-tollendo-ponens, in which the minor premiss $-r/r$ negates the alternative $r/-r$ in the major premiss, leaving the other alternative q as the only one which is possible. The use of Modus-tollendo-ponens makes it possible to derive q directly through the negation of $\frac{r}{-r}$ by $\frac{-r}{r}$.

The supposition of only one of the conjuncts could be pointed out in another manner. According to the rule of Conditional Proof, if q is derivable from assumptions which include $\frac{-r}{r}$, we can derive $\frac{\sim r \supset q}{r \supset q}$ from any further assumption on which q may depend.[12] In this piece of

11. Copi, *op. cit.*, p. 62.

12. A. N. Prior, *Formal Logic*, Clarendon Press, Second Edition (1962), p. 320.

inference, q depends on the further assumption that $\frac{r}{\sim r}$ is false and this could be made explicit when, through the application of the rule of Conditional Proof, we obtain the Law of the Denial of the Antecedent.

I

→1	r	11
→2	$\sim r$	12
3	$r \vee q$	Add.
4	$\sim r$	12
5	q	D.S.
6	$\sim r \supset q$	C.P.
7	$r \supset . \sim r \supset q$	C.P.

II

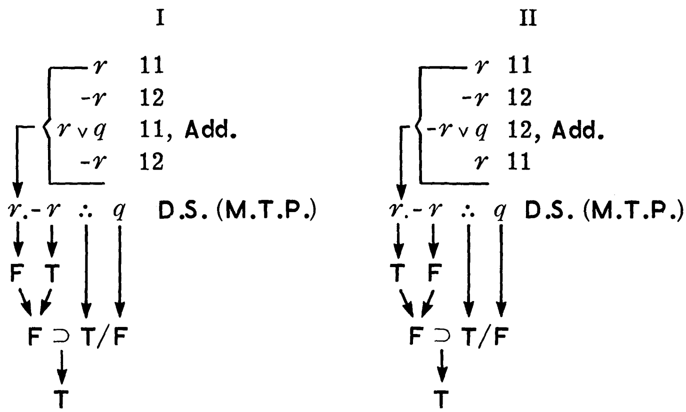
→1	$\sim r$	12
→2	r	11
3	$\sim r \vee q$	Add.
4	r	11
5	q	D.S.
6	$r \supset q$	C.P.
7	$\sim r \supset . r \supset q$	C.P.

The Law of the Denial of the Antecedent, $p \supset . - p \supset q$, states that if the antecedent p is false, any consequence, for example q, follows. But, q could be derived through the application of this law only when p could be denied by its contradictory $-p$. In formulation I, $-r$ is asserted true and this denies the truth of the antecedent r, while in formulation II, r is asserted true and this denies the truth of $-r$. Hence, the derivation through the use of Addition and Disjunctive Syllogism should not be artificially compartmentalised, but viewed as two interdependent steps from which the Law of the Denial of the Antecedent is derivable. This means, that the supposition of $\frac{r}{-r}$, in the derivation of $\frac{r \vee q}{-r \vee q}$, implies its own denial, $\frac{-r}{r}$, which enables q.

In Copi's formulation, the elementary valid arguments of Addition, $r \therefore r \vee q$ and Modus-tollendo-ponens,

$$\begin{array}{l} r \vee q \\ \underline{-r\quad} \\ \therefore q \end{array}$$

are logically prior to the construction of the contradiction, $r.-r$. They do not follow the contradiction as Scherer would prefer to place them. Continuing from steps 11 and 12, Copi uses these elementary valid arguments to decide which, r or $-r$, is true and with this decision, the contradiction is introduced $\left(\frac{r\ .\ -r}{T/F/\ F/T}\right)$ from which q is derived. These arguments have to be taken together as serving the same purpose, and in this context, it is obvious, that the supposition that $\frac{-r}{r}$ is true is itself the supposition that $\frac{r}{-r}$ is false. If Copi's comment, that from a contradiction any conclusion can validly be deduced, appearing between steps 13 and 14, is to be interpreted in a manner consistent with the rest of his account, $r.-r$ has to be regarded tautologously false, which means that r and $-r$ could only have opposite truth values. In the light of the recognition of the falsity of $r.-r$, his formulation may be represented as follows:



Hence, how could Copi be criticized as supposing the truth of r and $-r$ either conjunctively or separately? The assignment of "F" to $r.r$, as Scherer himself points out, makes the inference, $(r.-r) \supset q$ valid.

Although Copi's formulation does not contain an explicit denial of the contradiction, it is based on the tautological falsehood of the contradiction. When his derivation of q goes on through the contradiction, he recognises its logical falsehood by assigning "F" to it. In the context of the assignment of this value to the contradiction should be explicitly deny it? Should the line, $-(r.-r)$, occur in his type of formulation?

Scherer's alternative formulation clothes reductio in a Modus-tollens. But, is the form of Modus-tollens adequate to manifest to the maximum, the logical make-up of reductio? The conclusion in a Modus-tollens would always be a denial of the antecedent of the implicative premiss. Hence, when reductio is of this form, it would naturally be, "a pointing out to the irrationality of the initial assumption."[13] The denial of the contradiction, $(r.-r)$, permits us to deny $-q$, the initial assumption and hence, $--q$ would result. $-(r.-r)$ leads to $- -q$, but not q. Both in Scherer's simpler and more sophisticated versions of reductio, the derivation obtained through $-(r.-r)$ is the denial of the initial assumption, $-(-q)$ or $-(p.-q)$.

The derivation of the original conclusion, p or $p \supset q$, is merely an appendage to the conclusion, $--q$ or $-(p.q)$, obtained by Modus-tollens and hence it does not follow from the logical falsehood of the contradiction itself. In his first formulation $-(r.-r)$ leads to $-(-q)$ and q is obtained through Double Negation, while in his second formulation $-(r.-r)$ leads to $-(p.-q)$ and $p \supset q$ is derived according to the transformation rule of substitution by definition, $-(p.q) =_{df} (p \supset q)$. If one is satisfied with merely denying the initial assumption, the form of Modus-tollens is adequate. One

13. Scherer, *op. cit.*, p. 252.

Scherer's Formulation of Reductio

Formulation I[14]

	p	
	$-q$	
	.	
	.	
	.	
	$r.-r$	
1	$-q \supset (r.-r)$	C.P.
2	.	
3	.	
4	.	
5	.	
6	.	
7	.	
8	$-(r.-r)$	De.M.
9	$-(-q)$	M.T.
10	q	D.N.

Formulation II[15]

$(p.-q)$	
.	
.	
.	
$r.-r$	
$(p.-q) \supset (r.-r)$	C.P.
$-(r.-r)$	
$-(p.-q)$	M.T.
$\therefore p \supset q$	$-(p, -q) =_{df} (p \supset q)$

could agree with Hamblin, that reductio-ad-falsum[16] would be a more precise name for this type of derivation by Modus-tollens. Even Scherer, who gives reductio a Modus-tollens form, is not satisfied with the negative conclusion reached by it, $-(-q)$ or $-(p.-q)$, but takes the trouble to obtain q or $p \supset q$. Reductio in the Modus-tollens mould does not seem adequate to express the poignancy of the argument, which is founded on the logical make-up of the contradiction—it is so "absurd,"[17] "non-sensical,"[18] and "troublesome"[19] that it could lead to any conclusion, even the original conclusion, q. This seems to be more adequately expressed in Copi's formulation, $-q \supset. (r.-r) \supset q$, because contradiction is enabled to be, not only the necessary but also the sufficient basis, for the derivation of the conclusion, q. Thus, Copi's recognition, that $r.-r$ is tautologously false, makes the contradiction more central to reductio than Scherer's formulation, where $-(r.-r)$ is necessary, but not sufficient, to derive q. Copi, in

14. *Ibid.*, p. 248.

15. *Ibid.*, pp. 247 and 248 (footnote).

16. C. L. Hamblin, *Fallacies*, Methuen & Co. Ltd., London (1970), p. 78.

17. Patrick Suppes, *Introduction to Logic*, D. Van Nostrand Company, Inc., Princeton, New Jersey (1957), p. 34.

18. L. H. Hackstaff, *Systems of Formal Logic*, R. Reidel Publishing Company, Dordrecht-Holland (1966), p. 171.

19. W. Van Orman Quine, *Methods of Logic*, Routledge & Kegan Paul, London (1952), pp. 173-174.

introducing the subject of reductio, quotes Euclid's use of reductio,[20] the method of derivation in which the denial of the contradiction leads to the denial of the assumption that leads to the contradiction. But, Copi is emphatic that in reductio it is "deducing the argument's conclusion from the contradiction itself."[21] Deduction of the argument's conclusion from the contradiction itself is possible, not when the contradiction is explicitly denied and the formulation is of Modus-tollens type, but when the contradiction is accepted as tautologously false and the formulation is of the type $-q \supset. (r.-r) \supset q$. Then, the tautologously false $r.-r$ is itself enough to render q. Copi's selection of such a formulation of reductio, rather than the typical Modus-tollens type, seems to be a deliberate choice. The 'absurdity' of the contradiction is given a more decisive role to play and this makes his formulation typically reductio-ad-*absurdum*. Hence, it is only too pertinent to ask whether Scherer has been fair by Copi. Scherer's alternative formulation is really unnecessary and cannot therefore be preferred.

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20. Copi, *op. cit.*, p. 61.

21. *Ibid.*, p. 62.