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# GENERALISED LOGIC

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**1.** The logically unsophisticated will often protest when those who are more sophisticated draw out the consequences of their statements. If the discussion is pursued, it may be found that they are resistant to accepting the excluded middle law. Moreover, they may go further, by refusing to concede that a certain statement is or is not true and refusing to concede that it is or is not false, though admitting that it cannot be both true and false. Those who are more sophisticated have a powerful armoury for avoiding such pitfalls. This includes the adjectives of degree and emphasis and such devices as "partly the one thing and partly the other," "true or false but we don't know which," "true in one sense and false in another," "classes versus criteria," "extension versus intension" and in the last resort "neither true nor false" and "too vague to mean anything."

The one thing that is not attempted is to take the unsophisticated seriously, that is to say, to attempt to construct a logic in which there is a middle term that is not necessarily incompatible with truth or with falsity, though these remain incompatible with one another. In the present paper it will be shown that, contrary to expectation, such a logic can be constructed and that it is an interesting and very radical generalisation of elementary logic.

Take the matter from another point of view, by considering existing systems that modify or abrogate the excluded middle law. On the one hand, there are systems such as those of Heyting [1] and of Fitch [2] in which $ApNp$ is not a theorem, but in which there is no third term "$?p$". On the other hand, there are such systems as those discussed by Rosser and Turquette [3], in which there are three or more terms, but in which the terms are incompatible with one another, in the sense that any proposition takes one and only one value. Comparing these two groups of systems, it may be asked whether a system could be constructed that, unlike the systems of the first group, includes a third or middle term, but unlike the systems of the second group is such that the middle term is not incompatible with $p$ or with $Np$. The system of the present paper is such a system.

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This logic will be called "generalised logic," in distinction, of course, from "general logic," which will always be referred to as "elementary logic." The middle term will be symbolised by "$?p$" and pronounced "vague-$p$," the middle value being called "vagueness." It should particularly be noted that when discussing generalised logic such words as "proposition" and "value" take on a wider meaning, to include examples that would be excluded in discussions of conventional logic; for example, from some points of view the third "value" is not a true value. Generalised logic has the following properties.

(1) There is a third value, giving the theorem $AAp?pNp$. (2) The notion of value is generalised, by allowing that whereas $KpNp$ is disallowed, neither $Kp?p$ nor $K?pNp$ are disallowed. (3) Although $CKpNpq$, $EKpNpKqNq$, etc. are theorems, $NKpNp$ is not a theorem (as $KpNp$ may be merely vague). If $EqKpNp$ then $q$ is said to be "excluded." Exclusion is more general and weaker, than falsity, in particular, $CNqEqKpNp$ is a theorem, but its converse is not a theorem. (4) There is a class calculus corresponding to the propositional calculus. (5) For a domain of propositions that do not take the third value, the axioms and rules revert to a complete set for the Boole-Schroeder logic.

Three fallacies are worth noting, as they confuse thought when the logic is first studied. (1) Vague terms may not be substituted for non-vague terms and vice versa. For example, given that $\phi(p, q, r, \ldots)$ is a theorem one may not infer $\phi(?p, q, r, \ldots)$, the substitution rules debar this. (2) Given $Epq$, one may infer neither $ENpNq$ nor $E?p?q$ (in default, of course, of other premises) and one may not infer any of these three forms from any other. (3) The system includes the axiom $EpNNp$ but the rules of formation and substitution debar generalisation of this axiom to $E\phi NN\phi$ for any function $\phi$.

The intuitive basis of generalised logic is, indeed, so different from that of elementary logic that an intuitive derivation of the primitives of generalised logic is desirable. This intuitive derivation forms the next section of this paper and is followed by a formal development of the system.

**2.** In designating the items of the system, D = Definition, A = Axiom, Ru = Underived rule, R = Derived rule and T = Theorem. To facilitate tracing items among the intuitive derivations, the further designation "I" is used for these items. Apart from a distinction among the variables that will shortly be described, the only departure from the usual symbolism is the introduction of "?" for "is vague." To facilitate translation from the propositional to the class calculus, implication is not used in the axioms and rules but equivalence and for this reason the primitives may be rather more numerous than necessary. In any case, however, the aim is to achieve intelligibility rather than brevity.

The available basis for discussion is:

$AAp?pNp$ I1.
$p$ and $Np$ are incompatible, but $?p$ is incompatible with neither $p$ nor $Np$ I2.

Clearly, there is a distinction here, between the logic of vagueness, on the one hand and the logic of truth and falsity, on the other. In accordance with this distinction, one may expect to encounter axioms and theorems for whose variables one may not indifferently substitute formulae containing occurrences of "?", on the one hand, and formulae not containing occurrences of "?", on the other. This expectation is, indeed, realised and to uphold the required distinction it is necessary to work with two kinds of variable, the "restricted variables" (r.v.'s for brevity) and the "general variables" (g.v.'s for brevity). The letters $p$, $q$, $r$, $s$, are reserved for the r.v.'s and the letters $f$, $g$, $h$, $i$, are reserved for the g.v.'s. Before formulating a substitution rule, two types of formulae will now be defined.

> Type 1 formulae are those well formed formulae that contain no occurrences of "?" and no g.v.'s. Type 2 formulae are those well formed formulae that contain at least one occurrence of "?" and/or at least one g.v. I3.

It is doubtful whether every well formed formula is intuitively interpretable, or even formally significant: for example, I can offer no interpretation of $?Kp?p$ and no inference from it. To distinguish between the logic of truth and falsity, on the one hand and the logic of vagueness, on the other, type 1 formulae must be confined to that type under substitution for their variables and this is achieved by the following rule (due to Prior in [4], Appendix C).

> Formulae of either type can be substituted for the g.v.'s in an axiom, theorem, or rule, but only type 1 formulae can be substituted for the r.v.'s in an axiom, theorem, or rule. R1, I4.

It is fairly obviously safe to formulate the following axioms and rules in g.v.'s (they contain no occurrences of "?" or "$N$").

The associative and commutative laws are valid for conjunctions and disjunctions of g.v.'s A3, A4, A5, A6, I5.
$EKfAghAKfgKfh$ and $EAfKghKAfgAfh$ A7, A8, I6.
$EKfff$ and $EAfff$ A1, A2, I7.
If $Efg$ then $Egf$ Ru2, I8.
If $Efg$ and $Egh$, then $Efh$ Ru3, I9.
If $f$ and $Efg$ then $g$ Ru6, I10.
If $Efg$ then $EKhfKhg$ and $EAhfAhg$ Ru4, Ru5, I11.
$Cfg = EKfgf$ Definition D1, I12.

The rules of the system are completed by adding a weak case of the deduction theorem to justify the method of subordinate proof (see [2]). Granted the usual intuitive notion of implication, it is evident that subordinate proof will be valid, but it is admittedly odd to base an underived rule on a definition (I12). The oddity is trivial, however, for it could be removed by replacing each equivalence among the primitives by a pair of implications, replacing I12 by "*Efg = KCfgCgf*" and adding the rule "if *Cfg* and *Cgf*, then *KCfgCgf*."

Subordinate proofs will be placed in square brackets. No rule will be cited to justify the hypothesis as this is invariably the first step within the brackets and the rule is invariably Ru7 (I13, below). Also, no rule will be cited to justify the conclusion, as it is invariably the first step after closure of the brackets, invariably comprises "*C* (first step within brackets) (last step within brackets)" and the rule is invariably Ru7.

Hypotheses are allowed and any proposition infered from an hypothesis is implied by it. Ru7, I13.

Turning now to axioms that contain occurrences of "?" or "*N*," it is found, not surprisingly, that with the exception of one variable in A9 and A10 they must be formulated in r.v.'s.

Firstly, notice that the enlargement of the scope of the logic to include for vagueness introduces no assymmetry between *p* and *Np*: to negate *Np* is still to assert *p* and to negate *p* is still to assert *Np*. Moreover, ?*p* stands in precisely the same relation to *p* and to *Np*.

*ENNpp* and *E?Np?p* A17, A18, I14.

Now reconsider I1. There must be conditions under which ?*p* is excluded, yielding *ApNp*. Similarly, there must be conditions under which *p* is excluded, yielding *A?pNp* and conditions under which *Np* is excluded, yielding *Ap?p*. It might be supposed that this notion of exclusion was simply negation of the relevant term, but in fact, it is weaker than negation. This can be seen by considering *A?pNp*, which excludes *p* but is weaker than *Np*, or by considering *Ap?p*, which excludes *Np* but is weaker than *NNp*, i.e., *p*. It would seem as if one were committed to a fourth value but fortunately, as will be seen shortly, exclusion can be specified without introducing a new primitive. Meanwhile, the notion of exclusion must be further investigated.

Bearing in mind that falsity is stronger than exclusion, if *Kpq* is not merely vague or excluded, but false, then either *p* is false or *q* is false and similarly, if *p* is false or *q* is false, then *Kpq* is false. That is to say, De Morgan's theorem is valid, provided it is formulated as a type 1 formula.

*ENKpqANpNq* and *ENApqKNpNq* A13, A14, I15.

But by I4 one may substitute *Np* for *q* to obtain:

*ENKpNpApNp* I16.

Which is startling—if there is such a thing in logic—because clearly *ApNp* is not a theorem and therefore, by I16, neither is *NKpNp*. However, one may already conjecture that I16 should be interpreted as entailing that the conventional laws of excluded middle and contradiction are valid if and only if ?*p* is excluded. Moreover, a further hint is obtained by substituting

$KpNp$ in I1 (by I4) giving $AAKpNp?KpNpNKpNp$ and suggesting (though the proof is not yet available) that $A?KpNpNKpNp$ will be a theorem and that although $KpNp$ is not false it is valid to eliminate it from the former function, to obtain the latter, because it is excluded.

Consider, now, in what sense, if any, $?Kpq$ and $?Apq$ are value functions. In particular, might there be a state of affairs where $p$ and $q$ were both true, where neither of them were vague, yet where $Kpq$ were vague and/or $Apq$ were vague? This would amount to positing that the connective itself could be vague and although it is conceivable that a system could be constructed to include for vague connectives, this is a further generalisation with which we are not concerned. But therefore:

$C?KpqA?p?q$ and $C?ApqA?p?q$ I17.

And from this one can prove (using I14):

$C?KpNp?p$ and $C?ApNp?p$ I18.

Also, bearing in mind that $?Kpq$ and $?Apq$ are value functions in the sense described above, I19 below is self evident and from this I20 can be proved. Finally I21 can be proved from I18 and I20 (using R6, which is proved independently of the present considerations).

$CK?p?q?Kpq$ and $CK?p?q?Apq$ I19.
$C?p?KpNp$ and $C?p?ApNp$ I20.
$EE?p?KpNp?ApNp$ I21.

Comparing I21 with I14 and I16 it can be seen that neither $NKpNp$ nor $ApNp$ are theorems because both may be merely vague and that they are both vague if and only if $p$ is vague. Now while this is to say that where propositions are vague, contradictions are vague, it is not to deny that a contradiction implies anything:

$CKpNpf$ and $EKKpNpfKpNp$ and $EKpNpKqNq$ A11, I22.

The first and second members of I22 are equivalent by I12 and the third is derivable from the first by substituting $KqNq$ for $f$, etc. Also, using I22 with the two lemmas "if $Afg$ and $Cgh$, then $Afh$" and "$CfAfg$" (T31 and T15), both of which can be proved independently of the present discussion, one obtains

$EAKpNpff$ I23.

Also, it will be shown in the next section that $CNqEqKpNp$ (T39), but I do not think that the converse of this theorem can be proved (if it can, the system is inconsistent). Therefore statements equivalent to a contradiction have, by I22, I23, T39 (and the absence of its converse), the properties of of exclusion described in the discussion following I14. These various theorems specify exclusion and a proposition equivalent to a contradiction will be said to be excluded. It will be convenient to reserve the letter "$s$" for asserting exclusion ($EpKsNs$, etc.) but it will be understood that this usage has no logical significance. The following further axioms are intuitively obvious:

*EKAAp?pNpff* and *EAAAp?pNpfAAp?pNp* A9, A12, I24.

Let us now seek the De Morgan analogue of *?Kpq* and *?Apq*. It has already been mentioned that there is a class calculus corresponding to the propositional calculus and the De Morgan analogues can most clearly be intuited by analogy with Venn-type diagrams for the class calculus.

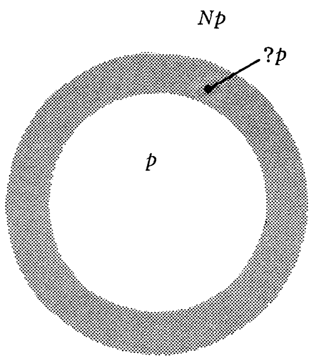


Figure 1.

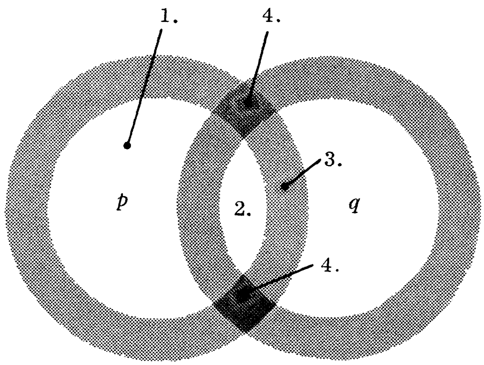


Figure 2.

Figure 1 illustrates the diagrams. In Figure 2, 1. is *KpNq*, 2. is *Kpq*, 3. is *K?pq*, 4. is *K?p?q*, and so on. It is easy to see that:

*E?KpqAAK?pqK?p?qKp?q* A15, I25.
*E?ApqAAK?pNqK?p?qKNp?q* A16, I26.

This completes the primitives of the system.

**3.** In summary, the primitives of the system are:

| | | | |
|---|---|---|---|
| *Cfg = EKfgf* | | | I12, D1. |
| *EKfff* | I7, A1. | *EAfff* | I7, A2. |
| *EKfgKgf* | I5, A3. | *EAfgAgf* | I5, A4. |
| *EKfKghKKfgh* | I5, A5. | *EAfAghAAfgh* | I5, A6. |
| *EKfAghAKfgKfh* | I6, A7. | *EAfKghKAfgAfh* | I6, A8. |
| *EKAAp?pNpff* | I24, A9. | *EAKsNsff* | I23, A10. |
| *EKKsNsfKsNs* | I22, A11. | *EAAAp?pNpfAAp?pNp* | I24, A12. |
| *ENKpqANpNq* | I15, A13. | *ENApqKNpNq* | I15, A14. |
| *E?KpqAAK?pqK?p?qKp?q* | I25, A15. | *E?ApqAAK?pNqK?p?qKNp?q* | I26, A16. |
| *ENNpp* | I14, A17. | *E?Np?p* | I14, A18. |
| Substitution rule | I4, Ru1. | | |
| If *Efg*, then *Egf* | I8, Ru2. | | |
| If *Efg* and *Egh*, then *Efh* | I9, Ru3. | | |
| If *Efg*, then *EKhfKhg* | I11, Ru4. | | |
| If *Efg*, then *EAhfAhg* | I11, Ru5. | | |
| If *f* and *Efg*, then *g* | I10, Ru6. | | |
| See I13 (subordinate proof) | I13, Ru7. | | |

The primitives of the class calculus can be obtained from these primitives by deleting Ru6 and Ru7 and translating the remaining primitives into class notation.

The following theorems and rules are easily proved, here the proofs will be omitted.

R1. If *Efg* and *Ehi*, then *EAfhAgi*
R2. If *Efg* and *Ehi*, then *EKfhKgi*
R3. If *EfAgh* and *Ehi* then *EfAgi*
T1. *Eff* T2. *ENKpNpApNp*
T3. *E?KpNp?p* T4. *E?ApNp?p*
T5. *E?KpNp?ApNp* T6. *ENApNpKsNs*
T7. *EA?KsNsNKsNsAAp?pNp* T8. *E?Apq?KNpNq*

For the intuitive validity of T7, see T25, below.

T9. *E?Kpq?ANpNq* T10. *EAKpq?KpqKAp?pAq?q*
T11. *EAApKA?pNpANq?qqAAr?rNr* T12. *EAANpKA?ppAq?qNqAAr?rNr*
T13. *EfAfKfg* T14. *CKfgf*
R4. If *Cfg* and *Cgh* then *Cfh*
R5. If *Cpq* then *EKpNqKsNs*
R6. *Cfg* and *Cgf* if and only if *Efg*
T15. *CfAfg* T16. *Cff*
T17. *EKpNpKqNq* T18. *CKsNsf*
T19. *EAAp?pNpAAq?qNq* T20. *CfAAq?qNq*
T21. *EAANANpNqNANpqKp?qp*
R7. *Cfg* if and only if *EAfgg*

So far, Ru6 and Ru7 have not been used in the proofs of rules or theorems, so that the above twenty-one theorems and seven rules can be translated into class notation without becoming invalid. Most or all of the following theorems, down to T50, do, however, require Ru6 or Ru7 for their proof.

R8. If *f* and *Cfg* then *g*
T22. *CgCfg*
*Proof.* [1. *g* [2. *f* 3. *g* (1)] 4. *cfg*] 5. *CgCfg*
T23. *CfEfAAp?pNp*
*Proof.* [1. *f* 2. *CfCAAp?pNpf* (T22) 3. *CAAp?pNpf* (1, 2, R8) 4. *CfAAp?pNp* (T20) 5. *EfAAp?pNp* (3, 4, R6)] 6. *CfEfAAp?pNp*
T24. *AAp?pNp*
T25. *A?KsNsNKsNs*

Intuitively, this last amounts to "contradictions are vague or false" and is central to the point that exclusion is weaker than falsity and to the notion of vagueness: where propositions are vague, contradictions may be merely vague.

T26. *AApKA?pNpA?qNqq*
T27. *AANpKA?ppA?qqNq* T28. *EEffAAp?pNp*

R9. *f* and *g* if and only if *Kfg*

T29. *CKfCfgg*
T30. *EKCfgCgfEfg*
T31. *CKAfgCghAfh*
T32. *CCfgCCghCfh*
T33. *CfCgKfg*
T34. *CKCfgChiCAfhAgi*
T35. *CCfKghKCfgCfh*
T36. *CCghCAfgAfh*
T37. *ECKfghCfCgh*
T38. *CAEfKsNsEgKsNsEKfgKsNs*
T39. *EEAfgKsNsKEfKsNsEgKsNs*
T40. *CpENpKsNs*
T41. *CpCNpf*
T42. *CCfpEKfNpKsNs*
T43. *CCfpCCfNpEfKsNs*
T44. *CANpfCpf*
T45. *CCpfAA ?pNpf*
T46. *ECfgEAfgg*
T47. *CKApfNpf*
T48. *ECAfghKCfhCgh*
T49. *CEpNp ?p*
T50. *CCNppAp ?p*

It is a reasonable conjecture that if $\phi_n$ is an $n$-variable theorem of Boolean propositional logic *CKK . . . E ?*$p_1$*KsNsE ?*$p_2$*KsNs . . . E ?*$p_n$*KsNs*$\phi_n$ will be a theorem of generalised logic, but I have been unable to show this. A piecemeal comparison with the relevant theorems of Principia Mathematica is of some interest. About two thirds of these theorems are theorems of generalised logic and interesting comparisons can be made with nearly all the remainder, the following being a selection of examples.

| PRINCIPIA MATHEMATICA | GENERALISED LOGIC |
|---|---|
| 1.01. *Cpq = ANpq* Definition | T51. *ECpA ?pfAA ?pNpf* |
| 2.01. *CCpNpNp* | T52. *CCpNpEpKsNs* and |
| | T53. *CCpNpA ?pNp* |
| 2.03. *CCpNqCqNp* | T54. *CCpNqCqA ?pNp* |
| 2.5. *CNCpqCNpq* | T55. *CNCpqA ?pCNpq* |
| 2.521. *CNCpqCqp* | T56. *CNCpqA ?pCqp* and |
| | T57. *CNCpqA ?qCqp* |
| 2.68. *CCCpqqApq* | T58. *CCCpA ?pffApf* |
| 2.85. *CCApqAprApCqr* | T59. *CCApfApgAAp ?pCfg* |
| 3.24. *NKpNp* | T60. *A ?pNKpNp* |
| 4.83. *EKCpqCNpqq* | T61. *EKKCpfC ?pfCNpff* |
| 5.13. *ACpqCqp* | T62. *AA ?pCpfCfp* |

Most of these theorems are not difficult to prove. Perhaps the only form that presents some difficulty is the form *CNCpq*$\phi$, as in T55 to T57. Theorems of this form can be proved from derivatives of T54. For example, the proof of T56 is: (1) *CCNpqCNqA ?pp* (T54, etc.). (2) *CCNpCpqCNCpqA ?pp* (1, etc.). (3) *CNpCpq* (T41, etc.). (4) *CNCpqA ?pp* (2, 3, R8). (5) *CpCqp* (T22). (6) *CNCpqA ?pCqp* (4, 5, T31, R4).

**4**. Consider the relation of generalised propositional logic to Boolean propositional logic, or of generalised class logic to Boolean class logic. To do so, add the axiom *E ?pKsNs* to the generalised logic and substitute r.v.'s for all g.v.'s. Then this axiom can be used to eliminate all occurrences of "?" and A10, A11, can be used to eliminate redundant occurrences of *KsNs*, leaving every axiom, rule, and theorem an axiom, rule, or theorem, of Boolean logic. Moreover, with both the resulting propositional logic and the resulting class logic it is easy to prove a complete set for the corresponding Boolean logic.

These results define the sense in which generalised logic is more general than Boolean logic, for the addition of $E?pKsNs$ makes the system degenerate. Also, they show that generalised logic is incomplete and that it can only be completed by degenerating into Boolean logic.

Now consider the part of the system that is formulated in g.v.'s only, without r.v.'s. This part of the system contains no occurrences of "?" or "$N$." One may ask whether this sub-system is a complete system for $K$, $A$ and $C$ (and $E$). I have been unable to prove this and believe that it is incomplete, but it is easy to show that a complete $K$-$A$-$C$ system results from the addition of the axiom $CCCfgff$ to the system. Also, as far as I can judge the addition of $CCCfgff$ to the system of generalised logic as a whole does not make it inconsistent.

It is therefore tempting to add $CCCfgff$ to generalised logic, but for two reasons I have not done so in the present paper. Firstly, $CCCfgff$ is not intuitively obvious in the sense in which A1 to A8 are intuitively obvious. Secondly, it is probably possible to make additions to the axioms containing "?" and such strengthening of the system may not be consistent with $CCCfgff$ but may be more interesting. The axiom remains, however, a likely candidate for addition to the system and defines the sense in which the $K$-$A$-$C$ sub-system of g.v.'s is incomplete.

It may be asked whether generalised propositional logic can be extended to a generalised functional logic. This is not the time and place for an exposition, but it will, perhaps, be of interest to mention that a system of first order functions of one variable can be constructed by distinguishing between general functions, $F_1, F_2, \ldots$ and restricted functions, $f_1, f_2, \ldots$ and there seems every reason to suppose that this can be extended to any number of variables and to higher functions. It is, perhaps, worth mentioning that $(\exists x)\, F(x)$ cannot be adequately defined by $\sim(x) \sim F(x)$ because, for example, if $?f(x)$ is substituted for $F(x)$ the result is $\sim(x) \sim ?f(x)$ and there are no propositional axioms or rules for handling $\sim ?p$. Existential quantification can, however, be handled by the method of instantiation by constants. Because $(\exists x)\, F(x)$ cannot be defined and because there are two types of function, the system is inclined to proliferate axioms and elementary theorems; probably a higher functional calculus of any number of variables will be cumbersome.

Finally, consider the status of the Law of Contradiction and of the Logical Paradoxes, in the context of generalised logic.

By the Law of Contradiction one might mean A: contradictions are false, ($NKpNp$), or B: contradictions are excluded, ($EKpNpKqNq$, $EKKpNpfKpNp$, $EAKpNpff$, and $CKpNpf$). In Boolean logic both are the case, in generalised logic only B is the case. In either logic a paradox is the case when $EpNp$. In Boolean logic this entails a contradiction, i.e., enables one to infer $p$ and therefore $KpNp$ and therefore anything. In generalised logic one has $EEpNpKEpKsNsENpKsNs$ and $CEpNp?p$ and $E?p?KpNp$ and

$E?KpNp?ApNp$, etc. Thus $p$ and $Np$ are equivalent if and only if both are excluded and in that event $p$ is vague, as, indeed, is $KpNp$, but a paradox does not entail a contradiction.

For a sentence, $p$, to be paradoxical (i.e., for $EpNp$) it is necessary but not sufficient that it should be vague and it is both necessary and sufficient that both $p$ and $Np$ should be excluded. It need not occasion surprise that deductive systems can be constructed for domains of propositions some of which conform to these conditions. A paradox generates a contradiction in a badly constructed logica magna, because although its domain includes paradoxical propositions, its axioms and rules are only valid for a domain of non-vague propositions. In a conventional system, the use of techniques for eluding the paradoxes is not an arbitrary meta-logical intrusion, but the necessary limitation of the domain of the system to one as narrow, or narrower, than the domain for which the axioms and rules are valid. Also, the domain of the relevant axioms and rules is not arbitrary, because if it is widened to include for vague propositions the propositional logic of the system becomes incomplete. Boolean logic, however, is incomplete in another sense, namely, that it does not include vague propositions.

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