Notre Dame Journal of Formal Logic
Volume 21, Number 4, October 1980

Location of Some Modal Systems

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Georgacarakos [1] described as Modal Family $\mathscr{J}$ the intersections of the trivial system with the irregular systems *S7*, *S8*, and *S9*. He axiomatized these intersections using

P1 $\quad LLp \supset (q \prec Lq)$
Q1 $\quad MLLp \supset (q \prec Lq)$

to define systems *J1* (= *S3* + *P1*), *J2* (= *S3* + *Q1*), and *J3* (= *S3.5* + *P1*). This note relates them to appropriate extensions of *S3* in [3], thereby finding their patterns of modalities and subsuming some of Georgacarakos's proofs.

First, the system *12p* includes *J1*. For, the *12p* semantic condition ([2], p. 78) is that every normal world y is either related to a nonnormal world (in which case LLp is always false at y), or related only to itself (in which case $(q \prec Lq)$ is always true at y); so *P1* is *12p*-valid.

Second, *J2* includes the system *10p*. For:

(1)	$(Mp \supset Lq) \prec (LMp \prec q)$	[*S3*]
(2)	$LMLLp \prec (q \supset Lq)$	[1, p/LLp, $q/(q \supset Lq)$, *Q1*]
(3)	$LMLLMp \prec (Mq \supset q)$	[2, p/Mp, $q/{\sim}q$, $S2^0$]
(4)	$(Lp \prec (p \supset LLq)) \prec (Lp \prec q)$	[*S2*]
(5)	$LMLLMp \prec Mp$	[3, $q/LLMp$, 4]
(6)	$LMLLMp \prec p$	[3, q/p, 5, *C2*]

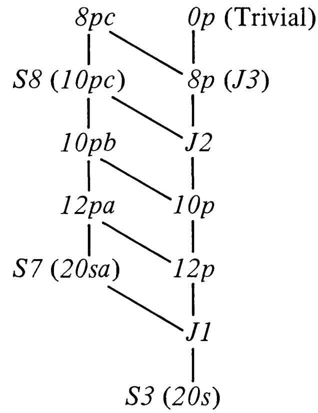
which is an axiom for *10p* ([3], p. 275).

Now, $J3 = 12r + J1 \subseteq 12r + 12p = 8p$ ([3], p. 273), and $J3 = 12r + J2 \supseteq 12r + 10p = 8p$. Hence *J3* is precisely the system *8p*.

Each line in the accompanying diagram indicates that the upper system includes the lower. The systems *14q* and *18r* also fall between *12p* and *20s*, but they are independent of *J1*, for Algebras 2.2 and 2.3 of [3] show that *J1* is not

Received September 25, 1978; revised February 12, 1980

included in either *14q* or *18r*; and Algebras 4.2 and 4.3 show that it does not include either of them or *12p*. So *J1* has the full *20s* modalities. *J2* has *10p* modalities, but Algebra 4.1 of [3] shows that *10p* does not include *J2*. Hence and from [1] and [3], all systems in the diagram are distinct.



This particular diagram correctly shows both joins and intersections. The only join requiring comment is *10pc* = *20sa* + *J2*, which follows from:

$MM{\sim}(q \supset q) \supset LMMp$
$$[Q1,\ p/{\sim}p,\ q/(q \supset q),\ C2]$$

[3], Table 7 includes the intersection $8p = 8pc \cap 0p$, which Georgacarakos writes as $J3 = S9 \cap$ **PC**. Also $P1[p/(p \supset p)]$ shows that $LL(p \supset p)$ strengthens *J1* to *0p*, so ([3], p. 282) $J1 = (J1 + 6s) \cap J1a = 0p \cap 20sa$. Again, $Q1[p/(p \supset p)]$ shows that the *S3.1* axiom $MLL(p \supset p)$ strengthens *J2* to *0p*, so ([3], p. 283) $J2 = (J2 + S3.1) \cap J2c = 0p \cap 10pc$. These are the intersection results obtained by Georgacarakos. His family $\mathscr{J}$ could be enlarged by intersecting *0p* with all the irregular systems of [3], Figure 4.

REFERENCES

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