Notre Dame Journal of Formal Logic
Volume XIX, Number 4, October 1978
NDJFAM

THE ONTOLOGICAL THEOREM

CHARLES D. BROWN

Charles Hartshorne presented a proof of the Ontological Argument containing the postulate, $p \rightarrow \sim\Diamond\sim p$, which he calls "Anselm's Principle."[1] Few of us, however, would accept even the weaker expression, $p \supset \sim\Diamond\sim p$, as plausible, and most of us would reject it on intuitive grounds.[2] If we do deny it, then strange things happen in any standard modal system that is sufficiently complex (*e.g.*, Feys' System T) to contain an equivalent of the rule:

(**R**) $\vdash \alpha \therefore \vdash \Diamond\beta \supset \Diamond(\alpha \cdot \beta)$.

For example:

(1)	$\sim(p \supset \sim\Diamond\sim p)$	
(2)	$\vdash p \cdot \Diamond\sim p$	1, **PC**
(3)	$\vdash p$	2, **PC**
(4)	$\vdash \Diamond \sim p \supset \Diamond(p \cdot \sim p)$	3, **R**
(5)	$\vdash \Diamond \sim p$	2, **PC**
(6)	$\vdash \Diamond(p \cdot \sim p)$	4,5, **PC**
(7)	$\vdash \sim(p \supset \sim\Diamond\sim p) \supset \Diamond(p \cdot \sim p)$	1-6, **PC**
(8)	$\vdash \sim\Diamond(p \cdot \sim p) \supset (p \supset \sim\Diamond\sim p)$	7, **PC**

Let us call (8) the Ontological Theorem. If we assume the law of Non-Contradiction then we are forced by the Ontological Theorem to accept

1. Charles Hartshorne, *The Logic of Perfection* (La Salle: Open Court Publishing Company, 1962), pp. 49ff, esp. p. 51.

2. Nicholas Rescher, in "On the formalization of two modal theses," *Notre Dame Journal of Formal Logic*, vol. II (1961), pp. 154-157, formulates "No 'mere fact' or 'merely contingent proposition' entails a necessary proposition" as a tautology by imbedding the consequent ($\sim\Box q$) as a conjunctive component ($\Diamond\sim q$) of the antecedent of the strict implication. His formulation does avoid the *consequentia mirabilis*, which was his purpose in having $\Diamond\sim q$ in the antecedent; but the Ontological Theorem in no way involves the *consequentia mirabilis*, either.

Received April 1, 1976

$\vdash p \supset \sim\Diamond\sim p$ as well as, by substitution $(\sim p/p)$, $\vdash \sim p \supset \sim\Diamond p$ and, by **PC** rules, $\vdash \Diamond p \supset \sim\Diamond\sim p$ and $\vdash \Diamond\sim p \supset \sim\Diamond p$. The Rule of Necessitation yields "Anselm's Principle" from $\vdash p \supset \sim\Diamond\sim p$ and the definition of Strict Implication. If we intuitively reject $p \supset \sim\Diamond\sim p$, we must reject Non-Contradiction and, since $\sim\Diamond(p\cdot\sim p) \equiv \sim\Diamond\sim(\sim p \vee p) \equiv \sim\Diamond\sim(p \supset p)$, the Excluded Middle and Identity laws as well.

"Anselm's Principle" is highly dubious as a *postulate*, but perhaps it gains some respectability as a consequent of the law of Non-Contradiction. The most questionable assumption in the proof of the Ontological Argument then becomes $\Diamond p$. Note that, unlike other rationales of the Ontological Argument, the Ontological Theorem does not depend upon any analysis of such hypostitized attributes within p as existence or perfection, nor is it restricted to any such referent as God. Thus, the familiar rejoinder to the Ontological Argument, that the conceivability of God's existence implies only the inconceivability of His non-existence, has no relevance to the Ontological Theorem.

Should we attempt to evade the Ontological Argument by denying $\sim\Diamond(p\cdot\sim p)$, thus calling into question the *logical truth* of $\sim(p\cdot\sim p)$ by admitting $\Diamond(p\cdot\sim p)$, then we similarly call into question the Excluded Middle and Identity Laws, respectively, by admitting the equivalencies, $\Diamond\sim(\sim p \vee p)$ and $\Diamond\sim(p \supset p)$. Consequently, since the Ontological Theorem is equivalent by Impl. to $\Diamond(p\cdot\sim p) \vee (p \supset \sim\Diamond\sim p)$, we are forced by the above considerations into the dilemma of, on the one hand, questioning Non-Contradiction, Identity and the Excluded Middle (together with their logical consequences) or, on the other hand, accepting the implication of a thing's necessity from its existence or mere possibility, and the credibility of the Ontological Argument.

Auburn University
Auburn, Alabama