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SOME REMARKS ON METAPHYSICS AND THE MODAL LOGICS F*F

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1 *Kant's Questions 'On the Possibility of Metaphysics'* Kant's 'Prolegomena' [1] sets, and attempts an answer to the following three basic questions. They are:

Q1. 'Is Metaphysics possible at all?'
Q2. 'How is Metaphysics possible as a science?'
Q3. 'How is Metaphysics possible in general?'

Near the end of [1] Kant affirms this:

> And thus I conclude the analytical solution of the main question I set up myself: how is metaphysics possible in general? by ascending from where its use is really given, at least in its consequences, to the grounds of its possibility.

When we reflect on Kant's questions and this central idea of the 'grounds of its possibility', a question underlying and presupposed by all three is that of the logical possibility of metaphysics. Kant did not consider this latter question, nor did he feel any need to ask it, since what Kant accepted as 'logic'—a variety of the traditional Aristotelian logic—was considered by him to be a closed body of knowledge. However, in recent times, the appearance of many (presumed) alternative logics of propositions, e.g., Łukasiewicz [2], [3], Rosser and Turquette [4], and more especially alternative modal propositional logics, e.g., Lewis S1-S5 [5], Łukasiewicz [6], turn this question of the logical possibility of metaphysics into one of fundamental importance. We approach it by way of this question:

Q4. Is there a formal logic of propositions that can serve as a logical grounding for a certain class of metaphysics?

In this paper I will attempt to provide several positive answers to Q4 by appealing to the modal logics **F*F** [7]. Also, some main features of the class of metaphysics to be associated with some of these logics will be outlined.

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2 *Truth, Knowledge, and Belief* The semantics **F***, [7], resolves the idea of truth into the two notions C_1-truth and C_2-truth, i.e., into the two senses in which a proposition can be said to be 'true'.

The C_1-truth-value or relative truth-value of a proposition is its truth-value based on human assessment. In [7] we said a C_1-truth-value of a proposition was one, in quite general terms, derived or based on experience, and we include mathematical and logical propositions as well.

Against the view (i.e., Ayer [8]) that mathematical and logical truths are necessary and certain, we can appeal to the history of mathematics and logic to find plenty of counter-examples against the view that all mathematical and logical truths acceptable at some time, inevitably remain so. The views of von Neumann [9], Brouwer [10], Bourbaki [11], and Whitehead [12] provide further documented evidence from working mathematicians rather than a practising philosopher,

> that it is hardly possible to believe in the existence of an absolute, intuitable concept of mathematical rigor, dissociated from all human experience. Whatever philosophical or epistemological preferences anyone may have in this respect, the mathematical fraternities' actual experiences with its subject give little support to the assumption of the existence of an a priori concept of mathematical rigor. [9]

Von Neumann in [9], referring to Brouwer and Weyl comments:

> It is difficult to overestimate the significance of these events. In the third decade of the twentieth century two mathematicians—both of them of the first magnitude, and as deeply and fully conscious of what mathematics is, or is for, or is about, as anybody could be—actually proposed that the concept of mathematical rigor, of what constitutes an exact proof should be changed!

Also, Brouwer in [10]:

> An a priori character was so consistently ascribed to the laws of theoretical logic that until recently these laws, including the principles of excluded middle, were applied without reservation even in mathematics of infinite systems and we did not allow ourselves to be disturbed by the consideration that the results obtained in this way are in general not open, either practically or theoretically, to any empirical corroboration.

And another mathematician N. Bourbaki [11]:

> Historically speaking, it is of course quite untrue that mathematics is free from contradiction; noncontradiction appears as a goal to be achieved, not as a God-given quality that has been granted us once and for all. . . .modern examples such as the development of the infinitesimal calculus, the theory of series, the theory of sets, all point to the same conclusion. Contradictions do occur; but they cannot be allowed to subsist if the distinction between true and false, proved and unproved is to keep its meaning. Theories have no sharply drawn line between these contradictions which occur in the daily work of every mathematician, beginner or master of his craft, as the result of more or less easily detected mistakes, and the major paradoxes which provide food for logical thought for decades and sometimes

centuries. Absence of contradiction in mathematics as a whole or in any given branch of it, thus appears as an empirical fact, rather than as a metaphysical principle . . .

Whitehead's views on 'number' expressed in both his essays in [12] are especially important, since Whitehead, as co-author of the *Principia Mathematica,* has been intimately concerned with one attempt to provide a satisfactory definition and treatment of 'number'. The fact that he tends to distrust the naive view that even simple arithmetical propositions such as 'One and one make two' are clearly 'necessary and certain' tends to discredit Ayer's simple-minded account of '$7+5 = 12$' in his chapter, "A Priori" [8].

It is perhaps too often forgotten that significant works (and hence significant propositions) in the mathematics, the logics, and the sciences arise always in an historical setting, and are consequently works formed *relative* to a given factual realm. Such a given factual realm depends upon:

1. the condition of human consciousness and current works available at the time (and perhaps even place) in the relevant field,
2. dialogue between figures in a community and also, very likely,
3. earlier works belonging to a tradition, a tradition that may very likely reach far back into the past (this is especially true of works in formal logic).

The factual realm is open-ended. In 1 above 'condition of human consciousness' is meant to be applied both to the organic life of individual persons and also to the organic life of communities.

To discuss C_2-truth, we introduce the postulate of a Supreme Being or the Divine. This is taken as a primitive notion. The justification for making such an assumption is that it clarifies the notions of C_2-truth, C_2-knowledge and, as will be shown later in sections **4** and **6**, it helps to illuminate the concept of human freedom, i.e., gives the concept sharp edges. The C_2-truth-value, or absolute truth-value, of a proposition is its truth-value based on Divine assessment. For some time now, propositions have usually been classified into: (1) synthetic, a posteriori, empirical propositions, e.g., of the sciences, and (2) analytic, a priori, necessary propositions, e.g., of mathematics and logics. A radical departure is made in the semantics $\mathbf{F^*}$ [7], since here we put all our propositions in one basket. The idea of a necessary, certain truth, usually identified as the analytic or apodeictic truths of mathematics and logic, is located in the idea of C_2-truth. But here, *every* proposition considered under C_2 has a C_2-truth-value and so what can be taken as an example of a C_2-truth can be found in propositions quite generally and not exclusively in ones of the mathematical and logical varieties.

We now turn to C_1-belief, C_2-belief, C_1-knowledge, and C_2-knowledge. We begin with knowledge. C_1-knowledge is the kind of knowledge amenable to man and is associated with C_1-truth or propositions under C_1. C_2-knowledge is knowledge amenable to the Divine and is associated with C_2-truth or propositions under C_2. As in the case of knowledge, we need to consider two kinds of belief. C_1-belief is human belief about matters considered under C_1, and C_2-belief is human belief about matters considered under C_2. The basic difference in these two kinds of belief can be illustrated by this simple example. We consider the proposition:

$$\text{The sun will rise tomorrow.} \tag{1}$$

First, under C_1, to believe that this proposition is C_1-true allows us the possibility to believe that (1) could become C_1-knowledge. Under C_2, to believe (1) is C_2-true requires us to believe that (1) is C_2-true despite the fact that this could never become possible knowledge for us. Further remarks on C_2-knowledge and the idea of the Divine are made in section **5**. Thus, quite generally, to believe under C_1 that a proposition is C_1-true means at the same time we can believe that this could become possible C_1-knowledge, whereas to believe under C_2 that a proposition is C_2-true means we believe despite the fact that this could never become possible knowledge for us. C_2-beliefs are of central importance for metaphysics here, and this is discussed in section **3**.

How do the views of truth according to conceptualism and realism relate to the idea of the categories? The notion of C_2-truth and C_2-truth-value generally is a realist conception of truth, which is given a fixed reference point through the idea of the Divine. The notion of C_1-truth is perhaps more problematic, but is probably best related to the conceptualist view of truth. If we turn to mathematical truth, conceptualism is best located in intuitionism, as expounded by Brouwer, Heyting and Weyl, and the intuitionist's view of truth is the one that is of importance for the idea of C_1-truths in mathematics. We discuss here one example of a proposition in the intuitionist-classicist debate, set out in Heyting's "Disputation" [13], and show how this problem is handled in **F*F**.

> Int: . . . Let us compare two definitions of natural numbers, say k and l.
>
> I. k is the greatest prime such that $k - 1$ is also a prime, or $k = 1$ if such a number does not exist.
>
> II. l is the greatest prime such that $l - 2$ is also a prime, or $l = 1$ if such a number does not exist.
>
> Classical mathematics neglects altogether the obvious difference in character between these definitions. k can actually be calculated ($k = 3$), whereas we possess no method for calculating l, as it is not known whether the sequence of pairs of twin primes p, $p + 2$ is finite or not. Therefore intuitionists reject II as a definition of an integer; they consider an integer to be well-defined only if a method for calculating it is given. Now this line of thought leads to the rejection of the principle of excluded middle, for if the sequence or twin primes were either finite or not finite, II would define an integer.
>
> . . . .
>
> Class: That is to say, as long as we do not know if there exists a last pair of twin primes, II is not a definition of an integer, but as soon as this problem is solved, it suddenly becomes such a definition. Suppose on January 1, 1970 it is proved that an infinity of twin primes exists; from that moment $l = 1$. Was $l = 1$ before that date or not? (Menger, 1930).

> Int: A mathematical assertion affirms the fact that a certain mathematical construction has been effected. It is clear that before the construction was made, it had not been made. Applying this remark to your example, we see that before Jan. 1, 1970 it had not been proved that $l = 1$. But this is not what you mean. It seems to me that in order to clarify the sense of your question you must again refer to metaphysical concepts: to some world of mathematical things existing independently of our knowledge, where "$l = 1$" is true in some absolute sense. But I repeat that mathematics ought not to depend upon such notions as these. In fact all mathematicians and even intuitionists are convinced that in some sense mathematics bear upon eternal truths, but when trying to define precisely this sense, one gets entangled in a maze of metaphysical difficulties. The only way to avoid them is to banish them from mathematics. This is what I meant by saying that we study mathematical constructions as such and that for this study classical logic is inadequate.

If we consider the proposition 'The number defined by II above is 1' (2) then under C_1, we should affirm this as C_1-indeterminate. Thus we do not discount the fact that the C_1-truth-value of (2) may at some future date become C_1-determinate or that it may remain C_1-indeterminate. However, in tune perhaps with the classicist, there seems sufficient grounds for believing that (2) is C_2-determinate. This is perhaps a more satisfactory way of handling the above issue than the one advocated by the intuitionist who feels compelled to avoid the problem by banishing the 'metaphysical difficulties'.

We can note that the intuitionist criticism of classical mathematics has undeniably shown that in much of it two quite different kinds of belief (what I explicate as C_1-belief and C_2-belief) are *indiscriminately* mixed up in some parts of classical mathematics; the above issue provides one concrete example. We give just two further quotations to support this view:
H. Weyl [14]:

> Brouwer opened our eyes and made us see how far classical mathematics nourished by a belief in the 'absolute' that transcends all human possibilities or realization, goes beyond such statements as can claim real meaning and truth founded on evidence.

A. Heyting [15]:

> But I must still make one remark which is essential for a correct understanding of our intuitionist position: we do not attribute an existence independent of our thought, i.e., a transcendental existence, to the integers or to any other mathematical objects. . . . Their existence is guaranteed only insofar as they can be determined by thought. They have properties only insofar as they can be discerned in them by thought. But this possibility of knowledge is revealed to us only by the act of knowing itself. Faith in transcendental existence, unsupported by concepts, must be rejected as a means of mathematical proof. As I will shortly illustrate more fully by an example, this is the reason for doubting the law of the excluded middle.

3 *The Essential Content of Metaphysics Based on* **F*F** The essential content of metaphysics based on the logics **F*F** of [7] is supplied by the answers to this question:

Q5. Which propositions should we believe under C_2 to be C_2-true, and what are the rational grounds for those C_2-beliefs?

What makes the C_2-beliefs rational will depend of course upon the C_1-knowledge and C_1-beliefs that support the C_2-beliefs. We can view metaphysics here as concerned with the establishment of a body or canon of C_2-beliefs, and we can note that the logics **F*F** have already advanced this programme. For example, if we are constructing a metaphysics on the basis of **W*W** and we ask which logical truths should we believe to be C_2-true, then the logic **W*W** provides the answer—namely the **W**C_2-tautologies or the **W**C_2-provable wff. In section **5** we discuss the question of alternative metaphysics associated with the logics **F*F**, but here we note some common features:

1. We will certainly be concerned with C_2-beliefs in mathematics, the sciences, and the logics. Hence the metaphysician will be concerned with making evaluative criticism of individual works in these disciplines, from a special standpoint, i.e., his concern for establishing and locating rational C_2-beliefs and presentations of the rational grounds.
2. In general, the body of C_2-beliefs will be subject to change. It is a corpus that from the very start does not pretend to be anything other than an historical document of a certain kind.
3. The idea of a classic—'News that stays news'—and its role and significance, will be stressed.
4. The idea of the common, or what is common to all, will be a sustaining category in all these metaphysics, because when we seek an answer to Q5 above, whole stocks of propositions of the everyday claim ascendency.
5. Ontologies to be associated with the logics **F*F**, on the one hand, will not be ignorant of origins, especially Aristotle, while on the other, will not be alien in spirit to Heidegger's views of ontology as expressed in [9], i.e., the function of ontology is 'to make being manifest itself'. Concerning reality, or the real, there will be two basic concepts: the C_1-real and C_2-real, where the C_1-real has two sub-divisions,

(i) C_1-real linked with C_1-knowledge and C_1-belief.
(ii) C_1-real linked with C_2-belief (as well as C_1-knowledge and C_1-belief).

Kant's view of the 'essential content of metaphysics' is summarised in [1] as follows:

> But the generation of knowledge a priori, both according to intuition and according to concepts, and finally the generation of synthetic propositions a priori in philosophical knowledge, constitutes the essential content of metaphysics.

In contrast to Kant's difficulties in [1] of discovering just one of these true synthetic propositions which at the same time can be labelled metaphysical, from the metaphysical standpoint here, we are confronted with an alarming abundance. Every proposition, considered under C_2, is a metaphysical proposition. Naturally, the metaphysician will be more interested in significant propositions and significant C_2-beliefs (as implied by the phrase 'evaluative criticism' used in 1 above).

4 *Human Freedom* In formal logic, as in so many other creative activities, we are set on forging a weapon. We are now about to test the logics **F*F** against the idea of human freedom. I want to consider the idea in the form of freedom now for a future event. We associate the idea of human freedom with the idea of alternative possibilities that are de facto open.

Q6. Relative to the logics **F*F** [7], what is the logical possibility of human freedom?

For this question we need only consider propositions under C_2. We divide **F*F** into the two groups (1) **Ł*Ł**, **W*W**, **S*S** and (2) **D*D**, **E*E**.

Suppose we wish to discuss the hypothetical question of whether a Mr. X is free or not free now to eat bread tomorrow. We consider the proposition:

'Mr. X will eat breat tomorrow', denoted by 'P'. Considering group (1), P is C_2-true or C_2-false now, and so Mr. X is not free now for that event. Considering (2) if P is C_2-indeterminate, we have from [7] MP is C_2-true and LP is C_2-false, and so belief in freedom here is logically possible.

This discussion is comparable to Łukasiewicz [10], [2]. Since we are concerned with propositions under C_2, and C_2-truth-values, the question of Mr. X's or any one else's knowledge or lack of knowledge about the facts of the matter is quite irrelevant to the logical problem of freedom. The notion of human freedom discussed here is an absolute one. If we consider the particular class of propositions about future events, then we can divide these into two classes:

(1) Those propositions that are C_2-determinate, i.e., C_2-true or C_2-false now, which are associated with future events determinate now.
(2) Those propositions that are C_2-indeterminate now, which are associated with future events indeterminate now.

We can see that according to class (1) there are only determinate events in the world, but according to (2) there are the two different kinds. Consequently we can refer to the logics of class (1) as deterministic, and those of class (2) as indeterministic. If this is the correct logical treatment of the concept of human freedom, and if **D*D** or **E*E** is the correct logical structure of the world, then we have two important consequences:

1. The way man exercises his freedom determines to some extent what will become C_2-true (or C_2-false).
2. It is not logically possible to know that you are free now for a future event, since this would require C_2-knowledge. Hence, we can only achieve at best rational C_2-beliefs in relation to human freedom in the above sense.

5 *Alternative Metaphysics* Alternative metaphysics based on **F*F** can arise in two distinct ways:

A. Alternative metaphysics grounded on different formal logics.

B. Alternative metaphysics grounded on the same formal logic but differing in other respects.

Associated with A and B are the two questions:

QA. What is the correct logical grounding for a metaphysics?
QB. Relative to a given formal logic, which is the best metaphysics?

Considering QA, the functors 'T' and 'F' and the remarks given in **3.3** of [7], there seem to me only two genuine alternative logics, **S*S** or **E*E**. On the basis of section **4** and A we get two divisions:

M1. Deterministic metaphysics grounded on **Ł*Ł**, **W*W** or **S*S**.
M2. Indeterministic metaphysics grounded on **D*D** or **E*E**.

Further sub-divisions occur when we consider the law of excluded middle (**LEM**). This we present in the two forms:

(1) **LEM** under C_1 For all propositions P, P OR NOT-P is C_1-true.
(2) **LEM** under C_2 For all propositions P, P OR NOT-P is C_2-true.

(Note that the 'OR' here is not the same as the truth-functional 'or' that occurs in the semantics **F*** in [7].)

If we consider (2), the difficult case is where P can assume the value C_2-indeterminate. On the basis of the semantics **F*** in [7], and the discussion in section **4**, above, the C_2-indeterminate value is not thought of as an intrinsic one, since it is associated only with propositions that will become C_2-determinate. Hence I would argue that **LEM** under C_2 should be assumed a law that holds in all these logics **F*F** (although in general, there is no wff in $\mathbf{F}_2$ that corresponds to it).

We consider (1) now. (1) holds in **Ł*Ł**. For the other logics, if we consider the second-order proposition—'p is C_2-true', then this is C_1-indeterminate, and is of an intrinsic kind. Hence **LEM** (1) does not hold. However, if we restrict (1) to range over only first-order propositions (and refer to it as **RLEM**) then we need to consider whether there are first-order propositions that are intrinsically C_1-indeterminate (i.e., will never become C_1-determinate). That there may be such propositions is by no means chimerical in view of:

1. The existence of absolutely undecidable propositions in the sense of Church [16].
2. Propositions associated with indeterminacies in quantum mechanics [17].

On 1, Emil Post notes in [21]:

> A fundamental problem is the question of the existence of absolutely undecidable propositions that is, propositions which in some a priori fashion can be said to have a determined truth-value, and yet cannot be proved or disproved by any valid logic . . . . For to the writer it is axiomatic that if the truth-value of $\phi(n)$ "is determined" for each natural number n, the truth-value of $(\exists n)\phi(n)$ and $(n)\phi(n)$ . . . "is determined"—whether determinable by us or not.

> The writer cannot overemphasize the fundamental importance to mathematics of the existence of absolutely unsolvable combinatory problems . . . . The fundamental new thing is that for [these] combinatory problems the given set of instruments [for solving the problems] is in effect the only humanly possible set.

And, on 2, H. Reichenbach argues forcibly in [17] for the need to admit a third truth-value, on a par with 'truth' and 'falsity':

> It is possible to introduce an intermediate truth-value which may be called *indeterminacy* and to coordinate this truth-value to the group of statements which in the Bohr-Heisenberg interpretation are called *meaningless*.

Further details are given on p. 145 of [17].

Propositions of types 1 and 2, under C_1 should be affirmed C_1-indeterminate. The question remains (concerning **LEM**) whether these are intrinsic indeterminacies or not. This question is quite independent of the logical status of these propositions under C_2. Thus even for group (1) M1, we could conceivably still have intrinsic indeterminacies under C_1. (In the Post passage above, we see again an appeal to C_2-belief.)

For all these logics, except **L*L**, we can get three sub-divisions within both M1 and M2 depending on **RLEM** under C_1. The three cases are:

(1) We assume **RLEM** holds under C_1.
(2) We assume **RLEM** does not hold under C_1.
(3) We leave the question open.

The discussion of **LEM** (2) is comparable to Prior's discussion of Aristotle's idea of a third truth-value and the law of excluded middle in [18], pp. 240-250. From what has been said so far, a suitable family name for the formal logics **F*F** (**F** = **L**, **W**, **S**, **D**, **E**) would be transcendental modal logics.

6 *Omniscience and the Idea of an Omniscient Being* We present the idea of an omniscient being provisionally as this: it is the notion of a Supreme Being or the Divine who has C_2-knowledge and knows everything that is possible C_2-knowledge. We discuss the possibilities under two cases corresponding to M1 and M2 of section **5**.

For M1, every proposition is either C_2-true or C_2-false. The totality of such C_2-knowledge is independent of time or timeless. For M1, there seems no difficulty in admitting the logical possibility of the idea of such an omniscient being, and this kind of omniscience.

For M2, every proposition is either C_2-true, C_2-false, or C_2-indeterminate. Here omniscience requires that for every proposition its C_2-truth-value is known. Also, unlike M1, the totality of possible C_2-knowledge is dependent on time. Thus, provided we are prepared to think along the lines of an organic conception of the Divine, the idea of an omniscient being seems logically possible in M2.

It may be worth recalling one medieval discussion of this sort of problem in Ockham [19]. Boehner has an article on Ockham's treatise in [20], and Prior discusses it in [18], pp. 240-250. Ockham does provisionally entertain the idea of a third truth-value—'neuter'—to account for freedom and future contingent events and discusses Aristotle's use of this third value for that purpose. However, Ockham dismisses it on the grounds that it would conflict with God's omniscience, arguing that as a consequence there would be some possible knowledge which God could not have. This sort of argument (and others have suggested it) is mistaken in that it applies a deterministic concept of omniscience (such as the one above for M1) to an indeterministic world and consequently leads to a false notion of what constitutes possible C_2-knowledge. The point here is that the composition of the world according to M1 is different from that of M2. One possible difference being that in the latter human freedom is a commodity, and as argued above, what constitutes omniscience is different in relation to these two worlds.

Considering a proposition that is C_2-indeterminate now. The possible C_2-knowledge now linked with such a proposition is the knowledge that it is C_2-indeterminate. The fact that our omniscient being does not know which way the proposition will become determinate is not a deficiency in C_2-knowledge, for the simple reason that this is not possible C_2-knowledge in that world!

7 *Divine Freedom, Christianity and Judaism* By an analogous argument to that given in section **4**, we can suggest another source of C_2-indeterminacies: those connected with Divine freedom. We are thinking here of Divine freedom now for future events in the world. Propositions associated with such events could assume the truth-value C_2-indeterminate. Clearly this idea of Divine freedom is logically possible in M2, but not in M1.

As a special case of Q4 in section **1**, we can also consider the question of suitable logical groundings for historical Christianity and historical Judaism. From what has already been said in sections **4**, **5**, and **6**, we can see that the ideas of the Divine and the absence of human freedom, associated with M1 are far removed from both the Judaic and Christian conceptions of God and belief in freedom, but that M2 offers something more promising. The discussion of omniscience in M2 indicates that an organic conception of the Divine is appropriate; one to which we can attribute both being and becoming, becoming that is, in relation to the C_2-knowledge of history or what goes on in the world. In M1, the idea of becoming in this sense does not seem attributable to the Divine. Hence the concept of the Divine in M2 seems not entirely foreign to the anthropomorphic conception of God in the Old Testament—'the God of Abraham and of Isaac and of Jacob'. Martin Buber in [22] speaks of Christianity and Judaism not as 'two expressions of faith, but of two kinds of faith'. Buber lays great stress on the difference between the Pauline-Johannine 'pistis'—'It is true', and the Judaic 'emunah'—'We believe and know'. The concept of C_2-belief perhaps finds a natural expression in the Judaic 'emunah', but the Pauline-Johannine 'pistis' is, I think, more problematic. Nevertheless, in principle, **D*D** or **E*E** may be suitable logical candidates for these historical religions.

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