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A NOTE CONCERNING THE NOTION OF MEREOLOGICAL CLASS

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1 In mereology we have a number of equivalences which in various ways characterize the notion of mereological class. Some of these equivalences have been used, in some systems of mereology, as definitions while others have been proved in these systems as theorems. In the present note I shall be concerned with the following three equivalences:

E1 $[Aa] \therefore A \varepsilon A : [B] : B \varepsilon a \,.\supset.\, B \varepsilon \mathbf{el}(A) : [B] : B \varepsilon \mathbf{el}(A) \,.\supset.\, [\exists CD] . C \varepsilon a . D \varepsilon \mathbf{el}(B) . D \varepsilon \mathbf{el}(C) \,:\equiv.\, A \varepsilon \mathbf{Kl}(a)$

E2 $[Aa] \therefore A \varepsilon A : [B] : [\exists C] . C \varepsilon \mathbf{el}(A) . C \varepsilon \mathbf{el}(B) \,.\equiv.\, [\exists DE] . D \varepsilon a . E \varepsilon \mathbf{el}(B) . E \varepsilon \mathbf{el}(D) \,:\equiv.\, A \varepsilon \mathbf{Kl}(a)$

E3 $[Aa] :: A \varepsilon A \therefore [B] \therefore A \varepsilon \mathbf{el}(B) \,.\equiv:\, [C] : C \varepsilon a \,.\supset.\, C \varepsilon \mathbf{el}(B) \,\therefore\equiv.\, A \varepsilon \mathbf{Kl}(a)$

Equivalence *E1*, which is due to Leśniewski, is normally used as a definition in systems of mereology in which the notion of mereological element serves as the only undefined mereological notion.[1] Thus, for instance, *E1* is used as a definition in the system based on the following single axiom:

AA1 $[AB] :: A \varepsilon \mathbf{el}(B) \,.\equiv\because\, B \varepsilon B \because [Ca] \because [D] \therefore D \varepsilon C \,.\equiv:\, [E] : E \varepsilon a \,.\supset.\, E \varepsilon \mathbf{el}(D) : [E] : E \varepsilon \mathbf{el}(D) \,.\supset.\, [\exists FG] . F \varepsilon a . G \varepsilon \mathbf{el}(E) . G \varepsilon \mathbf{el}(F) :: B \varepsilon \mathbf{el}(B) . B \varepsilon a \,::\supset.\, A \varepsilon \mathbf{el}(C)$[2]

It is not difficult to see that *E1* is, in a sense, embedded in *A1*, whose meaning becomes clearer once we have realized that the set of presuppositions consisting of *AA1* and *E1* is inferencially equivalent to the set of presuppositions consisting of *E1* and

AA1.1 $[AB] \therefore A \varepsilon \mathbf{el}(B) \,.\equiv:\, B \varepsilon B : [a] : B \varepsilon \mathbf{el}(B) . B \varepsilon a \,.\supset.\, A \varepsilon \mathbf{el}(\mathbf{Kl}(a))$

With the aid of symbols we state this equivalence thus: $\{AA1, E1\} \rightleftarrows \{AA1.1, E1\}$, and we note that in $\{AA1, E1\}$ *E1* can be regarded as a definition whereas in $\{AA1.1, E1\}$ it cannot be so regarded in view of the fact that the notion of '**Kl**' already occurs in *AA1.1*. Consequently, $\{AA1.1, E1\}$ must be treated as an axiom system involving two undefined mereological notions, i.e., '**el**' and '**Kl**'.

In 1954 I noticed that *E2* could be used as the definition of '**Kl**' in a

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system of mereology which is inferentially equivalent to the one based on *AA1*.[3] However, in order to be able to use *E2* as a definition, we have to replace *AA1* by *BA1*, which reads as follows:

BA1 $[AB] :: A \,\varepsilon\, \mathbf{el}(B) \;.\!\equiv\!:\cdot:\; B \,\varepsilon\, B :\cdot: [CDa] :\cdot: [E] \therefore E \,\varepsilon\, D \;.\!\equiv\!:\; [F] : [\exists G]\,.$
$G \,\varepsilon\, \mathbf{el}(E)\,.\,G \,\varepsilon\, \mathbf{el}(F) \;.\!\equiv\!.\; [\exists HI]\,.\,H \,\varepsilon\, a\,.\,I \,\varepsilon\, \mathbf{el}(F)\,.\,I \,\varepsilon\, \mathbf{el}(H) :: B \,\varepsilon\, \mathbf{el}(B)\,.$
$B \,\varepsilon\, \mathbf{el}(C)\,.\,C \,\varepsilon\, a \;::\!\supset\!.\; A \,\varepsilon\, \mathbf{el}(D)$[4]

Now, it is easy to prove that $\{BA1, E2\}$ is inferentially equivalent to the set of presuppositions consisting of *E2* and

BA1.1 $[AB] \therefore A \,\varepsilon\, \mathbf{el}(B) \;.\!\equiv\!:\; B \,\varepsilon\, B : [a] : B \,\varepsilon\, \mathbf{el}(B)\,.\,B \,\varepsilon\, \mathbf{el}(C)\,.\,C \,\varepsilon\, a \;.\!\supset\!.$
$A \,\varepsilon\, \mathbf{el}(\mathbf{Kl}(a)))$

As in the case of $\{AA1.1, E1\}$ we can regard $\{BA1.1, E2\}$ as an axiom system involving '**el**' and '**Kl**' as two undefined mereological notions.

E3 is a slight simplification of a thesis first proved within the framework of a system of mereology by Tarski.[5] So far as I know, no one has yet suggested an axiom system with '**el**' as the only primitive mereological notion and such that within its framework *E3* could be used as the definition of '**Kl**'. It will be proved in the sequel that a system of this sort can be based on the following single axiom:

CA1 $[AB] \because\!: A \,\varepsilon\, \mathbf{el}(B) \;.\!\equiv\!::\; B \,\varepsilon\, B :: \sim\!(B \,\varepsilon\, \mathbf{el}(B))\,.\!\vee\!:\cdot: [C] : C \,\varepsilon\, \mathbf{el}(A) \;.\!\supset\!.$
$C \,\varepsilon\, \mathbf{el}(B) :\cdot: [Ca] :\cdot: [D] :: D \,\varepsilon\, C \;.\!\equiv\!\therefore\; D \,\varepsilon\, D \therefore [E] \therefore D \,\varepsilon\, \mathbf{el}(E) \;.\!\equiv\!:\; [F] :$
$F \,\varepsilon\, a \;.\!\supset\!.\; F \,\varepsilon\, \mathbf{el}(E) \;::\!\supset\!\therefore\; [G] : G \,\varepsilon\, \mathbf{el}(A) \;.\!\supset\!.\; [\exists HI]\,.\,H \,\varepsilon\, a\,.\,I \,\varepsilon\, \mathbf{el}(G)\,.$
$I \,\varepsilon\, \mathbf{el}(H) \;:\!\equiv\!.\; A \,\varepsilon\, \mathbf{el}(C)$

$\{CA1, E3\}$ can easily be shown to be inferentially equivalent to the set of presuppositions consisting of *E3* and

CA1.1 $[AB] :\cdot: A \,\varepsilon\, \mathbf{el}(B) \;.\!\equiv\!::\; B \,\varepsilon\, B :: \sim\!(B \,\varepsilon\, \mathbf{el}(B))\,.\!\vee\!\therefore [C] : C \,\varepsilon\, \mathbf{el}(A) \;.\!\supset\!.$
$C \,\varepsilon\, \mathbf{el}(B) \therefore [a] \therefore [C] : C \,\varepsilon\, \mathbf{el}(A) \;.\!\supset\!.\; [\exists DE]\,.\,D \,\varepsilon\, a\,.\,E \,\varepsilon\, \mathbf{el}(C)\,.$
$E \,\varepsilon\, \mathbf{el}(D) \;:\!\equiv\!.\; A \,\varepsilon\, \mathbf{el}(\mathbf{Kl}(a))$

CA1.1 and *E3* being regarded as axioms with '**el**' and '**Kl**' as undefined mereological terms.

My principal aim is to prove that two systems of mereology, System 𝔄 based on *AA1* as the only axiom and System ℭ based on *CA1* as the only axiom, are inferentially equivalent to one another. I shall also prove that System 𝔅 based on *BA1* as the only axiom is inferentially equivalent to System 𝔄. I wish to do this on the present occasion because the paper referred to in footnote 4 is no longer easily available.

2 In order to achieve our aim we deduce, within the framework of System 𝔄, the following theses:

AA1 $[AB] :: A \,\varepsilon\, \mathbf{el}(B) \;.\!\equiv\!:\cdot:\; B \,\varepsilon\, B :\cdot: [Ca] :\cdot: [D] \therefore D \,\varepsilon\, C \;.\!\equiv\!:\; [E] : E \,\varepsilon\, a \;.\!\supset\!.$
$E \,\varepsilon\, \mathbf{el}(D) : [E] : E \,\varepsilon\, \mathbf{el}(D) \;.\!\supset\!.\; [\exists FG]\,.\,F \,\varepsilon\, a\,.\,G \,\varepsilon\, \mathbf{el}(E)\,.\,G \,\varepsilon\, \mathbf{el}(F) ::$
$B \,\varepsilon\, \mathbf{el}(B)\,.\,B \,\varepsilon\, a \;::\!\supset\!.\; A \,\varepsilon\, \mathbf{el}(C)$ [Axiom]

AD1(=E1) $[Aa] \therefore A \,\varepsilon\, A : [B] : B \,\varepsilon\, a \;.\!\supset\!.\; B \,\varepsilon\, \mathbf{el}(A) : [B] : B \,\varepsilon\, \mathbf{el}(A) \;.\!\supset\!.\; [\exists CD]\,.$
$C \,\varepsilon\, a\,.\,D \,\varepsilon\, \mathbf{el}(B)\,.\,D \,\varepsilon\, \mathbf{el}(C) \;:\!\equiv\!.\; A \,\varepsilon\, \mathbf{Kl}(a)$ [Definition]

AD2 $[AB]:[\exists C].A\,\varepsilon\,\mathbf{Kl}(\mathbf{el}(C)\cap\mathbf{el}(B))\ .\!\equiv\!.\ A\,\varepsilon\,\mathbf{ingr}(B)$ [Definition]

AT1 $[AB]:A\,\varepsilon\,\mathbf{el}(B)\ .\!\supset\!.\ B\,\varepsilon B$ [follows from *AA1*]

AT2 $[Aa]:A\,\varepsilon a\ .\!\supset\!.\ A\,\varepsilon\,\mathbf{el}(A)$ [from *AA1*]

AT3 $[Ea]::E\,\varepsilon a\ .\!\supset\!\therefore\ [A]\therefore A\,\varepsilon\,\mathbf{Kl}(a)\ .\!\equiv\!:\ [B]:B\,\varepsilon a\ .\!\supset\!.\ B\,\varepsilon\,\mathbf{el}(A):[B]:$
$B\,\varepsilon\,\mathbf{el}(A)\ .\!\supset\!.\ [\exists CD].C\,\varepsilon a\,.D\,\varepsilon\,\mathbf{el}(B)\,.D\,\varepsilon\,\mathbf{el}(C)$ [*AD1*; *AT1*]

AT4 $[ABa]:A\,\varepsilon\,\mathbf{el}(B)\,.B\,\varepsilon a\ .\!\supset\!.\ A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))$ [*AA1*; *AT2*; *AT3*]

AT5 $[Aa]:A\,\varepsilon a\ .\!\supset\!.\ A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))$ [*AT4*; *AT2*]

AT6 $[Aa]:A\,\varepsilon a\ .\!\supset\!.\ \mathbf{Kl}(a)\,\varepsilon\,\mathbf{Kl}(a)$ [*AT5*; *AT1*]

AT7 $[Aa]:A\,\varepsilon a\ .\!\supset\!.\ A=\mathbf{Kl}(A)$

PR $[Aa]::\mathrm{Hp}(1)\ .\!\supset\!\therefore$

(2) $A\,\varepsilon\,\mathbf{el}(A):$ [*AT2*; 1]

(3) $[B]:B\,\varepsilon A\ .\!\supset\!.\ B\,\varepsilon\,\mathbf{el}(A):$ [2]

(4) $[B]:B\,\varepsilon\,\mathbf{el}(A)\ .\!\supset\!.\ [\exists DE].D\,\varepsilon A\,.E\,\varepsilon\,\mathbf{el}(B)\,.E\,\varepsilon\,\mathbf{el}(D)\therefore$ [*AT1*; *AT2*]

(5) $A\,\varepsilon\,\mathbf{Kl}(A)\,.$ [*AD1*; 1; 3; 4]

(6) $A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(A))\,.$ [*AT4*; 2; 1]

(7) $\mathbf{Kl}(A)\,\varepsilon\,\mathbf{Kl}(A)\,.$ [*AT1*; 6]

$A=\mathbf{Kl}(A)$ [5; 7]

AT8 $[AB]\therefore B\,\varepsilon B:[a]:B\,\varepsilon\,\mathbf{el}(B)\,.B\,\varepsilon a\ .\!\supset\!.\ A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))\ :\!\supset\!.\ A\,\varepsilon\,\mathbf{el}(B)$

PR $[AB]\therefore\mathrm{Hp}(2)\ \therefore\!\supset\!.$

(3) $B\,\varepsilon\,\mathbf{el}(B)\,.$ [*AT1*; 1]

(4) $A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(B))\,.$ [2; 3; 1]

(5) $B=\mathbf{Kl}(B)\,.$ [*AT7*; 1]

$A\,\varepsilon\,\mathbf{el}(B)$ [4; 5]

AT9(=AA1.1) [*AT1*; *AT4*; *AT8*]

AT10 $[Aa]:A\,\varepsilon a\ .\!\supset\!.\ A=\mathbf{Kl}(\mathbf{el}(A))$

PR $[Aa]::\mathrm{Hp}(1)\ .\!\supset\!\therefore$

(2) $[B]:B\,\varepsilon\,\mathbf{el}(A)\ .\!\supset\!.\ [\exists CD].C\,\varepsilon\,\mathbf{el}(A)\,.D\,\varepsilon\,\mathbf{el}(B)\,.D\,\varepsilon\,\mathbf{el}(C)\therefore$ [*AT2*]

(3) $A\,\varepsilon\,\mathbf{Kl}(\mathbf{el}(A))\,.$ [*AD1*; 1; 2]

(4) $A\,\varepsilon\,\mathbf{el}(A)\,.$ [*AT2*; 1]

(5) $\mathbf{Kl}(\mathbf{el}(A))\,\varepsilon\,\mathbf{Kl}(\mathbf{el}(A))$ [*AT6*; 4]

$A=\mathbf{Kl}(\mathbf{el}(A))$ [3; 5]

AT11 $[ABC]:A\,\varepsilon\,\mathbf{el}(B)\,.B\,\varepsilon\,\mathbf{el}(C)\ .\!\supset\!.\ A\,\varepsilon\,\mathbf{el}(C)$

PR $[ABC]:\mathrm{Hp}(2)\ .\!\supset\!.$

(3) $A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(\mathbf{el}(C)))\,.$ [*AT4*; 1; 2]

(4) $C\,\varepsilon C\,.$ [*AT1*; 2]

(5) $C=\mathbf{Kl}(\mathbf{el}(C))\,.$ [*AT10*; 4]

$A\,\varepsilon\,\mathbf{el}(C)$ [3; 5]

AT12 $[AB]\therefore A\,\varepsilon A:[C]:C\,\varepsilon\,\mathbf{el}(A)\ .\!\supset\!.\ [\exists D].D\,\varepsilon\,\mathbf{el}(C)\,.D\,\varepsilon\,\mathbf{el}(B)\ :\!\supset\!.$
$A\,\varepsilon\,\mathbf{Kl}(\mathbf{el}(A)\cap\mathbf{el}(B))$

PR $[AB]::\mathrm{Hp}(2)\ :\!\supset\!\therefore$

(3) $[C]:C\,\varepsilon\,\mathbf{el}(A)\cap\mathbf{el}(B)\ .\!\supset\!.\ C\,\varepsilon\,\mathbf{el}(A):$

(4) $[C]:C\,\varepsilon\,\mathbf{el}(A)\ .\!\supset\!.\ [\exists B].D\,\varepsilon\,\mathbf{el}(A)\cap\mathbf{el}(B)\,.E\,\varepsilon\,\mathbf{el}(C)\,.E\,\varepsilon\,\mathbf{el}(D)\therefore$
[2; *AT11*; *AT2*]

$A\,\varepsilon\,\mathbf{Kl}(\mathbf{el}(A)\cap\mathbf{el}(B))$ [*AD1*; 1; 2; 3]

AT13 $[ABC]:A\,\varepsilon\,\mathbf{Kl}(\mathbf{ingr}(B))\,.C\,\varepsilon\,\mathbf{el}(A)\ .\!\supset\!.\ [\exists DE].D\,\varepsilon\,\mathbf{el}(B)\,.$
$E\,\varepsilon\,\mathbf{el}(C)\,.E\,\varepsilon\,\mathbf{el}(D)$

PR $[ABC] \therefore \mathrm{Hp}(2) .\supset ::$

$[\exists DE] \therefore$

(3) $D \varepsilon \mathbf{ingr}(B) .$

(4) $E \varepsilon \mathbf{el}(C) .$ [*AD1*; 1; 2]

(5) $E \varepsilon \mathbf{el}(D) :$

$[\exists F] :$

(6) $D \varepsilon \mathbf{Kl}(\mathbf{el}(F) \cap \mathbf{el}(B)) .$ [*AD2*; 3]

$[\exists GH] .$

(7) $G \varepsilon \mathbf{el}(B) .$ [*AD1*; 6; 5]

(8) $H \varepsilon \mathbf{el}(E) .$

(9) $H \varepsilon \mathbf{el}(D) .$ [*AT11*; 8; 5]

(10) $H \varepsilon \mathbf{el}(C) ::$ [*AT11*; 8; 4]

$[\exists DE] . D \varepsilon \mathbf{el}(B) . E \varepsilon \mathbf{el}(C) . E \varepsilon \mathbf{el}(D)$ [7; 10; 9]

AT14 $[AB] : A \varepsilon \mathbf{el}(B) .\supset. A \varepsilon \mathbf{ingr}(B)$

PR $[AB] :: \mathrm{Hp}(1) .\supset \therefore$

(2) $[C] : C \varepsilon \mathbf{el}(A) \cap \mathbf{el}(B) .\supset. C \varepsilon \mathbf{el}(A) :$

(3) $[C] : C \varepsilon \mathbf{el}(A) .\supset. [\exists DE] . D \varepsilon \mathbf{el}(A) \cap \mathbf{el}(B) . E \varepsilon \mathbf{el}(C) . E \varepsilon \mathbf{el}(D) \therefore$ [*AT11*; 1; *AT2*]

(4) $A \varepsilon \mathbf{Kl}(\mathbf{el}(A) \cap \mathbf{el}(B)) .$ [*AD1*; 1; 2; 3]

$A \varepsilon \mathbf{ingr}(B)$ [*AD2*; 4]

AT15 $[AB] : A \varepsilon \mathbf{Kl}(\mathbf{ingr}(B)) .\supset. A = B$

PR $[AB] :: \mathrm{Hp}(1) .\supset \therefore$

(2) $[C] : C \varepsilon \mathbf{el}(B) .\supset. C \varepsilon \mathbf{el}(A) :$ [*AT14*; *AD1*; 1]

(3) $[C] : C \varepsilon \mathbf{el}(A) .\supset. [\exists DE] . D \varepsilon \mathbf{el}(B) . E \varepsilon \mathbf{el}(C) . E \varepsilon \mathbf{el}(D) \therefore$ [*AT13*; 1]

(4) $A \varepsilon \mathbf{Kl}(\mathbf{el}(B)) .$ [*AD1*; 1; 2; 3]

(5) $A \varepsilon \mathbf{el}(A) .$ [*AT2*; 4]

(6) $B \varepsilon B .$ [3; 5; *AT1*]

(7) $B = \mathbf{Kl}(\mathbf{el}(B)) .$ [*AT10*; 6]

$A = B$ [4; 7]

AT16 $[AB] \therefore A \varepsilon A : [C] : C \varepsilon \mathbf{el}(A) .\supset. [\exists B] . D \varepsilon \mathbf{el}(C) . D \varepsilon \mathbf{el}(B) :\supset. A \varepsilon \mathbf{el}(B)$[6]

PR $[AB] \therefore \mathrm{Hp}(2) :\supset.$

(3) $A \varepsilon \mathbf{Kl}(\mathbf{el}(A) \cap \mathbf{el}(B)) .$ [*AT12*; 1; 2]

(4) $A \varepsilon \mathbf{ingr}(B) .$ [*AD2*; 3]

(5) $A \varepsilon \mathbf{el}(\mathbf{Kl}(\mathbf{ingr}(B))) .$ [*AT5*; 4]

(6) $\mathbf{Kl}(\mathbf{ingr}(B)) \varepsilon \mathbf{Kl}(\mathbf{ingr}(B)) .$ [*AT1*; 5]

(7) $\mathbf{Kl}(\mathbf{ingr}(B)) = B .$ [*AT15*; 6]

$A \varepsilon \mathbf{el}(B)$ [5; 7]

AT17 $[ABCa] : A \varepsilon \mathbf{Kl}(a) . C \varepsilon \mathbf{el}(A) . C \varepsilon \mathbf{el}(B) .\supset. [\exists DE] . D \varepsilon a . E \varepsilon \mathbf{el}(B) . E \varepsilon \mathbf{el}(D)$

PR $[ABCa] : \mathrm{Hp}(3) .\supset.$

$[\exists DE] .$

(4) $D \varepsilon a .$

(5) $E \varepsilon \mathbf{el}(C) .$ [*AD1*; 1; 2]

(6) $E \varepsilon \mathbf{el}(D) .$

(7) $E \varepsilon \mathbf{el}(B) .$ [*AT11*; 5; 3]

$[\exists DE] . D \varepsilon a . E \varepsilon \mathbf{el}(B) . E \varepsilon \mathbf{el}(D)$ [4; 7; 6]

AT18 $[ABEa]:A\,\varepsilon\,\mathbf{Kl}(a).D\,\varepsilon\,a.E\,\varepsilon\,\mathbf{el}(B).E\,\varepsilon\,\mathbf{el}(D).\supset.[\exists C].C\,\varepsilon\,\mathbf{el}(A).$

$C\,\varepsilon\,\mathbf{el}(B)$

PR $[ABEa]:\mathrm{Hp}(4).\supset.$

(5) $D\,\varepsilon\,\mathbf{el}(A).$ [*AD1*; 1; 2]

(6) $E\,\varepsilon\,\mathbf{el}(A).$ [*AT11*; 4; 5]

$[\exists C].C\,\varepsilon\,\mathbf{el}(A).C\,\varepsilon\,\mathbf{el}(B)$ [6; 3]

AT19 $[AFGa]::[B]:[\exists C]:C\,\varepsilon\,\mathbf{el}(A).C\,\varepsilon\,\mathbf{el}(B).\equiv.[\exists DE].D\,\varepsilon\,a.E\,\varepsilon\,\mathbf{el}(B).$

$E\,\varepsilon\,\mathbf{el}(D)\therefore F\,\varepsilon\,a.G\,\varepsilon\,\mathbf{el}(F)\therefore\supset.[\exists H].H\,\varepsilon\,\mathbf{el}(G).H\,\varepsilon\,\mathbf{el}(A)$

PR $[AFGa]::\mathrm{Hp}(3)\therefore\supset.$

(4) $G\,\varepsilon\,\mathbf{el}(G).$ [*AT2*; 3]

$[\exists H].H\,\varepsilon\,\mathbf{el}(G).H\,\varepsilon\,\mathbf{el}(A)$ [1; 2; 4; 3]

AT20 $[AFa]::[B]:[\exists C].C\,\varepsilon\,\mathbf{el}(A).C\,\varepsilon\,\mathbf{el}(B).\equiv.[\exists DE].D\,\varepsilon\,a.$

$E\,\varepsilon\,\mathbf{el}(B).E\,\varepsilon\,\mathbf{el}(D)\therefore F\,\varepsilon\,a\therefore\supset.F\,\varepsilon\,\mathbf{el}(A)$

PR $[AFa]::\mathrm{Hp}(2)\therefore\supset\therefore$

(3) $[G]:G\,\varepsilon\,\mathbf{el}(F).\supset.[\exists H].H\,\varepsilon\,\mathbf{el}(G).H\,\varepsilon\,\mathbf{el}(A)\therefore$ [*AT19*; 1; 2]

$F\,\varepsilon\,\mathbf{el}(A)$ [*AT16*; 2; 3]

AT21 $[ACa]::[B]:[\exists C].C\,\varepsilon\,\mathbf{el}(A).C\,\varepsilon\,\mathbf{el}(B).\equiv.[\exists DE].D\,\varepsilon\,a.E\,\varepsilon\,\mathbf{el}(B).$

$E\,\varepsilon\,\mathbf{el}(D)\therefore C\,\varepsilon\,\mathbf{el}(A)\therefore\supset.[\exists DE].D\,\varepsilon\,a.E\,\varepsilon\,\mathbf{el}(C).E\,\varepsilon\,\mathbf{el}(D)$

PR $[ACa]::\mathrm{Hp}(2)\therefore\supset.$

(3) $C\,\varepsilon\,\mathbf{el}(C).$ [*AT2*; 2]

$[\exists DE].D\,\varepsilon\,a.E\,\varepsilon\,\mathbf{el}(C).E\,\varepsilon\,\mathbf{el}(D)$ [1; 2; 3]

AT22 $[Aa]\therefore A\,\varepsilon\,A:[B]:[\exists C].C\,\varepsilon\,\mathbf{el}(A).C\,\varepsilon\,\mathbf{el}(B).\equiv.[\exists DE].D\,\varepsilon\,a.$

$E\,\varepsilon\,\mathbf{el}(B).E\,\varepsilon\,\mathbf{el}(D):\supset.A\,\varepsilon\,\mathbf{Kl}(a)$

PR $[Aa]::\mathrm{Hp}(2):\supset\therefore$

(3) $[B]:B\,\varepsilon\,a.\supset.B\,\varepsilon\,\mathbf{el}(A):$ [*AT20*; 2]

(4) $[B]:B\,\varepsilon\,\mathbf{el}(A).\supset.[\exists DE].D\,\varepsilon\,a.E\,\varepsilon\,\mathbf{el}(B).E\,\varepsilon\,\mathbf{el}(D)\therefore$ [*AT21*; 2]

$A\,\varepsilon\,\mathbf{Kl}(a)$ [*AD1*; 1; 3; 4]

AT23(=E2) [*AT22*; *AT17*; *AT18*]

AT24 $[ABCa]:A\,\varepsilon\,\mathbf{el}(B).B\,\varepsilon\,\mathbf{el}(C).C\,\varepsilon\,a.\supset.A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))$

PR $[ABCa]:\mathrm{Hp}(3).\supset.$

(4) $A\,\varepsilon\,\mathbf{el}(C).$ [*AT11*; 1; 2]

$A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))$ [*AT4*; 4; 3]

AT25 $[AB]\therefore B\,\varepsilon\,B:[Ca]:B\,\varepsilon\,\mathbf{el}(B).B\,\varepsilon\,\mathbf{el}(C).C\,\varepsilon\,a.\supset.A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a)):\supset.$

$A\,\varepsilon\,\mathbf{el}(B)$

PR $[AB]\therefore\mathrm{Hp}(2):\supset.$

(3) $B\,\varepsilon\,\mathbf{el}(B).$ [*AT2*; 1]

(4) $A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(B)).$ [2; 3; 1]

(5) $B=\mathbf{Kl}(B).$ [*AT7*; 1]

$A\,\varepsilon\,\mathbf{el}(B)$ [4; 5]

AT26(=BA1.1) [*AT1*; *AT24*; *AT25*]

AT27 $[Ga]::G\,\varepsilon\,a.\supset\therefore[A]\therefore A\,\varepsilon\,\mathbf{Kl}(a).\equiv:[B]:[\exists C].C\,\varepsilon\,\mathbf{el}(A).C\,\varepsilon\,\mathbf{el}(B).\equiv.$

$[\exists DE].D\,\varepsilon\,a.E\,\varepsilon\,\mathbf{el}(B).E\,\varepsilon\,\mathbf{el}(D)$ [*AT23*; *AT2*; *AT1*]

AT28 $[ABCDa]\because A\,\varepsilon\,\mathbf{el}(B)\therefore[E]\therefore E\,\varepsilon\,D.\equiv:[F]:[\exists G].G\,\varepsilon\,\mathbf{el}(E).$

$G\,\varepsilon\,\mathbf{el}(F).\equiv.[\exists HI].H\,\varepsilon\,a.I\,\varepsilon\,\mathbf{el}(F).I\,\varepsilon\,\mathbf{el}(H)::B\,\varepsilon\,\mathbf{el}(B).B\,\varepsilon\,\mathbf{el}(C).$

$C\,\varepsilon\,a::\supset.A\,\varepsilon\,\mathbf{el}(D)$

PR $[ABCDa]\because\mathrm{Hp}(5)::\supset::$

(6) $[E] \therefore E \,\varepsilon\, \mathbf{Kl}(a) \;.{\equiv}{:}\; [F] : [\exists G] \,.\, G \,\varepsilon\, \mathbf{el}(E) \,.\, G \,\varepsilon\, \mathbf{el}(F) \;.{\equiv}.\; [\exists HI] \,.\, H \,\varepsilon\, a \,.$
$I \,\varepsilon\, \mathbf{el}(F) \,.\, I \,\varepsilon\, \mathbf{el}(H) ::$ [*AT27*; 5]

(7) $[E] : E \,\varepsilon\, D \;.{\equiv}.\; E \,\varepsilon\, \mathbf{Kl}(a) \therefore$ [2; 6]

(8) $A \,\varepsilon\, \mathbf{el}(\mathbf{Kl}(a)) \,.$ [*AT24*; 1; 4; 5]

$A \,\varepsilon\, \mathbf{el}(D)$ [Extensionality; 7; 8]

AT29 $[AB] ::\cdot\, B \,\varepsilon\, B \because [CDa] \because [E] \therefore E \,\varepsilon\, D \;.{\equiv}{:}\; [F] : [\exists G] \,.\, G \,\varepsilon\, \mathbf{el}(E) \,.$
$G \,\varepsilon\, \mathbf{el}(F) \;.{\equiv}.\; [\exists HI] \,.\, H \,\varepsilon\, a \,.\, I \,\varepsilon\, \mathbf{el}(F) \,.\, I \,\varepsilon\, \mathbf{el}(H) :: B \,\varepsilon\, \mathbf{el}(B) \,.\, B \,\varepsilon\, \mathbf{el}(C) \,.$
$C \,\varepsilon\, a \;::\supset.\; A \,\varepsilon\, \mathbf{el}(D) \;\because\supset.\; A \,\varepsilon\, \mathbf{el}(B)$

PR $[AB] ::\cdot\, \mathrm{Hp}(2) \;\because\supset::$

(3) $B \,\varepsilon\, \mathbf{el}(B) \therefore$ [*AT2*; 1]

(4) $[E] \therefore E \,\varepsilon\, \mathbf{Kl}(B) \;.{\equiv}{:}\; [F] : [\exists G] \,.\, G \,\varepsilon\, \mathbf{el}(E) \,.\, G \,\varepsilon\, \mathbf{el}(F) \;.{\equiv}.\; [\exists HI] \,.$
$H \,\varepsilon\, B \,.\, I \,\varepsilon\, \mathbf{el}(F) \,.\, I \,\varepsilon\, \mathbf{el}(H) ::$ [*AT27*; 1]

(5) $A \,\varepsilon\, \mathbf{el}(\mathbf{Kl}(B)) \,.$ [2; 4; 3; 1]

(6) $B = \mathbf{Kl}(B) \,.$ [*AT7*; 1]

$A \,\varepsilon\, \mathbf{el}(B)$ [5; 6]

AT30(=*BA1*) [*AT1*; *AT28*; *AT29*]

AT31 $[ABCa] : A \,\varepsilon\, \mathbf{Kl}(a) \,.\, A \,\varepsilon\, \mathbf{el}(B) \,.\, C \,\varepsilon\, a \;.{\supset}.\; C \,\varepsilon\, \mathbf{el}(B)$

PR $[ABCa] : \mathrm{Hp}(3) \;.{\supset}.$

(4) $C \,\varepsilon\, \mathbf{el}(A) \,.$ [*AD1*; 1; 3]

$C \,\varepsilon\, \mathbf{el}(B)$ [*AT11*; 4; 2]

AT32 $[ABDa] :: A \,\varepsilon\, \mathbf{Kl}(a) : [C] : C \,\varepsilon\, a \;.{\supset}.\; C \,\varepsilon\, \mathbf{el}(B) \therefore D \,\varepsilon\, \mathbf{el}(A) \;\therefore\supset.\; [\exists E] \,.$
$E \,\varepsilon\, \mathbf{el}(D) \,.\, E \,\varepsilon\, \mathbf{el}(B)$

PR $[ABDa] :: \mathrm{Hp}(3) \;\therefore\supset.$

$[\exists EF] \,.$

(4) $E \,\varepsilon\, a \,.$
(5) $F \,\varepsilon\, \mathbf{el}(D) \,.$ [*AD1*; 1; 3]
(6) $F \,\varepsilon\, \mathbf{el}(E) \,.$

(7) $E \,\varepsilon\, \mathbf{el}(B) \,.$ [2; 4]

(8) $F \,\varepsilon\, \mathbf{el}(B) \,.$ [*AT11*; 6; 7]

$[\exists E] \,.\, E \,\varepsilon\, \mathbf{el}(D) \,.\, E \,\varepsilon\, \mathbf{el}(B)$ [5; 8]

AT33 $[ABa] \therefore A \,\varepsilon\, \mathbf{Kl}(a) : [C] : C \,\varepsilon\, a \;.{\supset}.\; C \,\varepsilon\, \mathbf{el}(B) \;:{\supset}.\; A \,\varepsilon\, \mathbf{el}(B)$

PR $[ABa] :: \mathrm{Hp}(2) \;:{\supset}\therefore$

(3) $[C] : C \,\varepsilon\, \mathbf{el}(A) \;.{\supset}.\; [\exists D] \,.\, D \,\varepsilon\, \mathbf{el}(C) \,.\, D \,\varepsilon\, \mathbf{el}(B) \therefore$ [*AT32*; 1; 2]

$A \,\varepsilon\, \mathbf{el}(B)$ [*AT16*; 1; 3]

AT34 $[Aa] :: A \,\varepsilon\, \mathbf{Kl}(a) \;.{\supset}\therefore\; [B] \therefore A \,\varepsilon\, \mathbf{el}(B) \;.{\equiv}{:}\; [C] : C \,\varepsilon\, a \;.{\supset}.\; C \,\varepsilon\, \mathbf{el}(B)$
[*AT31*; *AT33*]

AT35 $[AB] : A \,\varepsilon\, \mathbf{el}(B) \;.{\supset}.\; B \,\varepsilon\, \mathbf{Kl}(A \cup B)$

PR $[AB] :: \mathrm{Hp}(1) \;.{\supset}\therefore$

(2) $B \,\varepsilon\, B \,.$ [*AT1*; 1]

(3) $B \,\varepsilon\, \mathbf{el}(B) :$ [*AT2*; 2]

(4) $[C] : C \,\varepsilon\, A \cup B \;.{\supset}.\; C \,\varepsilon\, \mathbf{el}(B) :$ [1; 3]

(5) $[C] : C \,\varepsilon\, \mathbf{el}(B) \;.{\supset}.\; [\exists DE] \,.\, D \,\varepsilon\, A \cup B \,.\, E \,\varepsilon\, \mathbf{el}(C) \,.\, E \,\varepsilon\, \mathbf{el}(D) \therefore$ [*AT1*; *AT2*]

$B \,\varepsilon\, \mathbf{Kl}(A \cup B)$ [*AD1*; 2; 4; 5]

AT36 $[AB] : A \,\varepsilon\, \mathbf{el}(B) \,.\, B \,\varepsilon\, \mathbf{el}(A) \;.{\supset}.\; A \,\varepsilon\, B$

PR $[AB] : \mathrm{Hp}(2) \;.{\supset}.$

(3) $A \,\varepsilon\, A \cup B \,.$ [1]

(4) $\mathbf{Kl}(A \cup B) \,\varepsilon\, \mathbf{Kl}(A \cup B) \,.$ [*AT6*; 3]

(5) $A\,\varepsilon\,\mathbf{Kl}(A\cup B)\,.$ [*AT35*; 2]

(6) $B\,\varepsilon\,\mathbf{Kl}(A\cup B)\,.$ [*AT35*; 1]

$A\,\varepsilon\,B$ [4; 5; 6]

AT37 $[Aa]::A\,\varepsilon\,A\therefore[B]\therefore A\,\varepsilon\,\mathbf{el}(B)\;.{\equiv}:\;[C]:C\,\varepsilon\,a\;.{\supset}.\;C\,\varepsilon\,\mathbf{el}(B)\;\therefore{\supset}.$
$A\,\varepsilon\,\mathbf{Kl}(a)$

PR $[Aa]::\mathrm{Hp}(2)\;\therefore{\supset}\therefore$

(3) $A\,\varepsilon\,\mathbf{el}(A):$ [*AT2*; 1]

(4) $[C]:C\,\varepsilon\,a\;.{\supset}.\;C\,\varepsilon\,\mathbf{el}(A)\therefore$ [2; 3]

(5) $\sim(A\,\varepsilon\,\mathbf{el}(\wedge))\,.$ [*AT1*]

(6) $[\exists C]\,.\,C\,\varepsilon\,a:$ [2; 5]

(7) $\mathbf{Kl}(a)\,\varepsilon\,\mathbf{Kl}(a):$ [*AT6*; 6]

(8) $[C]:C\,\varepsilon\,a\;.{\supset}.\;C\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))\therefore$ [*AD1*; 7]

(9) $A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))\,.$ [2; 8]

(10) $\mathbf{Kl}(a)\,\varepsilon\,\mathbf{el}(A)\,.$ [*AT33*; 7; 4]

$A\,\varepsilon\,\mathbf{Kl}(a)$ [*AT36*; 9; 10]

AT38(=E3) [*AT34*; *AT37*]

AT39 $[AFa]::[C]:C\,\varepsilon\,\mathbf{el}(A)\;.{\supset}.\;[\exists DE]\,.\,D\,\varepsilon\,a\,.\,E\,\varepsilon\,\mathbf{el}(C)\,.\,E\,\varepsilon\,\mathbf{el}(D)\therefore$
$F\,\varepsilon\,\mathbf{el}(A)\;\therefore{\supset}.\;[\exists D]\,.\,D\,\varepsilon\,\mathbf{el}(F)\,.\,D\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))$

PR $[AFa]::\mathrm{Hp}(2)\;\therefore{\supset}.$
$[\exists DE]\,.$

$$\left.\begin{array}{ll}(3) & D\,\varepsilon\,a\,.\\ (4) & E\,\varepsilon\,\mathbf{el}(F)\,.\\ (5) & E\,\varepsilon\,\mathbf{el}(D)\,.\end{array}\right\}\qquad[1;\,2]$$

(6) $D\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))\,.$ [*AT5*; 3]

(7) $E\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))\,.$ [*AT11*; 5; 6]

$[\exists D]\,.\,D\,\varepsilon\,\mathbf{el}(F)\,.\,D\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))$ [4; 7]

AT40 $[ABa]\therefore A\,\varepsilon\,\mathbf{el}(B):[C]:C\,\varepsilon\,\mathbf{el}(A)\;.{\supset}.\;[\exists DE]\,.\,D\,\varepsilon\,a\,.\,E\,\varepsilon\,\mathbf{el}(C)\,.$
$E\,\varepsilon\,\mathbf{el}(D)\;:{\supset}.\;A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))$

PR $[ABa]::\mathrm{Hp}(2)\;:{\supset}\therefore$

(3) $[C]:C\,\varepsilon\,\mathbf{el}(A)\;.{\supset}.\;[\exists D]\,.\,D\,\varepsilon\,\mathbf{el}(C)\,.\,D\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))\therefore$ [*AT39*; 2]

$A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))$ [*AT16*; 1; 3]

AT41 $[ACa]:A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))\,.\,C\,\varepsilon\,\mathbf{el}(A)\;.{\supset}.\;[\exists DE]\,.\,D\,\varepsilon\,a\,.\,E\,\varepsilon\,\mathbf{el}(C)\,.\,E\,\varepsilon\,\mathbf{el}(D)$

PR $[ACa]:\mathrm{Hp}(2)\;.{\supset}.$

(3) $\mathbf{Kl}(a)\,\varepsilon\,\mathbf{Kl}(a)\,.$ [*AT1*; 1]

(4) $C\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))\,.$ [*AT11*; 2; 1]

$[\exists DE]\,.\,D\,\varepsilon\,a\,.\,E\,\varepsilon\,\mathbf{el}(C)\,.\,E\,\varepsilon\,\mathbf{el}(D)$ [*AD1*; 3; 4]

AT42 $[AB]::A\,\varepsilon\,\mathbf{el}(B)\;.{\supset}\therefore\;[a]\therefore[C]:C\,\varepsilon\,\mathbf{el}(A)\;.{\supset}.\;[\exists DE]\,.\,D\,\varepsilon\,a\,.$
$E\,\varepsilon\,\mathbf{el}(C)\,.\,E\,\varepsilon\,\mathbf{el}(D)\;:{\equiv}.\;A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))$ [*AT40*; *AT41*]

AT43 $[A]\because[a]\therefore[C]:C\,\varepsilon\,\mathbf{el}(A)\;.{\supset}.\;[\exists DE]\,.\,D\,\varepsilon\,a\,.\,E\,\varepsilon\,\mathbf{el}(C)\,.\,E\,\varepsilon\,\mathbf{el}(D)\;:{\equiv}.$
$A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))\;::{\supset}.\;A\,\varepsilon\,A$

PR $[A]\because\mathrm{Hp}(1)\;::{\supset}\therefore$

(2) $[C]:C\,\varepsilon\,\mathbf{el}(A)\;.{\supset}.\;[\exists DE]\,.\,D\,\varepsilon\,A\,.\,E\,\varepsilon\,\mathbf{el}(C)\,.\,E\,\varepsilon\,\mathbf{el}(D)\therefore$ [*AT1*; *AT2*]

$A\,\varepsilon\,A$ [1; 4]

AT44 $[AB]::[C]:C\,\varepsilon\,\mathbf{el}(A)\;.{\supset}.\;C\,\varepsilon\,\mathbf{el}(B)\therefore[a]\therefore[C]:C\,\varepsilon\,\mathbf{el}(A)\;.{\supset}.\;[\exists DE]\,.$
$D\,\varepsilon\,a\,.\,E\,\varepsilon\,\mathbf{el}(C)\,.\,E\,\varepsilon\,\mathbf{el}(D)\;:{\equiv}.\;A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))\;\therefore{\supset}.\;A\,\varepsilon\,\mathbf{el}(B)$

PR $[AB]::\mathrm{Hp}(2)\;\therefore{\supset}.$

(3) $A \varepsilon A .$ [*AT43*; 2]
(4) $A \varepsilon \mathbf{el}(A) .$ [*AT2*; 3]
$A \varepsilon \mathbf{el}(B)$ [1; 4]
AT45(=*CA1.1*) [*AT1*; *AT2*; *AT11*; *AT42*; *AT44*]
AT46 $[ABCa] \because\because A \varepsilon \mathbf{el}(B) :: [D] :: D \varepsilon C \ .\!\equiv\!\therefore\ D \varepsilon D \therefore [E] : D \varepsilon \mathbf{el}(E) \ .\!\equiv\!:\ [F] :$
$F \varepsilon a \ .\!\supset\!.\ F \varepsilon \mathbf{el}(E) \because [D] : D \varepsilon \mathbf{el}(A) \ .\!\supset\!.\ [\exists EF] . E \varepsilon a . F \varepsilon \mathbf{el}(D) .$
$F \varepsilon \mathbf{el}(E) \because\!\supset\!.\ A \varepsilon \mathbf{el}(C)$
PR $[ABCa] \because\because \mathrm{Hp}(3) \ \because\!\supset\!\therefore$
(4) $A \varepsilon \mathbf{el}(\mathbf{Kl}(a)) :$ [*AT40*; 1; 3]
(5) $[D] : D \varepsilon \mathbf{Kl}(a) \ .\!\equiv\!.\ D \varepsilon C \therefore$ [2; *AT38*]
$A \varepsilon \mathbf{el}(C)$ [Extensionality; 5; 4]
AT47 $[ABCGa] \because\because A \varepsilon \mathbf{el}(B) :: [D] :: D \varepsilon C \ .\!\equiv\!\therefore\ D \varepsilon D \therefore [E] \therefore D \varepsilon \mathbf{el}(E) \ .\!\equiv\!:\ [F] :$
$F \varepsilon a \ .\!\supset\!.\ F \varepsilon \mathbf{el}(E) \because A \varepsilon \mathbf{el}(C) . G \varepsilon \mathbf{el}(A) \ \because\!\supset\!.\ [\exists EF] . E \varepsilon a .$
$F \varepsilon \mathbf{el}(G) . F \varepsilon \mathbf{el}(E)$
PR $[ABCGa] \because\because \mathrm{Hp}(4) \ \because\!\supset\!\therefore$
(5) $G \varepsilon \mathbf{el}(C) :$ [*AT11*; 4; 3]
(6) $[D] : D \varepsilon C \ .\!\equiv\!.\ D \varepsilon \mathbf{Kl}(a) \therefore$ [2; *AT38*]
(7) $G \varepsilon \mathbf{el}(\mathbf{Kl}(a)) .$ [Extensionality; 6; 5]
(8) $\mathbf{Kl}(a) \varepsilon \mathbf{Kl}(a) .$ [*AT1*; 7]
$[\exists EF] . E \varepsilon a . F \varepsilon \mathbf{el}(G) . F \varepsilon \mathbf{el}(E)$ [*AD1*; 8; 7]
AT48 $[AB] \because\because A \varepsilon \mathbf{el}(B) \ .\!\supset\!\because\ [Ca] \because [D] :: D \varepsilon C \ .\!\equiv\!\therefore\ D \varepsilon D \therefore [E] \therefore D \varepsilon \mathbf{el}(E) \ .\!\equiv\!:$
$[F] : F \varepsilon a \ .\!\supset\!.\ F \varepsilon \mathbf{el}(E) \ ::\!\supset\!\therefore\ [G] : G \varepsilon \mathbf{el}(A) \ .\!\supset\!.\ [\exists HI] . H \varepsilon a .$
$I \varepsilon \mathbf{el}(G) . I \varepsilon \mathbf{el}(H) \ :\!\equiv\!.\ A \varepsilon \mathbf{el}(C)$ [*AT46*; *AT47*]
AT49 $[AB] \because\because [C] : C \varepsilon \mathbf{el}(A) \ .\!\supset\!.\ C \varepsilon \mathbf{el}(B) \because [Ca] \because [D] :: D \varepsilon C \ .\!\equiv\!\therefore\ D \varepsilon D \therefore$
$[E] \therefore D \varepsilon \mathbf{el}(E) \ .\!\equiv\!:\ [F] : F \varepsilon a \ .\!\supset\!.\ F \varepsilon \mathbf{el}(E) \ ::\!\supset\!\therefore\ [G] : G \varepsilon \mathbf{el}(A) \ .\!\supset\!.$
$[\exists HI] . H \varepsilon a . I \varepsilon \mathbf{el}(G) . I \varepsilon \mathbf{el}(H) \ :\!\equiv\!.\ A \varepsilon \mathbf{el}(C) \ \because\!\supset\!.\ A \varepsilon \mathbf{el}(B)$
PR $[AB] \because\because \mathrm{Hp}(2) \ \because\!\supset\!\therefore$
(3) $[G] : G \varepsilon \mathbf{el}(A) \ .\!\supset\!.\ [\exists HI] . H \varepsilon A . I \varepsilon \mathbf{el}(G) . I \varepsilon \mathbf{el}(H) \therefore$ [*AT1*; *AT2*]
(4) $A \varepsilon \mathbf{el}(\mathbf{Kl}(A)) .$ [2; *AT38*; 3]
(5) $A = \mathbf{Kl}(A) .$ [*AT7*; 4]
(6) $A \varepsilon \mathbf{el}(A) .$ [4; 5]
$A \varepsilon \mathbf{el}(B)$ [1; 6]
AT50(=*CA1*) [*AT1*; *AT11*; *AT48*; *AT2*; *AT49*]

It is evident, from *AT30* and *AT23*, that any thesis derivable within the framework of System 𝔅 is also derivable within the framework of System 𝔄. And *AT50* and *AT38*, between them, show that within the framework of System 𝔄 we can derive any thesis derivable within the framework of System ℭ.

3 In this section a number of theses will be deduced within the framework of System 𝔅. It will appear from them that in this system we can prove any thesis which is derivable within the framework of System 𝔄. Our deductions proceed as follows:

BA1 $[AB] \because\because A \varepsilon \mathbf{el}(B) \ .\!\equiv\!\because\ B \varepsilon B \because [CDa] \because [E] \therefore E \varepsilon D \ .\!\equiv\!:\ [F] : [\exists G] .$
$G \varepsilon \mathbf{el}(E) . G \varepsilon \mathbf{el}(F) \ .\!\equiv\!.\ [\exists HI] . H \varepsilon a . I \varepsilon \mathbf{el}(F) . I \varepsilon \mathbf{el}(H) :: B \varepsilon \mathbf{el}(B) .$
$B \varepsilon \mathbf{el}(C) . C \varepsilon a \ ::\!\supset\!.\ A \varepsilon \mathbf{el}(D)$ [Axiom]

BD1(=E2) $[Aa]\therefore A\,\varepsilon\,A:[B]:[\exists C].C\,\varepsilon\,\mathbf{el}(A).C\,\varepsilon\,\mathbf{el}(B).\equiv.[\exists DE].D\,\varepsilon\,a.$

$E\,\varepsilon\,\mathbf{el}(B).E\,\varepsilon\,\mathbf{el}(D):\equiv.A\,\varepsilon\,\mathbf{Kl}(a)$ [Definition]

BT1 $[AB].A\,\varepsilon\,\mathbf{el}(B).\supset.B\,\varepsilon\,B$ [*BA1*]

BT2 $[Aa]:A\,\varepsilon\,a.\supset.A\,\varepsilon\,\mathbf{el}(A)$ [*BA1*]

BT3 $[Fa]::F\,\varepsilon\,a.\supset\therefore[A]\therefore A\,\varepsilon\,\mathbf{Kl}(a).\equiv:[B]:[\exists C].C\,\varepsilon\,\mathbf{el}(A).C\,\varepsilon\,\mathbf{el}(B).\equiv.$

$[\exists DE].D\,\varepsilon\,a.E\,\varepsilon\,\mathbf{el}(B).E\,\varepsilon\,\mathbf{el}(D)$ [*BD1*; *BT2*; *BT1*]

BT4 $[ABCa]:A\,\varepsilon\,\mathbf{el}(B).B\,\varepsilon\,\mathbf{el}(C).C\,\varepsilon\,a.\supset.A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a))$

[*BA1*; *BT3*; *BT2*]

BT5 $[Aa]:A\,\varepsilon\,a.\supset.\mathbf{Kl}(a)\,\varepsilon\,\mathbf{Kl}(a)$ [*BT4*; *BT2*; *BT1*]

BT6 $[Aa]:A\,\varepsilon\,a.\supset.A=\mathbf{Kl}(A)$

PR $[Aa]::\mathrm{Hp}(1).\supset\therefore$

(2) $[B]:[\exists C].C\,\varepsilon\,\mathbf{el}(A).C\,\varepsilon\,\mathbf{el}(B).\equiv.[\exists DE].D\,\varepsilon\,A.E\,\varepsilon\,\mathbf{el}(B).E\,\varepsilon\,\mathbf{el}(D)\therefore$

[1]

(3) $A\,\varepsilon\,\mathbf{Kl}(A).$ [*BT3*; 1; 2]

(4) $\mathbf{Kl}(A)\,\varepsilon\,\mathbf{Kl}(A).$ [*BT5*; 1]

$A=\mathbf{Kl}(A)$ [3; 4]

BT7 $[ABC]:A\,\varepsilon\,\mathbf{el}(B).B\,\varepsilon\,\mathbf{el}(C).\supset.A\,\varepsilon\,\mathbf{el}(C)$

PR $[ABC]:\mathrm{Hp}(2).\supset.$

(3) $C\,\varepsilon\,C.$ [*BT1*; 2]

(4) $A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(C)).$ [*BT4*; 1; 2; 3]

(5) $C=\mathbf{Kl}(C).$ [*BT6*; 3]

$A\,\varepsilon\,\mathbf{el}(C)$ [4; 5]

BT8 $[ABa]:A\,\varepsilon\,\mathbf{Kl}(a).B\,\varepsilon\,a.\supset.B\,\varepsilon\,\mathbf{el}(A)$

PR $[ABa]:\mathrm{Hp}(2).\supset.$

(3) $B\,\varepsilon\,\mathbf{el}(B).$ [*BT2*; 2]

(4) $B\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a)).$ [*BT4*; 3; 2]

(5) $\mathbf{Kl}(a)\,\varepsilon\,\mathbf{Kl}(a).$ [*BT1*; 4]

(6) $A=\mathbf{Kl}(a).$ [1; 5]

$B\,\varepsilon\,\mathbf{el}(A)$ [4; 6]

BT9 $[ABa]:A\,\varepsilon\,\mathbf{Kl}(a).B\,\varepsilon\,\mathbf{el}(A).\supset.[\exists CD].C\,\varepsilon\,a.D\,\varepsilon\,\mathbf{el}(B).D\,\varepsilon\,\mathbf{el}(C)$

[*BD1*; *BT2*]

BT10 $[Aa]\therefore A\,\varepsilon\,A:[B]:B\,\varepsilon\,a.\supset.B\,\varepsilon\,\mathbf{el}(A):[B]:B\,\varepsilon\,\mathbf{el}(A).\supset.[\exists CD].$

$C\,\varepsilon\,a.D\,\varepsilon\,\mathbf{el}(B).D\,\varepsilon\,\mathbf{el}(C):\supset.A\,\varepsilon\,\mathbf{Kl}(a)$

PR $[Aa]::\mathrm{Hp}(3):\supset\therefore$

(4) $[BC]:C\,\varepsilon\,\mathbf{el}(A).C\,\varepsilon\,\mathbf{el}(B).\supset.[\exists DE].D\,\varepsilon\,a.E\,\varepsilon\,\mathbf{el}(B).E\,\varepsilon\,\mathbf{el}(D):$

[3; *BT7*]

(5) $[BDE]:D\,\varepsilon\,a.E\,\varepsilon\,\mathbf{el}(B).E\,\varepsilon\,\mathbf{el}(D).\supset.[\exists C].C\,\varepsilon\,\mathbf{el}(A).$

$C\,\varepsilon\,\mathbf{el}(B)\therefore$ [2; *BT7*]

$A\,\varepsilon\,\mathbf{Kl}(a)$ [*BD1*; 1; 4; 5]

BT11(=E1) [*BT10*; *BT8*; *BT9*]

BT12 $[AB]\therefore B\,\varepsilon\,B:[a]:B\,\varepsilon\,\mathbf{el}(B).B\,\varepsilon\,a.\supset.A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a)):\supset.A\,\varepsilon\,\mathbf{el}(B)$

PR $[AB]\therefore\mathrm{Hp}(2):\supset.$

(3) $B\,\varepsilon\,\mathbf{el}(B).$ [*BT2*; 1]

(4) $A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(B)).$ [2; 3; 1]

(5) $B=\mathbf{Kl}(B).$ [*BT6*; 1]

$A\,\varepsilon\,\mathbf{el}(B)$ [4; 5]

BT13(=AA1.1) [*BT1*; *BT4*; *BT12*]

BT14 $[ABCa] \because A\,\varepsilon\,\mathbf{el}(B) \therefore [D] \therefore D\,\varepsilon\,C \;.{\equiv}:\; [E] : E\,\varepsilon\,a \;.{\supset}.\; E\,\varepsilon\,\mathbf{el}(D) : [E] :$
$E\,\varepsilon\,\mathbf{el}(D) \;.{\supset}.\; [\exists FG] . F\,\varepsilon\,a . G\,\varepsilon\,\mathbf{el}(E) . G\,\varepsilon\,\mathbf{el}(F) :: B\,\varepsilon\,a \;::{\supset}.\; A\,\varepsilon\,\mathbf{el}(C)$

PR $[ABCa] \because \mathrm{Hp}(3) \;::{\supset}::$

(4) $[D] \therefore D\,\varepsilon\,\mathbf{Kl}(a) \;.{\equiv}:\; [E] : E\,\varepsilon\,a \;.{\supset}.\; E\,\varepsilon\,\mathbf{el}(D) : [E] : E\,\varepsilon\,\mathbf{el}(D) \;.{\supset}.\; [\exists FG] .$
$F\,\varepsilon\,a . G\,\varepsilon\,\mathbf{el}(E) . G\,\varepsilon\,\mathbf{el}(F) ::$ [*BT11*; *BT13*]

(5) $[D] : D\,\varepsilon\,\mathbf{Kl}(a) \;.{\equiv}.\; D\,\varepsilon\,C \therefore$ [4; 2]

(6) $B\,\varepsilon\,\mathbf{el}(B) .$ [*BT13*; 3]

(7) $A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(a)) .$ [*BT13*; 1; 6; 3]

$A\,\varepsilon\,\mathbf{el}(C)$ [Extensionality; 5; 7]

BT15 $[CB] : C\,\varepsilon\,\mathbf{el}(B) \;.{\supset}.\; [\exists DE] . D\,\varepsilon\,B . E\,\varepsilon\,\mathbf{el}(C) . E\,\varepsilon\,\mathbf{el}(D)$

PR $[CB] : \mathrm{Hp}(1) \;.{\supset}.$

(2) $B\,\varepsilon\,B .$ [*BT13*; 1]

(3) $C\,\varepsilon\,\mathbf{el}(C) .$ [*BT13*; 1]

$[\exists DE] . D\,\varepsilon\,B . E\,\varepsilon\,\mathbf{el}(C) . E\,\varepsilon\,\mathbf{el}(D)$ [2; 3; 1]

BT16 $[AB] :: B\,\varepsilon\,B \because [Ca] \because [D] \therefore D\,\varepsilon\,C \;.{\equiv}:\; [E] : E\,\varepsilon\,a \;.{\supset}.\; E\,\varepsilon\,\mathbf{el}(D) : [E] :$
$E\,\varepsilon\,\mathbf{el}(D) \;.{\supset}.\; [\exists FG] . F\,\varepsilon\,a . G\,\varepsilon\,\mathbf{el}(E) . G\,\varepsilon\,\mathbf{el}(F) :: B\,\varepsilon\,\mathbf{el}(B) .$
$B\,\varepsilon\,a \;::{\supset}.\; A\,\varepsilon\,\mathbf{el}(C) \;\because{\supset}.\; A\,\varepsilon\,\mathbf{el}(B)$

PR $[AB] :: \mathrm{Hp}(2) \;\because{\supset}::$

(3) $[D] \therefore D\,\varepsilon\,\mathbf{Kl}(B) \;.{\equiv}:\; [E] : E\,\varepsilon\,B \;.{\supset}.\; E\,\varepsilon\,\mathbf{el}(D) : [E] : E\,\varepsilon\,\mathbf{el}(D) \;.{\supset}.\; [\exists FG] .$
$F\,\varepsilon\,B . G\,\varepsilon\,\mathbf{el}(E) . G\,\varepsilon\,\mathbf{el}(F) ::$ [*BT11*; 1; *BT13*]

(4) $B\,\varepsilon\,\mathbf{el}(B) .$ [*BT13*; 1]

(5) $A\,\varepsilon\,\mathbf{el}(\mathbf{Kl}(B)) :$ [2; 3; 4; 1]

(6) $[C] : C\,\varepsilon\,B \;.{\supset}.\; C\,\varepsilon\,\mathbf{el}(B) \therefore$ [4]

(7) $B\,\varepsilon\,\mathbf{Kl}(B) .$ [*BT11*; 1; 6; *BT15*]

(8) $\mathbf{Kl}(B)\,\varepsilon\,\mathbf{Kl}(B) .$ [*BT13*; 5]

(9) $B = \mathbf{Kl}(B) .$ [7; 8]

$A\,\varepsilon\,\mathbf{el}(B)$ [5; 9]

BT17(=AA1) [*BT14*; *BT16*]

It is evident, from *BT17* and *BT11*, that any thesis derivable within the framework of System $\mathfrak{A}$ can be derived within the framework of System $\mathfrak{B}$.

4 In this section we go on to show that any thesis derivable within the framework of System $\mathfrak{A}$ can be derived within the framework of System $\mathfrak{C}$. This we prove by deducing, within the framework of System $\mathfrak{C}$, the following theses:

CA1 $[AB] \because\!: A\,\varepsilon\,\mathbf{el}(B) \;.{\equiv}::\; B\,\varepsilon\,B :: {\sim}(B\,\varepsilon\,\mathbf{el}(B)) \;.\mathrm{v}\because\; [C] : C\,\varepsilon\,\mathbf{el}(A) \;.{\supset}.$
$C\,\varepsilon\,\mathbf{el}(B) \because [Ca] \because [D] :: D\,\varepsilon\,C \;.{\equiv}\therefore\; D\,\varepsilon\,D \therefore [E] \therefore D\,\varepsilon\,\mathbf{el}(E) \;.{\equiv}:\; [F] :$
$F\,\varepsilon\,a \;.{\supset}.\; F\,\varepsilon\,\mathbf{el}(E) \;::{\supset}\therefore\; [G] : G\,\varepsilon\,\mathbf{el}(A) \;.{\supset}.\; [\exists HI] . H\,\varepsilon\,a .$
$I\,\varepsilon\,\mathbf{el}(G) . I\,\varepsilon\,\mathbf{el}(H) \;:{\equiv}.\; A\,\varepsilon\,\mathbf{el}(C)$ [Axiom]

CD1(=E3) $[Aa] :: A\,\varepsilon\,A \therefore [B] \therefore A\,\varepsilon\,\mathbf{el}(B) \;.{\equiv}:\; [C] : C\,\varepsilon\,a \;.{\supset}.\; C\,\varepsilon\,\mathbf{el}(B) \;\therefore{\equiv}.\; A\,\varepsilon\,\mathbf{Kl}(a)$
[Definition]

CT1 $[AB] : A\,\varepsilon\,\mathbf{el}(B) \;.{\supset}.\; B\,\varepsilon\,B$ [*CA1*]

CT2 $[Aa] : A\,\varepsilon\,a \;.{\supset}.\; A\,\varepsilon\,\mathbf{el}(A)$ [*CA1*]

CT3 $[ABC] : A\,\varepsilon\,\mathbf{el}(B) . B\,\varepsilon\,\mathbf{el}(C) \;.{\supset}.\; A\,\varepsilon\,\mathbf{el}(C)$

PR $[ABC] : \mathrm{Hp}(2) \;.{\supset}.$

(3) $C\,\varepsilon\,C .$ [*CT1*; 2]

(4) $C \varepsilon \mathbf{el}(C).$ [*CT2*; 3]
$A \varepsilon \mathbf{el}(C)$ [*CA1*; 2; 4; 1]
CT4 $[AGa]:A \varepsilon a\,.\,G \varepsilon \mathbf{el}(A)\,.\supset.\,[\exists HI]\,.\,H \varepsilon a\,.\,I \varepsilon \mathbf{el}(G)\,.\,I \varepsilon \mathbf{el}(H)$ [*CT2*]
CT5 $[Aa]:A \varepsilon a\,.\supset.\,A \varepsilon \mathbf{el}(\mathbf{Kl}(a))$
PR $[Aa]::\mathrm{Hp}(1)\,.\supset\therefore$
(2) $A \varepsilon \mathbf{el}(A):$ [*CT2*; 1]
(3) $[G]:G \varepsilon \mathbf{el}(A)\,.\supset.\,[\exists HI]\,.\,H \varepsilon a\,.\,I \varepsilon \mathbf{el}(G)\,.\,I \varepsilon \mathbf{el}(H)\therefore$ [*CT4*; 1]
$A \varepsilon \mathbf{el}(\mathbf{Kl}(a))$ [*CA1*; 2; *CD1*; 3]
CT6 $[Aa]:A \varepsilon a\,.\supset.\,\mathbf{Kl}(a) \varepsilon \mathbf{Kl}(a)$ [*CT5*; *CT1*]
CT7 $[Aa]:A \varepsilon \mathbf{Kl}(a)\,.\supset.\,A = \mathbf{Kl}(a)$
PR $[Aa]:\mathrm{Hp}(1)\,.\supset.$
(2) $\sim(A \varepsilon \mathbf{el}(\Lambda)).$ [*CT1*]
(3) $[\exists C]\,.\,C \varepsilon a.$ [*CD1*; 1; 2]
(4) $\mathbf{Kl}(a) \varepsilon \mathbf{Kl}(a).$ [*CT6*; 3]
$A = \mathbf{Kl}(a)$ [1; 4]
CT8 $[Aa]:A \varepsilon a\,.\supset.\,A = \mathbf{Kl}(A)$
PR $[Aa]\because\mathrm{Hp}(1)\,.\supset::$
(2) $[B]\therefore A \varepsilon \mathbf{el}(B)\,.\equiv:\,[C]:C \varepsilon A\,.\supset.\,C \varepsilon \mathbf{el}(B)::$ [1]
(3) $A \varepsilon \mathbf{Kl}(A).$ [*CD1*; 1; 2]
$A = \mathbf{Kl}(A)$ [*CT7*; 3]
CT9 $[ABa]:A \varepsilon \mathbf{Kl}(a)\,.\,B \varepsilon a\,.\supset.\,B \varepsilon \mathbf{el}(A)$
PR $[ABa]:\mathrm{Hp}(2)\,.\supset.$
(3) $B \varepsilon \mathbf{el}(\mathbf{Kl}(a)).$ [*CT5*; 2]
(4) $A = \mathbf{Kl}(a).$ [*CT7*; 1]
$B \varepsilon \mathbf{el}(A)$ [3; 4]
CT10 $[ABa]:A \varepsilon \mathbf{Kl}(a)\,.\,B \varepsilon \mathbf{el}(A)\,.\supset.\,[\exists CD]\,.\,C \varepsilon a\,.\,D \varepsilon \mathbf{el}(B)\,.\,D \varepsilon \mathbf{el}(C)$
PR $[ABa]:\mathrm{Hp}(2)\,.\supset.$
(3) $A \varepsilon \mathbf{el}(A).$ [*CT2*; 1]
(4) $A = \mathbf{Kl}(a).$ [*CT7*; 1]
(5) $A \varepsilon \mathbf{el}(\mathbf{Kl}(a)).$ [3; 4]
$[\exists CD]\,.\,C \varepsilon a\,.\,D \varepsilon \mathbf{el}(B)\,.\,D \varepsilon \mathbf{el}(C)$ [*CA1*; 3; *CD1*; 5; 2]
CT11 $[A]:A \varepsilon A\,.\supset.\,A \varepsilon \mathbf{Kl}(\mathbf{el}(A))$
PR $[A]\because\mathrm{Hp}(1)\,.\supset::$
(2) $A \varepsilon \mathbf{el}(A)\therefore$ [*CT2*; 1]
(3) $[B]\therefore A \varepsilon \mathbf{el}(B)\,.\supset:\,[C]:C \varepsilon \mathbf{el}(A)\,.\supset.\,C \varepsilon \mathbf{el}(B)\therefore$ [*CA1*; *CT2*]
(4) $[B]\therefore[C]:C \varepsilon \mathbf{el}(A)\,.\supset.\,C \varepsilon \mathbf{el}(B)\,:\supset.\,A \varepsilon \mathbf{el}(B)::$ [2]
$A \varepsilon \mathbf{Kl}(\mathbf{el}(A))$ [*CD1*; 1; 3; 4]
CT12 $[AEFa]::[B]:B \varepsilon a\,.\supset.\,B \varepsilon \mathbf{el}(A)\therefore A \varepsilon \mathbf{el}(E)\,.\,F \varepsilon a\,\therefore\supset.\,F \varepsilon \mathbf{el}(E)$
PR $[AEFa]::\mathrm{Hp}(3)\,\therefore\supset.$
(4) $F \varepsilon \mathbf{el}(A).$ [1; 3]
$F \varepsilon \mathbf{el}(E)$ [*CT3*; 4; 2]
CT13 $[AEa]\therefore A \varepsilon A:[B]:B \varepsilon \mathbf{el}(A)\,.\supset.\,[\exists CD]\,.\,C \varepsilon a\,.\,D \varepsilon \mathbf{el}(B)\,.\,D \varepsilon \mathbf{el}(C):$
$[F]:F \varepsilon a\,.\supset.\,F \varepsilon \mathbf{el}(E)\,\therefore\supset.\,A \varepsilon \mathbf{el}(E)$
PR $[AEa]::\mathrm{Hp}(3)\,:\supset\therefore$
(4) $A \varepsilon \mathbf{el}(A).$ [*CT2*; 1]
(5) $[\exists F]\,.\,F \varepsilon \mathbf{el}(E).$ [2; 4; 3]
(6) $E \varepsilon E.$ [*CT1*; 5]

(7) $E \varepsilon \mathbf{Kl}(\mathbf{el}(E))\,.$ [*CT11*; 6]

(8) $E = \mathbf{Kl}(\mathbf{el}(E)) :$ [*CT7*; 7]

(9) $[B] : B \varepsilon \mathbf{el}(A) \;.\supset.\; [\exists CD] . C \varepsilon \mathbf{el}(E) . D \varepsilon \mathbf{el}(B) . D \varepsilon \mathbf{el}(C) \therefore$ [2; 3]

(10) $A \varepsilon \mathbf{el}(\mathbf{Kl}(\mathbf{el}(E)))\,.$ [*CA1*; 4; *CD1*; 9]

$A \varepsilon \mathbf{el}(E)$ [10; 8]

CT14 $[Aa] \therefore A \varepsilon A : [B] : B \varepsilon a \;.\supset.\; B \varepsilon \mathbf{el}(A) : [B] : B \varepsilon \mathbf{el}(A) \;.\supset.\; [\exists CD] .$
$C \varepsilon a . D \varepsilon \mathbf{el}(B) . D \varepsilon \mathbf{el}(C) \;:\supset.\; A \varepsilon \mathbf{Kl}(a)$

PR $[Aa] :: \mathrm{Hp}(3) \;:\supset\therefore$

(4) $[B] \therefore A \varepsilon \mathbf{el}(B) \;.\supset:\; [C] : C \varepsilon a \;.\supset.\; C \varepsilon \mathbf{el}(B) \therefore$ [*CT12*; 2]

(5) $[B] \therefore [C] : C \varepsilon a \;.\supset.\; C \varepsilon \mathbf{el}(B) \;:\supset.\; A \varepsilon \mathbf{el}(B) \therefore$ [*CT13*; 1; 3]

$A \varepsilon \mathbf{Kl}(a)$ [*CD1*; 1; 4; 5]

CT15(=*E1*) [*CT14*; *CT9*; *CT10*]

CT16 $[ABa] : A \varepsilon \mathbf{el}(B) . B \varepsilon a \;.\supset.\; A \varepsilon \mathbf{el}(\mathbf{Kl}(a))$ [*CT5*; *CT3*]

CT17 $[AB] \therefore B \varepsilon B : [a] : B \varepsilon \mathbf{el}(B) . B \varepsilon a \;.\supset.\; A \varepsilon \mathbf{el}(\mathbf{Kl}(a)) \;:\supset.\; A \varepsilon \mathbf{el}(B)$

PR $[AB] \therefore \mathrm{Hp}(2) \;:\supset.$

(3) $B \varepsilon \mathbf{el}(B)\,.$ [*CT2*; 1]

(4) $A \varepsilon \mathbf{el}(\mathbf{Kl}(B))\,.$ [2; 3; 1]

(5) $B = \mathbf{Kl}(B)\,.$ [*CT8*; 1]

$A \varepsilon \mathbf{el}(B)$ [4; 5]

CT18(=*AA1.1*) [*CT16*; *CT17*]

CT19(=*BT14*) [*CT15*; *CT18*]

CT20(=*BT15*) [*CT18*]

CT21(=*BT16*) [*CT15*; *CT18*; *CT20*]

CT22(=*AA1*) [*CT19*; *CT21*]

It is evident, from *CT22* and *CT15*, that any thesis derivable within the framework of System $\mathfrak{A}$ can be derived within the framework of System $\mathfrak{C}$. This result, together with the results obtained in sections **2** and **3**, show that System $\mathfrak{A}$, System $\mathfrak{B}$, and System $\mathfrak{C}$ are inferentially equivalent to one another. Note that *AA1* and *BA1* consist of twelve ontological units each, while *CA1* consists of fifteen such units. This may indicate that *CA1* can be shortened without preventing us from using *E3* as the definition of '**Kl**'.

NOTES

1. See Leśniewski [5], vol. 33 (1930), p. 82, see also Lejewski [2]. For a general introduction to mereology see Luschei [6], Sobociński [7] and Sobociński [8].

2. This is a slightly simplified version of an axiom first proposed by Sobociński. See Sobociński [7], vol. 2 (1950), p. 257, Sobociński [8], Lejewski [3].

3. See Lejewski [2].

4. *BA1* is a simplified version of the original single axiom proposed in Lejewski [2].

5. See Leśniewski [5], vol. 33 (1930), p. 87.

6. The proof of *AT16* involving the use of *AD2* as an auxiliary definition differs from the original proof given by Leśniewski and involving a different definition. See Leśniewski [5], vol. 31 (1928), pp. 274-277; see also Lejewski [2]. For a proof involving no auxiliary definition see Clay [1].

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