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SIGNIFICANCE RANGE THEORY

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1 *Introduction* The object of this paper* is to examine some aspects of significance range theory, assuming a background of class theory and introducing significance ranges as a special kind of class. Such a class theory is developed in [2], where a 3-valued significance logic (in fact, **S5** of [1]) is used to set up an axiomatic theory of classes and individuals, the class theory being similar to Mendelson's treatment of **NBG** in [6] and the theory of individuals being an extension of Leonard and Goodman's Calculus of Individuals [5]. The Abstraction Axiom B of this theory will be used for the purpose of generating significance ranges. The symbolism used in this paper is as follows:

$U', V', W', X', Y', Z', \ldots$	(variables over classes and individuals)
$U, V, W, X, Y, Z, \ldots$	(variables over classes)
$u', v', w', x', v', z', \ldots$	(variables over sets and individuals)
$u, v, w, x, y, z, \ldots$	(variables over sets)
$k, l, m, n, \ldots$	(variables over individuals)

$\circ$ (overlaps)
ϵ (is a member of).

The connectives and quantifiers from **S5** that are used are the following:

$\sim$	
1	0
0	1
n	n

&	1	0	n
1	1	0	n
0	0	0	n
n	n	n	n

$\vee$	1	0	n
1	1	1	1
0	1	0	0
n	1	0	n

$\supset$	1	0	n
1	1	0	n
0	1	1	1
n	1	1	1

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$\equiv$	1	0	n
1	1	0	n
0	0	1	1
n	n	1	1

T	
1	1
0	0
n	0

F	
1	0
0	1
n	0

S	
1	1
0	1
n	0

T_n	
1	1
0	n
n	n

$(\mathbf{A}X')\phi(X')$ is true if $\phi(X')$ is true for all classes and individuals X'. $(\mathbf{A}X')\phi(X')$ is nonsignificant if $\phi(X')$ is nonsignificant for some class or individual X'. $(\mathbf{A}X')\phi(X')$ is false otherwise.

$(\mathbf{S}X')\phi(X')$ is true if $\phi(X')$ is true for some class or individual X'. $(\mathbf{S}X')\phi(X')$ is nonsignificant if $\phi(X')$ is nonsignificant for all classes and individuals X'. $(\mathbf{S}X')\phi(X')$ is false otherwise.

The quantifiers **A** and **S** are defined similarly for the other variables. Restricted quantification is carried out as follows:

$$(\mathbf{A}s)\phi(s) =_{df} (\mathbf{A}X')(R(X') \supset \phi(X')).$$
$$(\mathbf{S}s)\phi(s) =_{df} (\mathbf{S}X')(T_nR(X') \ \&\ \phi(X')).$$

The variable s ranges over all the X''s such that $R(X')$ is true and there must be at least one such X'.

As in [2], the relation '$\circ$', meaning 'overlaps', is such that it is significant for individuals to overlap and nonsignificant otherwise, and the relation 'ϵ', meaning 'is a member of', is such that it is significant for individuals or classes to be members of classes and nonsignificant otherwise.

The form of the Abstraction Axiom B of [2] needed is:

$$(\mathbf{A}x_1', \ldots, x_m', X_1', \ldots, X_n')SS\phi(x_1', \ldots, x_m', X_1', \ldots, X_n')$$
$$\supset (\mathbf{S}X)(\mathbf{A}x_1', \ldots, x_m')(\langle x_1', \ldots, x_m'\rangle \epsilon X \equiv S\phi(x_1', \ldots, x_m', X_1', \ldots, X_n'))$$

where quantification in ϕ is over sets and individuals only, $x_1', \ldots, x_m'$, $X_1', \ldots, X_n'$ are all the free variables of ϕ, and X is not among them. Also, if $R(x')$ is a restricting predicate for a restricted quantification over x' in ϕ then $x_1', \ldots, x_m'$ must not occur free in $R(x')$.

Since SSp is valid in the logic, the above form simplifies to:

$$(\mathbf{S}X)(\mathbf{A}x_1', \ldots, x_m')(\langle x_1', \ldots, x_m'\rangle \epsilon X \equiv S\phi(x_1', \ldots, x_m', X_1', \ldots, X_n')),$$

with the same proviso.

Although this would seem to yield the most natural definition of a significance range from Axiom B, this does not specify the class X uniquely. Hence we will use the following definition: The unique class X such that

$$(\mathbf{A}z')(z' \epsilon X \equiv (\mathbf{S}x_1', \ldots, x_m')(T(z' = \langle x_1', \ldots, x_m'\rangle) \ \&$$
$$S\phi(x_1', \ldots, x_m', \overline{Y}_1', \ldots, \overline{Y}_n'))),$$

is a *significance range*. (ϕ, of course, contains quantification over sets and individuals only, $x_1', \ldots, x_m'$ are all the free variables of ϕ, $\overline{Y}_1', \ldots, \overline{Y}_n'$ are constants, and the above condition applies to restricted quantification.) We shall call this unique class X, the *significance range of* $\phi(x_1', \ldots, x_m', \overline{Y}_1', \ldots, \overline{Y}_n')$. We shall also call ϕ the *generating predicate* for X.

2 *The connectives and quantifiers used to define significance ranges*

2.1 *The initial problem* Since there are no restrictions on the connectives that can be used to construct ϕ, the unique class X, such that

$$(\mathbf{A}z')(z' \epsilon X \equiv (\mathbf{S}x'_1, \ldots, x'_m)(T(z' = \langle x'_1, \ldots, x'_m\rangle) \\ \& \; ST_n\phi(x'_1, \ldots, x'_m, \overline{Y}'_1, \ldots, \overline{Y}'_n))),$$

is the significance range of $T_n\phi$. But $ST_n p \equiv Tp$, and hence every class uniquely defined as a class of m-tuples for some predicate $\phi(x'_1, \ldots, x'_m, \overline{Y}'_1, \ldots, \overline{Y}'_n)$, which is significant for all substitutions into its free variables, is the significance range of $T_n\phi(x'_1, \ldots, x'_m, \overline{Y}'_1, \ldots, \overline{Y}'_n)$, i.e., every such class is a significance range.

This is an undesirable result as there are many examples in ordinary discourse of classes which are not significance ranges of any predicate, if one restricts the connectives used to construct predicates so that the predicates so formed can be interpreted in ordinary discourse. A simple example of such a class is the class consisting of a single member, say, a particular leaf of a tree or a number. Unless T_n is used in restricting the **S**-quantifier [1] p. 181, it cannot be used to construct a predicate which can be interpreted in ordinary discourse. T_n is an operator which can convert a false sentence to a nonsignificant one, so there must be something intrinsically nonsignificant about T_n. T_n has no interpretation on its own and it was only introduced to serve the purpose of restricting the **S**-quantifier and such restricting does not give an interpretation to T_n. The difference between classes and significance ranges on this point is that, by Theorem 1 of [2], all classes can be generated by predicates constructed using only the connectives $\sim$, &, and T and the quantifier **A**, whereas by introducing further connectives, such as T_n, one may define significance ranges which would not have been definable had these further connectives not been added. It remains to determine what connectives and quantifiers should be used in constructing predicates to generate significance ranges.

2.2 *Interpretation in ordinary discourse for the connectives and quantifiers* One of the requirements is that the predicates constructed using these connectives and quantifiers must be such that they can be interpreted in ordinary discourse. This should also apply to predicates used to generate classes, but the only reasons for allowing all connectives and quantifiers is that it simplifies the formal treatment not to place restrictions on them and no classes are formed which could not have been formed by using predicates with some interpretation in ordinary discourse. If there is no such interpretation of a predicate then there would be no such interpretation of the class generated by it nor of its significance range.

The connectives, $\sim$, &, and T, can be interpreted as 'not', 'and', and 'it is true that', respectively. As explained in [1], v can be used to formally construct $fX' \vee gX'$, which has the same value as $(f \text{ or } g)X'$, which can be interpreted in ordinary discourse as a predicate disjunction. The example given was 'x is a holiday or likes cheese'. The quantifier **A** can be interpreted as 'for all'. As explained in [1], the quantifier **S** can be interpreted as 'for some' as in the example, 'Something is happy' or 'For some X', X' is happy'. As also pointed out, $\vee$ can also be used to express **S**-quantification over a finite range. Given a predicate $A(X')$ such that $A(X')$ is true for some X', the quantifier **S** restricted by the predicate A can be interpreted as 'for some X' such that $A(X')$'. Formally this is represented as $(\mathbf{S}X')(T_nA(X')\ \&\ \phi(X'))$, where ϕ is the predicate which is quantified. Given a similar predicate $A(X')$, the quantifier **A** restricted by the predicate A can be interpreted as 'for all X' such that $A(X')$'. Formally this is represented as $(\mathbf{A}X')(A(X') \supset \phi(X'))$, where ϕ is the predicate which is quantified. The connective S can be interpreted as 'it is significant that'. The connective $\supset$, as well as being used to restrict the **A**-quantifier, can be interpreted as 'if it is true that . . ., then . . .'. This has to be interpreted in a similar way to the material implication of the two-valued propositional calculus, in that if the antecedent is not true then the implicational statement is vacuously true and if the antecedent is true then the implicational statement takes the value of the conclusion.

However, peculiar significance ranges can be formed by using the connective $\supset$. Form the significance range X such that $(\mathbf{A}z')(z' \epsilon X \equiv S(\phi_1(z') \supset \phi_2(z')))$. Hence $(\mathbf{A}z')(z' \epsilon X \equiv \sim T\phi_1(z') \vee .T\phi_1(z')\ \&\ S\phi_2(z'))$. Thus, X is the union of the $\sim T$-range of ϕ_1 and the intersection of the T-range of ϕ_1 and the significance range of ϕ_2. X is a peculiar sort of construct from these ranges. In the above example if the significance range of ϕ_2 is empty (which can sometimes occur, e.g., as the intersection of two disjoint significance ranges) then the significance range X is the $\sim T$-range of ϕ_1, i.e., the union of the $\sim S$-range of ϕ_1 and the F-range of ϕ_1. Take the example of the predicate $X' = 2$ in formal arithmetic. Its $\sim T$-range consists of everything except the number 2.

The purpose in formalising the notion of a significance range, apart from its tie-up with 3-valued significance logic, is to elucidate the informal notion of a sort of thing, that is, a class with some homogeneous content. In this paper, significance ranges are taken as elucidating a sort of thing or some sorts of thing, whereas atomic significance ranges (to be formally introduced later) are taken as elucidating a single sort of thing.[1]

Going back to the $\sim T$-range of $X' = 2$, this cannot be a significance range because of the inhomogeneity of its content, the number 2 being the same sort of thing as the number 3, say. So not all connectives used to construct predicates, which can be interpreted in ordinary discourse, can be used in the formation of significance ranges.

2.3 *Significance ranges constructed from other significance ranges* The foregoing argument in the case of $\supset$ suggests that significance ranges should be constructed from other significance ranges rather than from T-ranges, F-ranges, $\sim T$-ranges, and $\sim F$-ranges. This does seem plausible enough since classes such as the one above consisting of everything except the number 2 should not be able to influence the construction of significance ranges. In order to satisfy this property the connectives and quantifiers must produce predicates whose significance depends only on the significance of the atomic formulas in the predicates. For example, the connective & satisfies the property because $S(p \,\&\, q) \equiv Sp \,\&\, Sq$. In fact, what is required is for the connectives and quantifiers to be able to be used to form an 's-n sublogic'.

An s-n sublogic is obtained by grouping together the significant values, 1 and 0, and calling it the value s, while the nonsignificant value n remains intact. In order to be able to perform this on a connective or quantifier one must be able to consistently assign the value s or n in the 2-valued matrix of the connective and in the 2-valued description of the quantifier. One can do this for the connectives $\sim$, &, v, T, S and the restricted and unrestricted quantifiers **A** and **S** as follows:

$\sim$	
s	s
n	n

&	s	n
s	s	n
n	n	n

v	s	n
s	s	s
n	s	n

T	
s	s
n	s

S	
s	s
n	s

$(\mathbf{A}X')\phi(X')$ takes the value s if $\phi(X')$ takes the value s for all X' (restricted or unrestricted).
$(\mathbf{A}X')\phi(X')$ takes the value n if $\phi(X')$ takes the value n for some X' (restricted or unrestricted).
$(\mathbf{S}X')\phi(X')$ takes the value s if $\phi(X')$ takes the value s for some X' (restricted or unrestricted).
$(\mathbf{S}X')\phi(X')$ takes the value n if $\phi(X')$ takes the value n for all X' (restricted or unrestricted).

However, there is no consistent assignment for $\supset$.

$\supset$	s	n
s	s	s or n
n	s	s

If p is significant and q is nonsignificant then $p \supset q$ is significant or nonsignificant according to whether p is false or p is true, respectively. This is, in fact, what caused the problem about significance ranges generated by predicates containing $\supset$.

However, consider the connective N defined by the matrix:

N	
1	n
0	n
n	1

N can be consistently assigned values in an s-n sublogic as follows:

N	
s	n
n	s

where N satisfies the equivalence, $SNp \equiv \sim Sp$. Form the significance range

X such that $(\mathbf{A}z')(z' \epsilon X \equiv SN\phi(z'))$. By the equivalence, $(\mathbf{A}z')(z' \epsilon X \equiv \sim S\phi(z'))$. So the significance range of $N\phi(z')$ is the $\sim S$-range of $\phi(z')$.

But not every $\sim S$-range is a significance range. Consider the predicate 'X' is prime'. This has a significance range consisting of all natural numbers. Its $\sim S$-range would consist of all things which are not natural numbers. This can hardly be said to be a significance range because one cannot isolate the natural numbers from the other rational numbers or from the other real numbers so as to form a significance range without the natural numbers. Such a significance range would not exhaust one sort of thing or many sorts of things, without containing only a part of such a sort of thing. Hence the connective N, although it can consistently be assigned values in an s-n sublogic, cannot be used to form predicates which generate significance ranges.

2.4 *The positivity requirement* The preceding argument suggests that the connectives and quantifiers which can be used to form an s-n sublogic should be positive so that the significance of any predicate constructed using them depends positively on the significance ranges of the atomic wffs, i.e., it does not depend on the $\sim S$-ranges of any of the atomic wffs. The s-n sublogic of such monadic and dyadic connectives is represented by the following matrices:

Monadic:

(1)

s	s
n	s

(2)

s	s
n	n

(3)

s	n
n	n

Dyadic:

(1)

	s	n
s	s	s
n	s	s

(2)

	s	n
s	s	s
n	s	n

(3)

	s	n
s	s	s
n	n	n

(4)

	s	n
s	s	n
n	s	n

(5)

	s	n
s	s	n
n	n	n

(6)

	s	n
s	n	n
n	n	n

I have shown in [1] that $\sim$, &, $\vee$, and T, and connectives defined in terms of them, assuming that the constants 1, 0, and n are obtainable from the theory of classes and individuals, will not only exhaust all of the above possibilities for monadic and dyadic connectives but also exhaust all of the 'positive' connectives which can be used to form an s-n sublogic.

If a quantifier, when used over a finite domain, can be replaced by a "positive" connective which can be used in forming an s-n sublogic, then this quantifier can be defined in terms of **A** and **S**. Both **A** and **S** are "positive" because the significance of the quantified statements, $(\mathbf{A}X')\phi(X')$ and $(\mathbf{S}X')\phi(X')$, can be determined from the significance of the $\phi(X')$'s.

The same applies for restricted quantification since $(\mathbf{A}X')(A(X') \supset \phi(X'))$ takes the value s iff $\phi(X')$ takes the value s for all X' such that $A(X')$, and $(\mathbf{S}X')(T_nA(X')\ \&\ \phi(X'))$ takes the value s iff $\phi(X')$ takes the value s for some X' such that $A(X')$.

2.5 *The resulting significance ranges* Now that the connectives $\sim$, &, $\vee$, and T and the quantifiers **A** and **S** (restricted and unrestricted) have been characterised, let us see what significance ranges result from predicates constructed using them.

Since $S\sim p \equiv Sp$, the significance range of $\sim\phi(z')$ is the same as that of $\phi(z')$. So no new significance ranges can be introduced by using $\sim$. Since STp is valid, the significance range of $T\phi(z')$ is the universal class of all sets and individuals. Intuitively, there should be a significance range of all classes and individuals determined by using a predicate such as 'X' is a member of the null class (or any particular class)'. One of the drawbacks of the present theory is that one can only form classes of sets and individuals and not classes of classes and individuals, in general. Accepting this drawback, the significance range of all sets and individuals, obtained above, is the closest one can get to the desired significance range of all classes and individuals. The predicate $\phi(z')$ can also be a constant, taking the value 1, 0, or n. If it takes the value 1 or 0 then its significance range is the same as for $T\phi(z')$, as above. If it takes the value n then its significance range is empty. This is the same as would be produced by the intersection of two disjoint significance ranges, which will be considered next. Since $S(p\ \&\ q) \equiv Sp\ \&\ Sq$, the significance range of $\phi_1(z')\ \&\ \phi_2(z')$ is the intersection of the significance ranges of $\phi_1(z')$ and $\phi_2(z')$. If these two significance ranges exhaust some sorts of things then their intersection exhausts the common sorts of things, if it consists of anything at all.[1] For example, the predicate 'x' is blue and hard (physically firm)' has a significance range consisting of all material objects, which is the intersection of the significance ranges of all extended things and all material objects, these two significance ranges being determined by the predicates, 'x' is blue' and 'x' is hard (physically firm)', respectively. Consider also the predicate 'x' is blue and is a holiday'. Its significance range is empty because the significance ranges of 'x' is blue' and 'x' is a holiday' are disjoint. However, this empty significance range is quite all right because it is determined by the predicate 'x' is blue and is a holiday' and because in establishing the notion of significance range it is awkward to try to avoid including it.

Since $S(p \vee q) \equiv Sp \vee Sq$, the significance range of $\phi_1(z') \vee \phi_2(z')$ is the union of the significance ranges of $\phi_1(z')$ and $\phi_2(z')$. If these two significance ranges exhaust some sorts of things then their union exhausts these sorts of things.[1] For example, the predicate 'x' is blue or is a holiday' has a significance range consisting of all extended things and all days, which is the union of the significance range of all extended things, determined by the predicate 'x' is blue', and the significance range of all days, determined by the predicate 'x' is a holiday'.

Since $S(\mathbf{A}x')\phi(x', z') \equiv (\mathbf{A}x')S\phi(x', z')$, the significance range of $(\mathbf{A}x')\phi(x', z')$ is the intersection of the significance ranges of $\phi(x', z')$, for all x'. The significance range of $(\mathbf{A}x')\phi(x', z')$ would then exhaust all the common sorts of things present in all of the significance ranges of the $\phi(x', z')$'s.[1] For example, the predicate 'x' is similar to everything' or $(\mathbf{A}y')(x'$ is similar to $y')$ has an empty significance range because it is the intersection of many disjoint significance ranges of the predicates 'x' is similar to y'', for all y'. However the predicate 'x' likes everything' has a significance range consisting of all animals because it is the intersection of identical significance ranges of the predicates 'x' likes y'', for all y'.

Since $S(\mathbf{S}x')\phi(x', z') \equiv (\mathbf{S}x')S\phi(x', z')$, the significance range of $(\mathbf{S}x')\phi(x', z')$ is the union of the significance ranges of $\phi(x', z')$, for all x'. The significance range of $(\mathbf{S}x')\phi(x', z')$ would then exhaust all the sorts of things present in at least one of the significance ranges of the $\phi(x', z')$'s.[1] For example, the predicate 'x' is similar to something' has a universal significance range because it is the union of the significance ranges of the predicates, 'x' is similar to y'', for all y', and there is always something which is similar or not similar to x'. The predicate 'x' likes something' has a significance range consisting of all animals because it is the union of identical significance ranges of the predicates, 'x' likes y'', for all y'.

The significance ranges resulting from the use of restricted quantification are restricted unions and intersections of significance ranges, determined in a similar way to those obtained from using unrestricted quantification. So, the connectives $\sim$, &, $\vee$, and T and the quantifiers, $\mathbf{A}$ and $\mathbf{S}$ (restricted and unrestricted) are satisfactory for the purpose of determining significance ranges. The predicates constructed using them can be interpreted in ordinary discourse. Because they are "positive" connectives which can be used to form an s-n sublogic, the significance ranges of predicates formed using them depend only on the significance ranges of atomic predicates (and on the universal significance range). Hence, instead of significance range theory being just a theory of s-ranges of classes, according to the definition in the introduction, significance range theory is an independent subtheory of the theory of classes. That is, significance ranges do not depend on T-ranges, F-ranges, $\sim T$-ranges, $\sim F$-ranges, or even $\sim S$-ranges. Significance ranges have their own characterisation as being classes whose members exhaust one sort of thing or exhaust some sorts of things, assuming that these classes are classes of 1-tuples. Significance ranges of (ordered) n-tuples derive their character from the significance ranges of 1-tuples. Here, the empty and universal significance ranges must be included as well. Classes, on the other hand, can be made up of arbitrary members, where there is no necessity to exhaust a sort of thing just because one of that sort is a member. The formal definition of significance range is as follows:

The unique class X, such that $(\mathbf{A}z')(z' \in X \equiv (\mathbf{S}x_1', \ldots, x_m')(T(z' = \langle x_1', \ldots, x_m'\rangle) \,\&\, S\phi(x_1', \ldots, x_m', \overline{Y}_1', \ldots, \overline{Y}_n')))$, where ϕ is constructed using only the connectives $\sim$, &, $\vee$, and T and only the quantifiers $\mathbf{A}$ and $\mathbf{S}$

(restricted and unrestricted) and where quantification in ϕ is over sets and individuals only, is *the significance range of* $\phi(x'_1, \ldots, x'_m, \overline{Y}'_1, \ldots, \overline{Y}'_n)$. (The conditions on $x'_1, \ldots, x'_m, X$ are as in Section **1**.)

2.6 *Reduction of the connectives to* $\{\&, \vee\}$

Theorem 1 *If X is the significance range of $\phi(x'_1, \ldots, x'_m)$, constructed as above, then there is a predicate $\phi'(x'_1, \ldots, x'_m)$, constructed using only the connectives & and $\vee$ and only the quantifiers* **A** *and* **S** *(restricted and unrestricted), such that X is the significance range of $\phi'(x'_1, \ldots, x'_m)$.*

Proof: We first need to prove a corollary of Theorem 1, that is, for the ϕ and ϕ' of the theorem, $S\phi \equiv S\phi'$. Since ϕ and ϕ' both have the significance range X,

$$(\mathbf{A}z')(z' \epsilon X \equiv (\mathbf{S}x'_1, \ldots, x'_m)(T(z' = \langle x'_1, \ldots, x'_m\rangle) \,\&\, S\phi(x'_1, \ldots, x'_m)))$$

and

$$(\mathbf{A}z')(z' \epsilon X \equiv (\mathbf{S}x'_1, \ldots, x'_m)(T(z' = \langle x'_1, \ldots, x'_m\rangle) \,\&\, S\phi'(x'_1, \ldots, x'_m))).$$

Hence,

$$(\mathbf{A}z')((\mathbf{S}x'_1, \ldots, x'_m)(T(z' = \langle x'_1, \ldots, x'_m\rangle) \,\&\, S\phi(x'_1, \ldots, x'_m)) \equiv (\mathbf{S}x'_1, \ldots, x'_m)(T(z' = \langle x'_1, \ldots, x'_m\rangle) \,\&\, S\phi'(x'_1, \ldots, x'_m))).$$

Hence, put $z' = \langle y'_1, \ldots, y'_m\rangle$ and, since

$$(\mathbf{S}x'_1, \ldots, x'_m)(T(\langle y'_1, \ldots, y'_m\rangle = \langle x'_1, \ldots, x'_m\rangle) \,\&\, S\phi(x'_1, \ldots, x'_m)) \equiv S\phi(y'_1, \ldots, y'_m),$$

$$S\phi(y'_1, \ldots, y'_m) \equiv S\phi'(y'_1, \ldots, y'_m).$$

The proof of Theorem 1 is by induction on the number of connectives and quantifiers used in the construction of ϕ.

1. It is clear in the case of atomic wffs.

2. Let ϕ be $\sim\psi$. By the induction hypothesis, there is a ψ', constructed using only &, $\vee$, **A**, and S, such that $S\psi \equiv S\psi'$. Since $S\phi \equiv S\psi$, $S\phi \equiv S\psi'$. Let ϕ' be ψ', which is constructed in the required way.

3. Let ϕ be $\psi_1 \,\&\, \psi_2$. By the induction hypothesis, there are predicates ψ'_1 and ψ'_2, constructed using only &, $\vee$, **A**, and S, such that $S\psi_1 \equiv S\psi'_1$ and $S\psi_2 \equiv S\psi'_2$. Since $S\phi \equiv S\psi_1 \,\&\, S\psi_2$, $S\phi \equiv S\psi'_1 \,\&\, S\psi'_2$, and $S\phi \equiv S(\psi'_1 \,\&\, \psi'_2)$. Let ϕ' be $\psi'_1 \,\&\, \psi'_2$, which is constructed in the required way.

4. Let ϕ be $\psi_1 \vee \psi_2$. By the induction hypothesis, there are predicates ψ'_1 and ψ'_2, constructed using only &, $\vee$, **A**, and S, such that $S\psi_1 \equiv S\psi'_1$ and $S\psi_2 \equiv S\psi'_2$. Since $S\phi \equiv S\psi_1 \vee S\psi_2$, $S\phi \equiv S\psi'_1 \vee S\psi'_2$ and $S\phi \equiv S(\psi'_1 \vee \psi'_2)$. Let ϕ' be $\psi'_1 \vee \psi'_2$, which is constructed in the required way.

5. Let ϕ be $T\psi$. Since $S\phi \equiv ST\psi$, $S\phi$ is true and so ϕ' can be a conjunction of any significant atomic wffs of the class theory so that the free variables are the same as those of ϕ.

6. Let ϕ be $(\mathbf{A}x')\psi(x')$. By the induction hypothesis, there is a predicate $\psi'(x')$, constructed using only &, v, $\mathbf{A}$, and S, such that $S\psi(x') \equiv S\psi'(x')$. Since $S\phi \equiv (\mathbf{A}x')S\psi(x')$, $S\phi \equiv (\mathbf{A}x')S\psi'(x')$ and $S\phi \equiv S(\mathbf{A}x')\psi'(x')$. Let ϕ' be $(\mathbf{A}x')\psi'(x')$, which is constructed in the required way.

7. Let ϕ be $(\mathbf{S}x')\psi(x')$. By the induction hypothesis, there is a predicate $\psi'(x')$, constructed using only &, v, $\mathbf{A}$, and S, such that $S\psi(x') \equiv S\psi'(x')$. Since $S\phi \equiv (\mathbf{S}x')S\psi(x')$, $S\phi \equiv (\mathbf{S}x')S\psi'(x')$ and $S\phi \equiv S(\mathbf{S}x')\psi'(x')$. Let ϕ' be $(\mathbf{S}x')\psi'(x')$, which is constructed in the required way.

8. Let ϕ be $(\mathbf{A}x')(A(x') \supset \psi(x'))$. By the induction hypothesis, there is a predicate $\psi'(x')$, constructed using only &, v, $\mathbf{A}$, and S, such that $S\psi(x') \equiv S\psi'(x')$. Since $S\phi \equiv (\mathbf{A}x')(A(x') \supset S\psi(x'))$, $S\phi \equiv (\mathbf{A}x')(A(x') \supset S\psi'(x'))$ and $S\phi \equiv S(\mathbf{A}x')(A(x') \supset \psi'(x'))$. Let ϕ' be $(\mathbf{A}x')(A(x') \supset \psi'(x'))$, which is constructed in the required way.

9. Let ϕ be $(\mathbf{S}x')(T_nA(x') \mathbin{\&} \psi(x'))$. By the induction hypothesis, there is a predicate $\psi'(x')$, constructed using only &, v, $\mathbf{A}$, and S, such that $S\psi(x') \equiv S\psi'(x')$. Since $S\phi \equiv (\mathbf{S}x')(T_nA(x') \mathbin{\&} S\psi(x'))$, $S\phi \equiv (\mathbf{S}x')(T_nA(x') \mathbin{\&} S\psi'(x'))$ and $S\phi \equiv S(\mathbf{S}x')(T_nA(x') \mathbin{\&} \psi'(x'))$. Let ϕ' be $(\mathbf{S}x')(T_nA(x') \mathbin{\&} \psi'(x'))$, which is constructed in the required way.

This theorem shows that, for significance ranges consisting of 1-tuples, once one has a set of significance ranges obtained from atomic predicates, then by forming all the unions and intersections of these ranges, one can obtain all the significance ranges of predicates constructed from these atomic predicates. In the case of the relations $\circ$ and ϵ, the following significance ranges are obtained:

1. The significance range of $\circ$ is the class X such that $(\mathbf{A}z')(z' \epsilon X \equiv (\mathbf{S}x')(\mathbf{S}y')(T(z' = \langle x', y'\rangle) \mathbin{\&} S(x' \circ y')))$. Since $S(x' \circ y') \equiv I(x') \mathbin{\&} I(y')$, X is the class of all ordered pairs of individuals [2]. By taking either x' or y' as constant, the significance range of 1-tuples resulting from this will consist of all individuals, i.e., it will be the set I.

2. The significance range of ϵ is the class X such that $(\mathbf{A}z')(z' \epsilon X \equiv (\mathbf{S}x')(\mathbf{S}y')(T(z' = \langle x', y'\rangle) \mathbin{\&} S(x' \epsilon y')))$. X is the class of all ordered pairs, $\langle x', y'\rangle$, such that $x' \epsilon V$ (the class of all sets and individuals) and $y' \epsilon T$ (the class of all sets). If x' is taken as a constant then the significance range of 1-tuples resulting from this will be the class of all sets, T. If y' is taken as a constant then the significance range of 1-tuples resulting from this will be the class of all sets and individuals, V.

If one considers the atomic predicate $x' \epsilon k_0$, for some particular individual k_0, then its significance range is the null class, $\emptyset$. Thus the atomic predicates formed using $\circ$ and ϵ yield the four significance ranges consisting of 1-tuples, ϕ, I, T, and V. Notice here the unusual situation where one significance range is a difference of two others, i.e., $T = V - I$ or $I = V - T$. This is brought about by the rather technical use of the word "overlaps" where it is applied to all individuals.

Section **5** of this paper deals with more complicated relations.

3 *Atomic significance ranges*

3.1 *Definition of an atomic significance range* I want to define the notion of atomic significance range so that, if it consists of 1-tuples, it is characterised as a class whose members exhaust just one sort of thing. It is clear that the union of at least two mutually disjoint significance ranges cannot be an atomic significance range. So, let us define an *atomic significance range* as a nonempty significance range which cannot be expressed as the union of at least two mutually disjoint nonempty significance ranges.[2]

3.2 *Problem of determining whether a significance range is atomic* In order to find out whether a given significance range is atomic it is necessary to know *all* the significance ranges which are subclasses of it. Hence it depends on the different predicates in a language as to whether a significance range is atomic or not. With some technical languages, where all the predicates are well-defined, it is an easy task to determine which are the atomic significance ranges. For example, in the theory of classes and individuals of [2], if no other predicates are added to the original ones, ϵ and $\circ$, then the atomic significance ranges, consisting of 1-tuples, are the set of all individuals, I, and the class of all sets, T.

However, if one takes into account the predicates of ordinary discourse, one is not always sure of a significance range being atomic. For example, whether the significance range of material objects is atomic or not depends on whether there are significance ranges of animate or inanimate objects, say, or whether there are significance ranges of certain types of animate objects which exhaust all of the animate objects and there are significance ranges of certain types of inanimate objects which exhaust all of the inanimate objects. So it depends on the types of predicates one has in the language.

3.3 *Relation between atomic predicates and atomic significance ranges* One would suspect that there is some connection between atomic significance ranges and atomic predicates. However, there are atomic predicates yielding nonatomic significance ranges. For example, the predicate "owns a car"[3] has a significance range consisting of people and companies (including the state). This occurs because "owns" is a legal term and can only legally apply to people and companies. It is clear that people form a significance range because of predicates like "works in the library". Also companies form a significance range because of predicates like "merged with Consolidated".

There are compound predicates yielding what seem to be atomic significance ranges. For example, "x is blue or is hard" has a significance range consisting of all extended things, which is obtained by forming the union of the significance ranges of extended things and of material objects. This significance range seems to be atomic because there seems to be no predicate with all nonmaterial extended things as a significance range. Another example of a compound predicate with what seems to be an atomic significance range is "is prime and divisible by 2", with a significance range of natural numbers.

However, I think that given an atomic significance range there is an atomic predicate with this significance range. If an atomic significance range consists of just one sort of thing then, if it is distinguished as a significance range at all by means of predicates, there should be some atomic predicate which applies to things of this sort. The example given above, "is blue or is hard", bears this out as "is blue" has the same significance range as "is blue or is hard".

4 *Axiomatisation of significance range theory* I wish to add to the 3-valued theory of classes and individuals, definitions and axioms dealing with significance ranges. The predicate "x is a significance range" cannot be defined using the previous definition because quantification over predicates would be required and I wish to develop the theory as a first-order one. Quantification over significance ranges is essential and, to ensure this, I will introduce the primitive predicate R to read "is a significance range". Then add the following definitions:

$$(\mathbf{A}F)\phi(F) =_{df} (\mathbf{A}X')(R(X') \supset \phi(X'))$$
$$(\mathbf{S}F)\phi(F) =_{df} (\mathbf{S}X')(T_nR(X') \;\&\; \phi(X'))\;.$$

Let F, G, H, J, . . . be such variables ranging over significance ranges.

$$(\mathbf{A}f)\phi(f) =_{df} (\mathbf{A}X')(R(X') \;\&\; M(X') \supset \phi(X'))$$
$$(\mathbf{S}f)\phi(f) =_{df} (\mathbf{S}X')(T_n(R(X') \;\&\; M(X')) \;\&\; \phi(X'))\;.$$

Let f, g, h, j, . . . be such variables ranging over set significance ranges.

$$At(F) =_{df} F \neq \emptyset \;\&\; \sim(\mathbf{S}G)(\mathbf{S}H)(G \neq \emptyset \;\&\; H \neq \emptyset \;\&\; F = G \cup H \;\&\; G \cap H = \emptyset).$$

The class Axiom B can be extended so that ϕ can be constructed using the predicate constant R as well as the relation constants, $\circ$ and ϵ. This then allows the formation of the class of all set significance ranges as the unique class X such that $(\mathbf{A}z')(z' \epsilon X \equiv TR(z'))$. The informal definition of atomic significance range is equivalent to the formal definition because one can always form the union of all but one of the disjoint significance ranges in the event of there being more than two disjoint significance ranges.

Axioms

1. $SR(X) \;\&\; \sim SR(k)$.

2. $(\mathbf{S}F)(\mathbf{A}z')(z' \epsilon F \equiv (Sx'_1, \ldots, x'_m)(T(z' = \langle x'_1, \ldots, x'_m\rangle) \;\&\; S\phi(x'_1, \ldots, x'_m, \overline{Y}'_1, \ldots, \overline{Y}'_m)))$, *where ϕ is constructed using only the connectives $\sim$, &, $\vee$, and T and only the quantifiers* **A** *and* **S** *(restricted and unrestricted), where $x'_1, \ldots, x'_m$ are all the free variables of ϕ, and where ϕ contains quantification over sets and individuals only.*

3. $(\mathbf{A}f)(f \epsilon X \supset . f \subseteq V^m \;\&\; \sim f \subseteq V^{m+1}) \supset (\mathbf{S}g)(\mathbf{A}z')(z' \epsilon g \equiv (\mathbf{A}f)(f \epsilon X \supset z' \epsilon f))$.

4. $(\mathbf{A}f)(f \epsilon X \supset . f \subseteq V^m \;\&\; \sim f \subseteq V^{m+1}) \supset (\mathbf{S}G)(\mathbf{A}z')(z' \epsilon G \equiv (\mathbf{S}f)(f \epsilon X \;\&\; z' \epsilon f))$.

5. *For $i = 1, \ldots, k$, let P_i denote an (ordered) n_i-tuple where each of its elements are one of $x'_1, \ldots, x'_m$, with no repetitions. Each of $x'_1, \ldots, x'_m$ must appear in at least one of $P_1, \ldots, P_k$.* $(F_1 \subseteq V^{n_1} \;\&\; \sim F_1 \subseteq V^{n_1+1}) \;\&\; \ldots \;\&$

$(F_k \subseteq V^{n_k} \& \sim F_k \subseteq V^{n_k+1}) \supset (\mathbf{S}G)(\mathbf{A}z')(z' \in G \equiv (\mathbf{S}x'_1, \ldots, x'_m)(T(z' = \langle x'_1, \ldots, x'_m\rangle) \& P_1 \in F_1 \& \ldots \& P_k \in F_k))$.

6. *Same conditions as for Axiom 5.*

$$(F_1 \subseteq V^{n_1} \& \sim F_1 \subseteq V^{n_1+1}) \& \ldots \& (F_k \subseteq V^{n_k} \& \sim F_k \subseteq V^{n_k+1}) \supset (\mathbf{S}G)(\mathbf{A}z')(z' \in G \equiv (\mathbf{S}x'_1, \ldots, x'_m)(T(z' = \langle x'_1, \ldots, x'_m\rangle) \& (P_1 \in F_1 \vee \ldots \vee P_k \in F_k))).$$

7a. *Let x' be not in the first position of the ordered $(m + 1)$-tuple or let x' be in the first position and $(\mathbf{A}x')$ be unrestricted or let x' be in the first position and $(\mathbf{A}x')$ be restricted using the predicate, $x' =$ constant, for some constant.*

$$(\mathbf{S}G)(\mathbf{A}z')(z' \in G \equiv (\mathbf{S}x'_1, \ldots, x'_m)(T(z' = \langle x'_1, \ldots, x'_m\rangle) \& (\mathbf{A}x')(A(x') \supset \langle x'_1, \ldots, x', \ldots, x'_m\rangle \in F))).$$

7b. *Let the condition on* 7(a) *be not satisfied.*

$$F \subseteq V^{m+1} \& \sim F \subseteq V^{m+2} \supset (\mathbf{S}G)(\mathbf{A}z')(z' \in G \equiv (\mathbf{S}x'_1, \ldots, x'_m)(T(z' = \langle x'_1, \ldots, x'_m\rangle) \& (\mathbf{A}x')(A(x') \supset \langle x'_1, \ldots, x', \ldots, x'_m\rangle \in F))).$$

8a. *Same condition as for* 7(a) *except that* $(\mathbf{A}x')$ *is replaced by* $(\mathbf{S}x')$. *The axiom is as for* 7(a) *with* '$(\mathbf{A}x')(A(x')\supset$' *replaced by* '$(\mathbf{S}x')(T_n\mathbf{A}(x')\&$'.

8b. *Same condition as for* 7(b) *except that* $(\mathbf{A}x')$ *is replaced by* $(\mathbf{S}x')$. *The axiom is as for* 7(b) *with* '$(\mathbf{A}x')(A(x')\supset$' *replaced by* '$(\mathbf{S}x')(T_n\mathbf{A}(x')\&$'.

Commentary on the axioms Axiom 1 determines the significance range of the predicate R. Axiom 2 corresponds to the informal definition of a significance range, given in Section **2**. However, because of the lack of quantification over predicates, the axiom cannot ensure that there is such a wff ϕ for every significance range F. For this reason, one cannot in general define significance ranges in terms of other significance ranges, using Axiom 2. Hence there is a need for Axioms 3 to 8.

The condition, $(\mathbf{A}f)(f \in X \supset. f \subseteq V^m \& \sim f \subseteq V^{m+1})$, on Axioms 3 and 4 means that all the members of X consist of m-tuples, that is, they do not entirely consist of $(m + k)$-tuples, for some $k \geq 1$.[4] If a significance range consists of m-tuples, in this sense, then it can only be generated by predicates of the form $\phi(x'_1, \ldots, x'_m)$, where $x'_1, \ldots, x'_m$ are all the free variables of ϕ. In general, the purpose of the above condition is to ensure that, given that a significance range F is being defined in terms of some other significance ranges, if these significance ranges are replaced by their generating predicates using Axiom 2, then the significance range F is then generated by a predicate in the manner of Axiom 2.[5] However, in Axioms 3 and 4 where the intersection and union, respectively, of a class X of significance ranges are being defined as significance ranges, this is only achieved for X finite. In these axioms, the idea is extended to apply to an infinite number of significance ranges, thus allowing significance ranges to be formed that are not generated by predicates as in Axiom 2. The informal definition of a significance range is extended so that all intersections and unions of significance ranges of m-tuples are significance ranges and not just those which can be generated by the predicates ϕ of the given form. Note that, in Axiom 4, if X is a set then G is a set.

Axioms 5 and 6 are generalisations of the cases of forming finite intersections and finite unions, respectively, of significance ranges. Axiom 5 also deals with the case of forming Cartesian products of significance ranges.

Axioms 7 and 8 use restricted quantification in place of the conjunctions and disjunctions of Axioms 5 and 6, respectively. By putting $x' \epsilon V$ for $A(x')$, the restricted quantification can be made unrestricted. The conditions on Axioms 7 and 8 are such as to allow G to have a generating predicate if F has one. In Axiom 7, consider the case of F consisting of $(m + k)$-tuples, for $k > 1$. If x' is not in the first position, then F has a generating predicate of the form $\phi(y_1', \ldots, y_k', x_2', \ldots, x', \ldots, x_m')$ and G has a generating predicate $(\mathbf{A}x')(A(x') \supset \phi(y_1', \ldots, y_k', x_2', \ldots, x', \ldots, x_m'))$. If x' is in the first position, then F has a generating predicate of the form $\phi(y_1', \ldots, y_k', x_1', \ldots, x_m')$. If then $(\mathbf{A}x')$ is unrestricted then G has a generating predicate $(\mathbf{A}y_1', \ldots, y_k')\phi(y_1', \ldots, y_k', x_1', \ldots, x_m')$. If $(\mathbf{A}x')$ is restricted by x' = constant, where the constant must be some k-tuple $\langle \overline{y}_1', \ldots, \overline{y}_k'\rangle$, then G has a generating predicate $(\mathbf{A}y_1', \ldots, y_k')((y_1' = \overline{y}_1')$ & $\ldots$ & $(y_k' = \overline{y}_k') \supset \phi(y_1', \ldots, y_k', x_1', \ldots, x_m'))$. In Axiom 8, the generating predicates for G are constructed similarly.

Theorem 2 *If $F_1, \ldots, F_k$ consist of m-tuples, then $F_1 \cap \ldots \cap F_k$ is a significance range.*

Proof: In Axiom 5, put $n_i = m$, for $i = 1, \ldots, k$. Hence $(\mathbf{S}G)(\mathbf{A}z')(z' \epsilon G \equiv (\mathbf{S}x_1', \ldots, x_m')(T(z' = \langle x_1', \ldots, x_m'\rangle)$ & $\langle x_1', \ldots, x_m'\rangle \epsilon F_1$ & $\ldots$ & $\langle x_1', \ldots, x_m'\rangle \epsilon F_k))$ and $\langle x_1', \ldots, x_m'\rangle \epsilon G \equiv \langle x_1', \ldots, x_m'\rangle \epsilon F_1$ & $\ldots$ & $\langle x_1', \ldots, x_m'\rangle \epsilon F_k$. $G, F_1, \ldots, F_k$ are all contained in V^m and $G = F_1 \cap \ldots \cap F_k$.
Note: $F_1 \cap \ldots \cap F_k$ may consist of $(m + k)$-tuples for any $k \geqslant 0$. Also, if one of the F_i's is a set, then $F_1 \cap \ldots \cap F_k$ is a set.

Theorem 3 *If $F_1, \ldots, F_k$ consist of m-tuples, then $F_1 \cup \ldots \cup F_k$ is a significance range consisting of m-tuples.*

Proof: In Axiom 6, put $n_i = m$, for $i = 1, \ldots, k$. Hence $(\mathbf{S}G)(\mathbf{A}z')(z' \epsilon G \equiv (\mathbf{S}x_1', \ldots, x_m')(T(z' = \langle x_1', \ldots, x_m'\rangle)$ & $(\langle x_1', \ldots, x_m'\rangle \epsilon F_1 \vee \ldots \vee \langle x_1', \ldots, x_m'\rangle \epsilon F_k)))$ and $\langle x_1', \ldots, x_m'\rangle \epsilon G \equiv \langle x_1', \ldots, x_m'\rangle \epsilon F_1 \vee \ldots \vee \langle x_1', \ldots, x_m'\rangle \epsilon F_k$. $G, F_1, \ldots, F_k$ are all contained in V^m and $G = F_1 \cup \ldots \cup F_k$. Let $\sim F_i \subseteq V^{m+1}$, for some $i = 1, \ldots, m$. Let $\langle y_1', \ldots, y_m'\rangle \epsilon F_i$ & $\langle y_1', \ldots, y_m'\rangle \not\epsilon V^{m+1}$. Hence $\langle y_1', \ldots, y_m'\rangle \epsilon G$ and $\sim G \subseteq V^{m+1}$.
Note: If all of the F_i's are sets, then $F_1 \cup \ldots \cup F_k$ is a set.

Theorem 4 *If $F \subseteq V^m$, then $\mathfrak{D}_1(F), \ldots, \mathfrak{D}_m(F)$ are all significance ranges.*

Proof: By putting $x_m' \epsilon V$ for $A(x')$ in Axiom 8, $(\mathbf{A}z')(z' \epsilon G_1 \equiv (\mathbf{S}x_1', \ldots, x_{m-1}')(T(z' = \langle x_1', \ldots, x_{m-1}'\rangle)$ & $(\mathbf{S}x_m')(\langle x_1', \ldots, x_{m-1}', x_m'\rangle \epsilon F)))$. Hence $G_1 \subseteq V^{m-1}$ and $\langle x_1', \ldots, x_{m-1}'\rangle \epsilon G_1 \equiv (\mathbf{S}x_m')(\langle x_1', \ldots, x_m'\rangle \epsilon F)$. Using Axiom 8 again,

$(\mathbf{A}z')(z' \in G_2 \equiv (\mathbf{S}x'_1, \ldots, x'_{m-2})(T(z' = \langle x'_1, \ldots, x'_{m-2}\rangle) \,\&\, (\mathbf{S}x'_{m-1})(\langle x'_1, \ldots, x'_{m-2}, x'_{m-1}\rangle \in G_1)))$. Hence $G_2 \subseteq V^{m-2}$ and $\langle x'_1, \ldots, x'_{m-2}\rangle \in G_2 \equiv (\mathbf{S}x'_{m-1})(\langle x'_1, \ldots, x'_{m-1}\rangle \in G_1)$. Substituting for G_1, $\langle x'_1, \ldots, x'_{m-2}\rangle \in G_2 \equiv (\mathbf{S}x'_{m-1})(\mathbf{S}x'_m)(\langle x'_1, \ldots, x'_m\rangle \in F)$. Let this process continue until we form G_{m-1}, such that $G_{m-1} \subseteq V^1$ and $\langle x'_1\rangle \in G_{m-1} \equiv (\mathbf{S}x'_2)(\mathbf{S}x'_3) \ldots (\mathbf{S}x'_m)(\langle x'_1, \ldots, x'_m\rangle \in F)$. Hence $G_{m-1} = \mathfrak{D}_1(F)$ and $\mathfrak{D}_1(F)$ is a significance range. Similarly, by omitting the appropriate $\mathbf{S}$-quantifier, $\mathfrak{D}_2(F), \ldots, \mathfrak{D}_m(F)$ are all significance ranges.
Note: If F is a set, then $\mathfrak{D}_1(F), \ldots, \mathfrak{D}_m(F)$ are all sets. If F consists of m-tuples then $\mathfrak{D}_1(F)$ consists of 1-tuples.

Theorem 5 *If $F_1, \ldots, F_m$ all consist of 1-tuples, then, $F_1 \times \ldots \times F_m$ is a significance range consisting of m-tuples.*

Proof: In Axiom 5, put $k = m$ and $n_i = 1$ for all $i = 1, \ldots, m$. Then, $(\mathbf{A}z')(z' \in G \equiv (\mathbf{S}x'_1, \ldots, x'_m)(T(z' = \langle x'_1, \ldots, x'_m\rangle) \,\&\, x'_1 \in F_1 \,\&\, \ldots \,\&\, x'_m \in F_m))$. Hence $G \subseteq V^m$ and $G = F_1 \times \ldots \times F_m$. Let $G \subseteq V^{m+1}$. Then all members of G are of the form $\langle\langle y'_1, y'_2\rangle, y'_3, \ldots, y'_{m+1}\rangle$ and $F_1 \subseteq V^2$. By the condition of the theorem, $\sim F_1 \subseteq V^2$ and hence $\sim G \subseteq V^{m+1}$.

Theorem 6 *If $X_1 \times \ldots \times X_m$ is a significance range, then $X_1, \ldots, X_m$ are all significance ranges.*

Proof: Since $X_1 \times \ldots \times X_m \subseteq V^m$, by Theorem 4, $X_1, \ldots, X_m$ are significance ranges because $X_i = \mathfrak{D}_i(X_1 \times \ldots \times X_m)$, for $i = 1, \ldots, m$.

Theorem 7 *If $F_1, \ldots, F_m$ are atomic significance ranges consisting of 1-tuples, then $F_1 \times \ldots \times F_m$ is an atomic significance range (consisting of m-tuples).*

Proof: The proof is by induction on the number m:

(i) The theorem holds in the case of $m = 1$.

(ii) Assume that the theorem holds for Cartesian products of m atomic significance ranges and consider $F_1 \times \ldots \times F_{m+1}$. By Theorem 5, $F_1 \times \ldots \times F_{m+1}$ is a significance range. Let $F_1 \times \ldots \times F_{m+1}$ be nonatomic and let $F_1 \times \ldots \times F_{m+1} = G_1 \cup G_2$, where $G_1 \neq \emptyset$, $G_2 \neq \emptyset$, and $G_1 \cap G_2 = \emptyset$. The domains $\mathfrak{D}_i(G_1)$ and $\mathfrak{D}_i(G_2)$ are significance ranges, for all $i = 1, \ldots, m + 1$, because of Theorem 4. If $\mathfrak{D}_i(G_1) \cap \mathfrak{D}_i(G_2) = \emptyset$, for some $i = 1, \ldots, m + 1$, then, since $\mathfrak{D}_i(G_1) \cup \mathfrak{D}_i(G_2) = F_i$, $\mathfrak{D}_i(G_1) \neq \emptyset$ and $\mathfrak{D}_i(G_2) \neq \emptyset$, F_i would be nonatomic, which contradicts the condition of the theorem. Hence $\mathfrak{D}_i(G_1) \cap \mathfrak{D}_i(G_2) \neq \emptyset$, for all $i = 1, \ldots, m + 1$.

Let $\overline{y}_{m+1} \in \mathfrak{D}_{m+1}(G_1) \cap \mathfrak{D}_{m+1}(G_2)$. Then, for some $y'_1, \ldots, y'_m$, $\langle y'_1, \ldots, y'_m, \overline{y}'_{m+1}\rangle \in G_1$ and, for some $z'_1, \ldots, z'_m$, $\langle z'_1, \ldots, z'_m, \overline{y}'_{m+1}\rangle \in G_2$. Using Axiom 8, form the significance range H_1 such that $\langle y'_1, \ldots, y'_m\rangle \in H_1 \equiv (\mathbf{S}y'_{m+1})(T_n(y'_{m+1} = \overline{y}'_{m+1}) \,\&\, \langle y'_1, \ldots, y'_m, y'_{m+1}\rangle \in G_1)$ and $H_1 \subseteq V^m$. Similarly, form the significance range H_2 such that $\langle z'_1, \ldots, z'_m\rangle \in H_2 \equiv (\mathbf{S}z'_{m+1})(T_n(z'_{m+1} = \overline{y}'_{m+1}) \,\&\, \langle z'_1, \ldots, z'_m, z'_{m+1}\rangle \in G_2)$ and $H_2 \subseteq V^m$. $H_1 \neq \emptyset$ and $H_2 \neq \emptyset$. $H_1 \cup H_2 = F_1 \times \ldots \times F_m$. If $\langle y'_1, \ldots, y'_m\rangle \in H_1 \cap H_2$ then $\langle y'_1, \ldots, y'_m, \overline{y}'_{m+1}\rangle \in G_1 \cap G_2$. Since $G_1 \cap G_2 = \emptyset$ then $H_1 \cap H_2 = \emptyset$. Hence $F_1 \times \ldots \times F_m$ is nonatomic. This contradicts the induction hypothesis. Hence $F_1 \times \ldots \times F_{m+1}$ is atomic and the theorem is proved.

Theorem 8 *If $X_1 \times \ldots \times X_m$ is an atomic significance range and $X_i \cap V^2 = \emptyset$ for all $i = 1, \ldots, m$, then $X_1, \ldots, X_m$ are all atomic significance ranges.*

Proof: By Theorem 6, $X_1, \ldots, X_m$ are all significance ranges. Let X_i, for some $i = 1, \ldots, m$, be nonatomic. Then $X_i = G_1 \cup G_2$, where $G_1 \neq \emptyset$, $G_2 \neq \emptyset$ and $G_1 \cap G_2 = \emptyset$. Since $\sim G_1 \subseteq V^2$ and $\sim G_2 \subseteq V^2$, by Theorem 5, $X_1 \times \ldots \times X_{i-1} \times G_1 \times X_{i+1} \times \ldots \times X_m$ and $X_1 \times \ldots \times X_{i-1} \times G_2 \times X_{i+1} \times \ldots \times X_m$ are both significance ranges. Call them H_1 and H_2, respectively. $H_1 \neq \emptyset$, $H_2 \neq \emptyset$, $H_1 \cap H_2 = \emptyset$ and $H_1 \cup H_2 = X_1 \times \ldots \times X_m$. Hence $X_1 \times \ldots \times X_m$ is nonatomic, which is a contradiction. Hence $X_1, \ldots, X_m$ are all atomic.

Theorem 9 *For each member z' of a given set significance range f_1 such that $f_1 \subseteq V^m$ and $f_1 \cap V^{m+1} = \emptyset$, there is an atomic set significance range which is contained in all significance ranges with z' as a member.*

Proof: Form the class Y such that $(\mathbf{A}x')(x' \in Y \equiv (\mathbf{S}y)(T(x' = y) \,\&\, R(y) \,\&\, z' \in y \,\&\, y \subseteq f_1))$. Using Axiom 3, form the significance range h_1 which is the intersection of all the significance ranges which are members of Y. $h_1 \neq \emptyset$ because $f_1 \in Y$ and $z' \in h_1$. $(\mathbf{A}g)(g \in Y \supset h_1 \subseteq g)$. If G is any significance range such that $z' \in G$, then $G \cap f_1 \in Y$, $h_1 \subseteq G \cap f_1$, and $h_1 \subseteq G$. Hence h_1 is contained in all significance ranges with z' as a member. Let h_1 be nonatomic and $h_1 = g_1 \cup g_2$, where $g_1 \neq \emptyset$, $g_2 \neq \emptyset$ and $g_1 \cap g_2 = \emptyset$. $z' \in g_1$ or $z' \in g_2$ but not both. Let $z' \in g_1$. Since $g_1 \subseteq f_1$, $g_1 \in Y$ and $h_1 \subseteq g_1$. But $g_1 \subset h_1$, which is a contradiction. Hence h_1 is atomic and satisfies the theorem.

Theorem 10 *Given a nonempty class X of atomic significance ranges, consisting of m-tuples, so that the union F of these atomic significance ranges cannot be expressed as a disjoint union of two unions of atomic significance ranges from X then the union F is an atomic significance range.*

Proof: By Axiom 4, F is a significance range. Let F be nonatomic and $F = G_1 \cup G_2$, where $G_1 \neq \emptyset$, $G_2 \neq \emptyset$ and $G_1 \cap G_2 = \emptyset$. There is at least one member h of X such that some member of h is a member of G_1 and some member of h is a member of G_2. This is so, since if all members f of X are such that $f \subseteq G_1$ or $f \subseteq G_2$ then the union of the f's such that $f \subseteq G_1$ and the union of the f's such that $f \subseteq G_2$ would be disjoint, contradicting the condition on X. Consider the significance ranges $h \cap G_1$ and $h \cap G_2$. $h \cap G_1 \neq \emptyset$, $h \cap G_2 \neq \emptyset$, $(h \cap G_1) \cap (h \cap G_2) = \emptyset$ and $(h \cap G_1) \cup (h \cap G_2) = h$. Hence h is nonatomic, contradicting the condition on X. Hence F is atomic.

Corollary *If f_1 and f_2 are two atomic significance ranges, consisting of m-tuples, such that $f_1 \cap f_2 \neq \emptyset$ then $f_1 \cup f_2$ is atomic.*

Proof: In Theorem 10, let $X = \{f_1, f_2\}$.

Theorem 11 *Each nonempty nonatomic set significance range f, such that $f \subseteq V^m$ and $f \cap V^{m+1} = \emptyset$, is a union of mutually disjoint atomic significance ranges.*

Proof: By Theorem 9, each member z' of f is a member of an atomic set significance range which is contained in all significance ranges with z' as a member. Form the class Y such that

$$(\mathbf{A}x')(x' \epsilon\ Y \equiv (\mathbf{S}g)(T(x' = g) \ \&\ At(g) \\ \&\ (\mathbf{S}z')(z' \epsilon\ f \ \&\ z' \epsilon\ g \ \&\ (\mathbf{A}h)(z' \epsilon\ h \supset g \subseteq h)))).$$

Note that $(\mathbf{A}h)(z' \epsilon\ h \supset g \subseteq h) \equiv (\mathbf{A}H)(z' \epsilon\ H \supset g \subseteq H)$ because, if $z' \epsilon\ H$ then $z' \epsilon H \cap f$ and hence $g \subseteq H \cap f$ and $g \subseteq H$. By the proof of Theorem 9 all the members of Y consist of m-tuples. Using Axiom 4, form the union h of all the atomic significance ranges that are members of Y. Note that, since there is a unique atomic significance range belonging to Y for every member z' of f, Y is a set by the Axiom of Replacement. Hence the union h is a set.

$f \subseteq h$. Let $h - f \neq \not{O}$ and let $x' \epsilon\ h - f$. Then $x' \epsilon\ g_1$, where g_1 is some member of Y. Hence, there is some z_1' such that $z_1' \epsilon\ f \ \&\ z_1' \epsilon\ g_1 \ \&\ (\mathbf{A}h)(z_1' \epsilon\ h \supset g_1 \subseteq h)$. Consider the significance range, $g_1 \cap f$. Then $z_1' \epsilon\ g_1 \cap f$ and $g_1 \cap f \subset g_1$. Since $z_1' \epsilon\ g_1 \cap f \supset g_1 \subseteq g_1 \cap f$, there is a contradiction. Hence $f = h$. Then f is a union of atomic significance ranges. Let f be the disjoint union of unions of these atomic significance ranges, that is, $f = \bigcup_{k \epsilon I} f_k$, where I is an index set, and $f_k \cap f_l = \not{O}$ for $k \neq l$ and $k, l \ \epsilon\ I$. Let I have one member only. Then f cannot be expressed as a disjoint union of two unions of its atomic significance ranges. By Theorem 10, f is an atomic significance range, contradicting the condition on f. Hence I has at least two members. Each f_k cannot be expressed as a disjoint union of two unions of its atomic significance ranges. By Theorem 10, each f_k is an atomic significance range and hence f is a union of mutually disjoint atomic significance ranges.

5 *Significance ranges of homogeneous and heterogeneous relations*

5.1 *Significance ranges of 2-place homogeneous relations* Before treating n-place relations in general, I wish to consider the simpler case of 2-place relations. Goddard, in [3], pp. 155-162, distinguishes two types of relations: homogeneous and heterogeneous. He defines the *significant domain,* call it D_R, of the relation R such that $(\mathbf{A}z')(z' \epsilon\ D_R \equiv S(\mathbf{S}x')(z'Rx'))$. He also defines the *significant converse domain,* call it C_R, of the relation R such that $(\mathbf{A}z')(z' \epsilon\ C_R \equiv S(\mathbf{S}x')(x'Rz'))$. He defines R as *homogeneous* if $x'Ry'$ is significant for an *arbitrary* choice of x' from D_R and y' from C_R, i.e., $S(x'Ry') \equiv x' \epsilon\ D_R \ \&\ y' \epsilon\ C_R$, and $S(x'Ry') \equiv S(\mathbf{S}y')(x'Ry') \ \&\ S(\mathbf{S}x')(x'Ry')$.

If F is the significance range of R, i.e.,

$$(\mathbf{A}z')(z' \epsilon\ F \equiv (\mathbf{S}x')(\mathbf{S}y')(T(z' = \langle x', y'\rangle) \ \&\ S(x'Ry'))),$$

then $\langle x', y'\rangle \ \epsilon\ F \equiv S(x'Ry')$. The domain of F, $\mathfrak{D}(F)$, is defined by: $(\mathbf{A}x')(x' \epsilon\ \mathfrak{D}(F) \equiv (\mathbf{S}y')(\langle x', y'\rangle \epsilon\ F))$. Hence $x' \epsilon\ \mathfrak{D}(F) \equiv (\mathbf{S}y')S(x'Ry')$ and $x' \epsilon\ \mathfrak{D}(F) \equiv S(\mathbf{S}y')(x'Ry')$. Then $\mathfrak{D}(F) = D_R$. The range of F, $\mathfrak{R}(F)$, is defined by: $(\mathbf{A}y')(y' \epsilon\ \mathfrak{R}(F) \equiv (\mathbf{S}x')(\langle x', y'\rangle \epsilon\ F))$. Hence $y' \epsilon\ \mathfrak{R}(F) \equiv (\mathbf{S}x')S(x'Ry')$ and $y' \epsilon\ \mathfrak{R}(F) \equiv S(\mathbf{S}x')(x'Ry')$. Then $\mathfrak{R}(F) = C_R$.

By the homogeneity of R, $\langle x', y'\rangle \in F \equiv S(\mathbf{S}y')(x'Ry')$ & $S(\mathbf{S}x')(x'Ry')$ and $\langle x', y'\rangle \in F \equiv x' \in \mathfrak{D}(F)$ & $y' \in \mathfrak{R}(F)$. Hence $F = \mathfrak{D}(F) \times \mathfrak{R}(F)$. An alternative definition of homogeneity is that a relation R is homogeneous iff its significance range F satisfies the identity, $F = \mathfrak{D}(F) \times \mathfrak{R}(F)$.

5.2 *Significance ranges of n-place homogeneous relations* The above definition can be generalised to n-place relations: an n-place relation R is *homogeneous* iff its significance range F can be expressed as the Cartesian product of its domains, i.e., $F = \mathfrak{D}_1(F) \times \ldots \times \mathfrak{D}_n(F)$. Hence, in this case, $SR(x_1', \ldots, x_n') \equiv x' \in \mathfrak{D}_1(F)$ & $\ldots$ & $x_n' \in \mathfrak{D}_n(F)$.

Let us call a nonempty significance range F, consisting of n-tuples, *homogeneous* iff $F = \mathfrak{D}_1(F) \times \ldots \times \mathfrak{D}_n(F)$. In this case, by Theorem 4 or 6, $\mathfrak{D}_1(F), \ldots, \mathfrak{D}_n(F)$ are all significance ranges, but one can only ensure that $\mathfrak{D}_1(F)$ consists of 1-tuples. By Theorem 8, if F is atomic and $\mathfrak{D}_i(F) \cap V^2 = \emptyset$ for all $i = 1, \ldots, n$, then $\mathfrak{D}_1(F), \ldots, \mathfrak{D}_n(F)$ are all atomic.

Theorem 12 *If f is a nonatomic homogeneous set significance range, consisting of n-tuples, such that $\mathfrak{D}_i(f) \cap V^2 = \emptyset$, for all $i = 1, \ldots, n$, then f is a union of mutually disjoint atomic homogeneous significance ranges.*

Proof: If $\mathfrak{D}_i(f)$ is nonatomic, by Theorem 11, it is a union of mutually disjoint atomic significance ranges. Since f is nonatomic, by Theorem 7, not all the $\mathfrak{D}_i(f)$'s are atomic. Consider Cartesian products, $g_1 \times \ldots \times g_n$, where, if $\mathfrak{D}_i(f)$ is atomic, $g_i = \mathfrak{D}_i(f)$ and where, if $\mathfrak{D}_i(f)$ is nonatomic, g_i is one of the mutually disjoint atomic significance ranges in $\mathfrak{D}_i(f)$. There are at least two such Cartesian products. By Theorem 7, $g_1 \times \ldots \times g_n$ is an atomic significance range. Also $g_1 \times \ldots \times g_n$ is homogeneous since $\mathfrak{D}_i(g_1 \times \ldots \times g_n) = g_i$. Form the union X of all such Cartesian products. Since each member of $\mathfrak{D}_1(f) \times \ldots \times \mathfrak{D}_n(f)$ is a member of *exactly one* of such Cartesian products, $f = X$ and f is a union of mutually disjoint atomic homogeneous significance ranges.

Theorem 13 (α) *If F is an atomic homogeneous significance range, consisting of n-tuples, such that $\mathfrak{D}_i(F) \cap V^2 = \emptyset$, for all $i = 1, \ldots, n$, then F is a Cartesian product of atomic significance ranges.*
(β) *If f is a nonatomic homogeneous set significance range, consisting of n-tuples, such that $\mathfrak{D}_i(f) \cap V^2 = \emptyset$, for all $i = 1, \ldots, n$, then f is a union of mutually disjoint Cartesian products of atomic significance ranges.*

Proof: (α) Since $f = \mathfrak{D}_i(f) \times \ldots \times \mathfrak{D}_n(f)$, by Theorem 8, $\mathfrak{D}_1(f), \ldots, \mathfrak{D}_n(f)$ are all atomic significance ranges. (β) follows from the proof of Theorem 12, since the g_i's are atomic significance ranges.

5.3 *Significance ranges of 2-place heterogeneous relations* A 2-place relation R is *heterogeneous* iff it is not homogeneous. Let F be the significance range of a 2-place heterogeneous relation R. Let $x_0' \in \mathfrak{D}(F)$. Form the significance range H_2 such that $(\mathbf{A}y')(y' \in H_2 \equiv S(x_0'Ry'))$. Now form the significance range H_1 such that $(\mathbf{A}x')(x' \in H_1 \equiv (\mathbf{A}y')(S(x_0'Ry') \supset S(x'Ry')))$. Let $G_{x_0'} = H_1 \times H_2$. By Axiom 2, $G_{x_0'}$ is a significance range. $G_{x_0'}$ is a nonempty proper subclass of F and $G_{x_0'} = \mathfrak{D}(G_{x_0'}) \times \mathfrak{R}(G_{x_0'})$. Hence

$G_{x_0'}$ is the homogeneous significance range of the relation, $(\mathbf{A}y')(S(x_0'Ry') \supset (x'Ry'))$ & $x_0'Ry'$. One can also form a similar significance range $G_{y_0'}$ by choosing y_0' from $\mathcal{R}(F)$. Thus each member $x_0' \in \mathcal{D}(F)$ determines a homogeneous significance range $G_{x_0'} \subset F$ and also each member $y_0' \in \mathcal{R}(F)$ determines a homogeneous significance range $G_{y_0'} \subset F$. Given any $\langle x_0', y_0'\rangle \in F$, $\langle x_0', y_0'\rangle \in G_{x_0'}$ and $\langle x_0', y_0'\rangle \in G_{y_0'}$. Hence $F = \bigcup_{x_0' \in \mathcal{D}(F)} G_{x_0'}$, where $G_{x_0'} = \mathcal{D}(G_{x_0'}) \times \mathcal{R}(G_{x_0'})$, and also $F = \bigcup_{y_0' \in \mathcal{R}(F)} G_{y_0'}$, where $G_{y_0'} = \mathcal{D}(G_{y_0'}) \times \mathcal{R}(G_{y_0'})$. Hence the significance range F of a 2-place heterogeneous relation R is a union of homogeneous significance ranges.

5.4 *Significance ranges of n-place heterogeneous relations* An n-place relation R is *heterogeneous* iff it is not homogeneous. Call a nonempty significance range F, consisting of n-tuples, *heterogeneous* iff F is not homogeneous.

Theorem 14 *If F is a heterogeneous significance range, consisting of n-tuples, such that $\mathcal{D}|_i(F) \cap V^2 = \emptyset$, for all $i = 1, \ldots, n$, then F is a union of homogeneous significance ranges.*

Proof: Let $\bar{x}_1' \in \mathcal{D}_1(F)$. Hence there are sets or individuals $\bar{x}_2', \ldots, \bar{x}_n'$ such that $\bar{x}_2' \in \mathcal{D}_2(F), \ldots, \bar{x}_n' \in \mathcal{D}_n(F)$ and $\langle \bar{x}_1', \ldots, \bar{x}_n'\rangle \in F$. Form the significance range H_n such that $(\mathbf{A}x_n')(x_n' \in H_n \equiv (\mathbf{S}x_1', \ldots, x_{n-1}')(T_n(x_1' = \bar{x}_1'$ & $\ldots$ & $x_{n-1}' = \bar{x}_{n-1}')$ & $\langle x_1', \ldots, x_{n-1}', x_n'\rangle \in F))$. Hence $\bar{x}_n' \in H_n$ and $H_n \subseteq \mathcal{D}_n(F)$. Given H_n, we now form the significance range H_{n-1} such that

$$(\mathbf{A}x_{n-1}')(x_{n-1}' \in H_{n-1} \equiv (\mathbf{S}x_1', \ldots, x_{n-2}')(T_n(x_1' = \bar{x}_1' \;\&\; \ldots \;\&\; x_{n-2}' = \bar{x}_{n-2}') \;\&\; (\mathbf{A}x_n')(x_n' \in H_n \supset \langle x_1', \ldots, x_n'\rangle \in F))).$$

$\bar{x}_{n-1}' \in H_{n-1}$, since

$$(\mathbf{A}x_n')(x_n' \in H_n \supset \langle \bar{x}_1', \ldots, \bar{x}_{n-2}', \bar{x}_{n-1}', x_n'\rangle \in F).$$

Also $H_{n-1} \subseteq \mathcal{D}_{n-1}(F)$. Given H_n and H_{n-1}, we can form the significance range H_{n-2} such that

$$(\mathbf{A}x_{n-2}')(x_{n-2}' \in H_{n-2} \equiv (\mathbf{S}x_1', \ldots, x_{n-3}')(T_n(x_1' = \bar{x}_1' \;\&\; \ldots \;\&\; x_{n-3}' = \bar{x}_{n-3}') \;\&\; (\mathbf{A}x_{n-1}')(\mathbf{A}x_n')(x_{n-1}' \in H_{n-1} \;\&\; x_n' \in H_n \supset \langle x_1', \ldots, x_n'\rangle \in F))).$$

$\bar{x}_{n-2}' \in H_{n-2}$, since

$$(\mathbf{A}x_{n-1}')(x_{n-1}' \in H_{n-1} \equiv (\mathbf{A}x_n')(x_n' \in H_n \supset \langle \bar{x}_1', \ldots, \bar{x}_{n-2}', x_{n-1}', x_n'\rangle \in F)).$$

Also $H_{n-2} \subseteq \mathcal{D}_{n-2}(F)$. By induction, we can form the significance range H_1 such that

$$(\mathbf{A}x_1')(x_1' \in H_1 \equiv (\mathbf{A}x_2') \ldots (\mathbf{A}x_n')(x_2' \in H_2 \;\&\; \ldots \;\&\; x_n' \in H_n \supset \langle x_1', \ldots, x_n'\rangle \in F)).$$

$x_1' \in H_1$, since, by the definition of H_2,

$$(\mathbf{A}x_2')(x_2' \in H_2 \equiv (\mathbf{A}x_3') \ldots (\mathbf{A}x_n')(x_3' \in H_3 \;\&\; \ldots \;\&\; x_n' \in H_n \supset \langle \bar{x}_1', x_2', \ldots, x_n'\rangle \in F)).$$

Also $H_1 \subseteq \mathcal{D}_1(F)$. Hence nonempty significance ranges $H_1, \ldots, H_n$ can be formed so that $(\mathbf{A}x_1', \ldots, x_n')(x_1' \in H_1 \& \ldots \& x_n' \in H_n \supset \langle x_1', \ldots, x_n'\rangle \in F)$, for each member $\langle \bar{x}_1', \ldots, \bar{x}_n'\rangle$ of F.

Let $H_1 \times \ldots \times H_n = G$, where G is a significance range by Theorem 5. Since $\mathfrak{D}_i(G) = H_i$, for all $i = 1, \ldots, n$, G is a homogeneous significance range. Since for every member $\langle \bar{x}_1', \ldots, \bar{x}_n'\rangle$ of F, $\langle \bar{x}_1', \ldots, \bar{x}_n'\rangle$ is a member of such a homogeneous significance range G, and since all such homogeneous significance ranges G are contained in F, F is a union of homogeneous significance ranges.

Theorem 15 *If F is a heterogeneous significance range, consisting of n-tuples, such that $\mathfrak{D}_i(F) \cap V^2 = \emptyset$, for all $i = 1, \ldots, n$, then F is a union of Cartesian products of atomic significance ranges.*

Proof: In the proof of Theorem 14, the homogeneous significance ranges G are such that $\mathfrak{D}_i(G) \cap V^2 = \emptyset$, for all $i = 1, \ldots, n$. Hence by Theorems 13 and 14, the theorem follows.

Theorem 16 *If F is an atomic (heterogeneous) significance range, consisting of n-tuples, such that $\mathfrak{D}_i(F) \cap V^2 = \emptyset$, for all $i = 1, \ldots, n$, then $\mathfrak{D}_i(F)$ is an atomic significance range, for all $i = 1, \ldots, n$.*

Proof: Let $\mathfrak{D}_i(F)$ be nonatomic, for some $i = 1, \ldots, n$. Then $\mathfrak{D}_i(F) = G_1 \cup G_2$ where $G_1 \neq \emptyset$, $G_2 \neq \emptyset$ and $G_1 \cap G_2 = \emptyset$. Using Axiom 5, form the significance range H_1 such that

$$(\mathbf{A}z')(z' \in H_1 \equiv (\mathbf{S}x_1', \ldots, x_n')(T(z' = \langle x_1', \ldots, x_n'\rangle) \& \langle x_1', \ldots, x_n'\rangle \in F \& x_i' \in G_1)).$$

Similarly, using G_2 instead of G_1, form the significance range H_2. $H_1 \neq \emptyset$, $H_2 \neq \emptyset$, $H_1 \cap H_2 = \emptyset$ and $H_1 \cup H_2 = F$. Hence F is nonatomic, which is a contradiction. Hence $\mathfrak{D}_1(F), \ldots, \mathfrak{D}_n(F)$ are all atomic.

5.5 *Significance ranges of 2-place stratified heterogeneous relations* By Theorem 14, if F is a heterogeneous significance range, consisting of 2-tuples, such that $\mathfrak{D}(F) \cap V^2 = \emptyset$ and $\mathfrak{R}(F) \cap V^2 = \emptyset$, then $F = \bigcup_{x_0' \in \mathfrak{D}(F)} G_{x_0'}$, where $G_{x_0'} = \mathfrak{D}(G_{x_0'}) \times \mathfrak{R}(G_{x_0'})$. If the $\mathfrak{D}(G_{x_0'})$'s are such that $\mathfrak{D}(G_{x_0'}) \cap \mathfrak{D}(G_{x_1'}) = \emptyset$ or $\mathfrak{D}(G_{x_0'}) = \mathfrak{D}(G_{x_1'})$ for each $x_0' \neq x_1'$ and $x_0', x_1' \in \mathfrak{D}(F)$, then F is a union of mutually disjoint homogeneous significance ranges. At least two of the $\mathfrak{D}(G_{x_0'})$'s must be distinct, otherwise $\mathfrak{D}(G_{x_0'}) = \mathfrak{D}(F)$, for all $x_0' \in \mathfrak{D}(F)$, and F is homogeneous.

Define F as a *stratified heterogeneous significance range*, consisting of 2-tuples, if $\mathfrak{D}(F) \cap V^2 = \emptyset$, $\mathfrak{R}(F) \cap V^2 = \emptyset$, and either $\mathfrak{D}(G_{x_0'}) \cap \mathfrak{D}(G_{x_1'}) = \emptyset$ or $\mathfrak{D}(G_{x_0'}) = \mathfrak{D}(G_{x_1'})$, for each $x_0' \neq x_1'$ and $x_0', x_1' \in \mathfrak{D}(F)$, and either $\mathfrak{R}(G_{x_0'}) \cap \mathfrak{R}(G_{x_1'}) = \emptyset$ or $\mathfrak{R}(G_{x_0'}) = \mathfrak{R}(G_{x_1'})$, for each $x_0' \neq x_1'$ and $x_0', x_1' \in \mathfrak{D}(F)$.

Theorem 17 *If F is a stratified heterogeneous significance range, consisting of 2-tuples, then*

(α) $x' \in \mathfrak{D}(G_{x_0'}) \& y' \in \mathfrak{R}(F) - \mathfrak{R}(G_{x_0'}) \supset \langle x', y'\rangle \notin F$,

(β) $x' \in \mathfrak{D}(F) - \mathfrak{D}(G_{x_0'}) \& y' \in \mathfrak{R}(G_{x_0'}) \supset \langle x', y'\rangle \notin F$,

for each $x_0' \in \mathfrak{D}(F)$.

Proof: Note that $\mathfrak{D}(F) - \mathfrak{D}(G_{x_0'}) \neq \emptyset$ and $\mathfrak{R}(F) - \mathfrak{R}(G_{x_0'}) \neq \emptyset$ as otherwise F would be homogeneous. To prove (α), let $y' \epsilon\ \mathfrak{R}(G_{x_1'})$ where $\mathfrak{R}(G_{x_1'}) \cap \mathfrak{R}(G_{x_0'}) = \emptyset$ and $x_0' \neq x_1'$. Let $\langle x', y'\rangle \epsilon\ F$. Then $\langle x', y'\rangle \epsilon\ G_{x_2'}$ for some $x_2' \epsilon\ \mathfrak{D}(F)$. Hence $x' \epsilon\ \mathfrak{D}(G_{x_2'})$ and $y' \epsilon\ \mathfrak{R}(G_{x_2'})$. By the definition of stratified heterogeneity, $\mathfrak{D}(G_{x_0'}) = \mathfrak{D}(G_{x_2'})$ and $\mathfrak{R}(G_{x_1'}) = \mathfrak{R}(G_{x_2'})$. Hence $\mathfrak{R}(G_{x_0'}) \cap \mathfrak{R}(G_{x_2'}) = \emptyset$. By the construction of $G_{x_0'}$, $(\mathsf{A}y')(y' \epsilon\ \mathfrak{R}(G_{x_0'}) \equiv \langle x_0', y'\rangle \epsilon\ F)$. By the construction of $\mathfrak{D}(G_{x_2'})$, $(\mathsf{A}x')(x' \epsilon\ \mathfrak{D}(G_{x_2'}) \equiv (\mathsf{A}y')(\langle x_2', y'\rangle \epsilon\ F \supset \langle x', y'\rangle \epsilon\ F))$. Since $\mathfrak{D}(G_{x_0'}) = \mathfrak{D}(G_{x_2'})$, $x_0' \epsilon\ \mathfrak{D}(G_{x_2'})$ and $(\mathsf{A}y')(\langle x_2', y'\rangle \epsilon\ F \supset \langle x_0', y'\rangle \epsilon\ F)$. Hence $y' \epsilon\ \mathfrak{R}(G_{x_2'}) \supset y' \epsilon\ \mathfrak{R}(G_{x_0'})$ and $\mathfrak{R}(G_{x_2'}) \subseteq \mathfrak{R}(G_{x_0'})$, which is a contradiction. Hence $\langle x', y'\rangle \notin F$.

Similarly, for (β), if $x' \epsilon\ \mathfrak{D}(F) - \mathfrak{D}(G_{x_0'})$, then $x' \epsilon\ \mathfrak{D}(G_{x_1'})$, where $\mathfrak{D}(G_{x_0'}) \cap \mathfrak{D}(G_{x_1'}) = \emptyset$. The proof follows the same line as that for (α).

Corollary *If F is a stratified heterogeneous significance range, consisting of 2-tuples, then $\langle x', y'\rangle \epsilon\ F \supset .\ x' \epsilon\ \mathfrak{D}(G_{x_0'}) \equiv y' \epsilon\ \mathfrak{R}(G_{x_0'})$, for all $x_0' \epsilon\ \mathfrak{D}(F)$, and hence there is a one-one correspondence between the distinct* $\mathfrak{D}(G_{x_0'})$'s *and the distinct* $\mathfrak{R}(G_{x_0'})$'s.

Proof: By Theorem 17, $\langle x', y'\rangle \epsilon\ F \supset .\ x' \epsilon\ \mathfrak{D}(G_{x_0'}) \supset y' \epsilon\ \mathfrak{R}(G_{x_0'})$ and $\langle x', y'\rangle \epsilon\ F \supset .\ y' \epsilon\ \mathfrak{R}(G_{x_0'}) \supset x' \epsilon\ \mathfrak{D}(G_{x_0'})$. Hence $\langle x', y'\rangle \epsilon\ F \supset .\ x' \epsilon\ \mathfrak{D}(G_{x_0'}) \equiv y' \epsilon\ \mathfrak{R}(G_{x_0'})$.

Theorem 18 *If f is a stratified heterogeneous significance range, consisting of 2-tuples, then f is a union of mutually disjoint Cartesian products of atomic significance ranges.*

Proof: From the definition, it is clear that f is a union of mutually disjoint homogeneous significance ranges. From Theorem 13, each homogeneous significance range is either a Cartesian product of atomic significance ranges or a union of mutually disjoint Cartesian products of atomic significance ranges. Hence the theorem follows.

Examples of stratified heterogeneous relations: The relation, $x' = y'$, is a stratified heterogeneous relation because $S(x' = y') \equiv (I(x')\ \&\ I(y')) \vee (M(x')\ \&\ M(y'))$. '=' is ambiguous between individual identity and class identity.

The relation ϵ under Type Theory is a stratified heterogeneous relation. If F is its significance range, then $\mathfrak{D}(F)$ consists of individuals (type 0) and classes (of all types) and $\mathfrak{R}(F)$ consists of classes (of all types). $\mathfrak{D}(F)$ and $\mathfrak{R}(F)$ are "stratified" as follows: Individuals (of type 0) in $\mathfrak{D}(F)$ correspond with classes of type 1 in $\mathfrak{R}(F)$. For all n, classes of type n in $\mathfrak{D}(F)$ correspond with classes of type $n + 1$ in $\mathfrak{R}(F)$. Each correspondence is such that an arbitrary choice from the two components yields significance and if one chooses from noncorresponding classes then nonsignificance results. The relation ϵ is ambiguous between the relations $\epsilon_n (n = 0, 1, 2, \ldots)$ where $\mathrm{D}\epsilon_n$ consists of classes or individuals of type n and $\mathrm{C}\epsilon_n$ consists of classes of type $n + 1$.

The relations "is between" and "is next to" are stratified heterogeneous relations because they are ambiguous between "is spatially between" and "is temporally between" and between "is spatially next to" and "is temporally next to", respectively. The significant domains and significant converse domains of the "spatial" relations contain only things that occupy space, places, points, etc., whereas the significant domains and significant converse domains of the "temporal" relations contain only times, days of the week, etc.

5.6 *Significance ranges of n-place stratified heterogeneous relations* By Theorem 14, if F is a heterogeneous significance range, consisting of n-tuples, such that $\mathfrak{D}_i(F) \cap V^2 = \emptyset$, for all $i = 1, \ldots, n$, then F is a union of homogeneous significance ranges G, since, for each member $\langle \bar{x}_1', \ldots, \bar{x}_n' \rangle$ of F, such a significance range G can be constructed. Label them as $G_{\langle \bar{x}_1', \ldots, \bar{x}_n' \rangle}$. Hence $F = \bigcup_{\langle x_1, \ldots, x_n' \rangle \in F} G_{\langle x_1', \ldots, x_n' \rangle}$, where $G_{\langle x_1', \ldots, x_n' \rangle} = \mathfrak{D}_1(G_{\langle x_1', \ldots, x_n' \rangle}) \times \ldots \times \mathfrak{D}_n(G_{\langle x_1', \ldots, x_n' \rangle})$. Define F as a *stratified heterogeneous significance range*, consisting of n-tuples, if, $\mathfrak{D}_i(F) \cap V^2 = \emptyset$, for all $i = 1, \ldots, n$, and for all $i = 1, \ldots, n$, $\mathfrak{D}_i(G_{\langle x_1', \ldots, x_n' \rangle}) \cap \mathfrak{D}_i(G_{\langle y_1', \ldots, y_n' \rangle}) = \emptyset$ or $\mathfrak{D}_i(G_{\langle x_1', \ldots, x_n' \rangle}) = \mathfrak{D}_i(G_{\langle y_1', \ldots, y_n' \rangle})$, for each $\langle x_1', \ldots, x_n' \rangle \neq \langle y_1', \ldots, y_n' \rangle$ and $\langle x_1', \ldots, x_n' \rangle$, $\langle y_1', \ldots, y_n' \rangle \in F$.

Theorem 19 *If f is a stratified heterogeneous significance range, consisting of n-tuples, then f is a union of mutually disjoint Cartesian products of atomic significance ranges.*

Proof: Let $\langle x_1', \ldots, x_n' \rangle \in G_{\langle x_1', \ldots x_n' \rangle}$ and $\langle x_1', \ldots, x_n' \rangle \in G_{\langle y_1', \ldots, y_n' \rangle}$. Hence, for all $i = 1, \ldots, n$, $x_i' \in \mathfrak{D}_i(G_{\langle x_1', \ldots, x_n' \rangle})$ and $x_i' \in \mathfrak{D}_i(G_{\langle y_1', \ldots, y_n' \rangle})$. By the definition of stratified heterogeneity, $\mathfrak{D}_i(G_{\langle x_1', \ldots, x_n' \rangle}) = \mathfrak{D}_i(G_{\langle y_1', \ldots, y_n' \rangle})$, for all i. Hence $G_{\langle x_1', \ldots, x_n' \rangle} = G_{\langle y_1', \ldots, y_n' \rangle}$ and the homogeneous significance ranges G are either equal or disjoint. By using Theorem 13, the theorem follows.

Theorem 20 *All stratified heterogeneous significance ranges are nonatomic.*

Proof: Let F be a stratified heterogeneous significance range, consisting of n-tuples. There must be at least two distinct homogeneous significance ranges, $G_{\langle x_1', \ldots, x_n' \rangle}$. For any $\langle \bar{x}_1', \ldots, \bar{x}_n' \rangle \in F$, $F = G_{\langle \bar{x}_1', \ldots, \bar{x}_n' \rangle} \cup (F - G_{\langle \bar{x}_1', \ldots, \bar{x}_n' \rangle})$, where $G_{\langle \bar{x}_1', \ldots, \bar{x}_n' \rangle} \neq \emptyset$, $F \neq G_{\langle \bar{x}_1', \ldots, \bar{x}_n' \rangle}$, and $G_{\langle \bar{x}_1', \ldots, \bar{x}_n' \rangle} \cap (F - G_{\langle \bar{x}_1', \ldots, \bar{x}_n' \rangle}) = \emptyset$. Hence F is nonatomic.

6 *The finiteness argument*

6.1 *The presentation of the argument* The aim is to present an argument showing the finiteness of the number of significance ranges consisting of 1-tuples. This argument does not give a proof but gives some idea of the type of counterexamples it could have and establishes the general plausibility of there being only finitely many sorts of thing.

The method is by examining the sorts of thing with infinitely many members. The only possibility of infinitely many material objects is if they extend infinitely in space or if atoms are infinitely divisible. Even if either is the case then there would be only finitely many significance ranges of predicates relating to them. There are infinitely many type-sentences but only finitely many significance ranges containing type-sentences because of grammatical rules.

In mathematics, there are many concepts, such as sets, classes, points, lines, real numbers, etc., which give rise to infinite classes. Because mathematical theory is well-defined, it is possible to determine all the significance ranges that occur in it. For example, "x is prime" and "x is divisible by 2" determine the significance range of all natural numbers. "x is greater than 2.56" and "x is between 2 and 2.4" determine the significance range of all real numbers. "x has 4 subgroups" determines the significance range of all groups. Although conventional mathematics does not contain significance ranges, they can be introduced as indicated in the above examples.

Hence there are only finitely many such significance ranges with infinitely many members. Also there are only finitely many sorts of thing with infinitely many members because there are only finitely many mathematical concepts.

Now consider the sorts of thing with finitely many members. These would include people, days of the week, companies, etc. The number of such sorts of thing is finite. Hence the number of finite significance ranges is finite. If one forms a significance range F, with infinitely many members, it would be a finite union G of infinite significance ranges and finite significance ranges. F would take this form because it would be generated by a predicate which would be formed using conjunctions, disjunctions and quantifiers. Since there are only finitely many of the type of infinite significance range that is contained in G and also only finitely many finite significance ranges, the number of such significance ranges F is finite. Hence there are only finitely many significance ranges (consisting of 1-tuples).

Using this result, the number of significance ranges F, consisting of n-tuples, such that $\mathfrak{D}_i(F)$ consists of 1-tuples, for all $i = 1, \ldots, n$, is finite. This follows because $F \subseteq \mathfrak{D}_1(F) \times \ldots \times \mathfrak{D}_n(F)$, where the number of possible significance ranges for $\mathfrak{D}_i(F)$ is finite.

6.2 *The scope and limitations of the argument* The argument is subject to the predicates, relating to the sorts of thing with infinitely many members, being well-defined enough to be able to determine their significance ranges and to count the number of such significance ranges. However, artificial counterexamples can be constructed as in the theory of types. There are denumerably many significance ranges: individuals (type 0), classes of type 1, classes of type 2, etc. In fact, in some recent class theories, there are a transfinite number of types and hence a transfinite number of significance ranges. Firstly, these theories are artificial and do not represent the ordinary discourse notions of membership and of class. Secondly, nonsignificance in these theories is excluded by the formation rules or just replaced by falsity.

The argument is also subject to there being only finitely many sorts of thing with infinitely many members. An artificial counterexample to this would occur if one considered infinitely many mathematical concepts with specialised predicates relating to each so that these predicates generate significance ranges, corresponding to each such concept.

7 *Sommers' Principle*

7.1 *Statement of the Principle* There are many examples of two atomic significance ranges consisting of 1-tuples, which are disjoint or which are such that one is properly contained in the other. But it is extremely difficult to find atomic significance ranges consisting of 1-tuples, which properly intersect, i.e., atomic significance ranges F and G such that $F \cap G \neq \emptyset$, $F - G \neq \emptyset$ and $G - F \neq \emptyset$. Sommers, in [7] and in [8], develops a theory of significance ranges in a rather different way to that in this paper. In his theory, he affirms the equivalent of "Two atomic significance ranges consisting of 1-tuples, do not properly intersect". Hence I will call this *Sommers' Principle*. In Sommers' theory, the two significance ranges are atomic because he "locates" a significance range in different places if the predicate determining it is ambiguous. For example, on p. 177 of [7], he gives 'reasonable' two locations, one according to the use of 'reasonable' in "A man is reasonable" and the other according to the use of 'reasonable' in "An argument is reasonable".

7.2 *The argument in its favour* Let us examine a case where Sommers' Principle fails. The intersection of the two atomic significance ranges is a significance range, not necessarily atomic. It consists of some sorts of thing and is properly contained in two distinct atomic significance ranges both consisting of just one sort of thing. In ordinary discourse, when some sorts of thing are all of some other sort of thing, that is, they have a more general classification, then any other more general classification is more general or less general than the first. That is, there is a total ordering of classifications of things which are members of some significance range, or, indeed, of any particular thing at all.

Consider the example of some sorts of thing which are all material objects. The significance range could be determined by the disjunction of the predicates "is a member of the British Cabinet", with the significance range of people, and "is a good creamer", with a significance range of animals. The members of this significance range can be generally classified as material objects, extended things, substances (as in Aristotle's theory of substance), and individuals. Some of these classifications may not be atomic significance ranges but anyway each one in the sequence contains all earlier ones in the sequence. This is an example of the total ordering of classifications.

There is another example of this in mathematics. Consider the sequence of classifications, natural numbers, rational numbers, algebraic numbers, real numbers, complex numbers, and mathematical objects. Here again, all natural numbers are rational numbers, all rational numbers are algebraic numbers, all algebraic numbers are real numbers, all real numbers are complex numbers, and all complex numbers are mathematical objects.

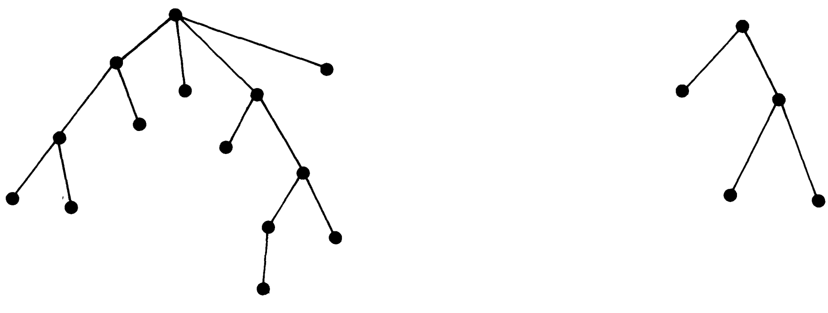
This argument is in terms of sorts of thing but this would also be borne out in the case of atomic significance ranges as determined by predicates. The total ordering of classifications supports Sommers'

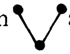
Principle because the negation of the Principle entails that there is a significance range whose elements can be classified in two different ways such that one is not more general than the other.

There is some difficulty in finding counterexamples to the Principle especially in ordinary discourse. However, one would be able to artificially construct counterexamples by specifying the significance ranges of artificially constructed predicates in such a way as to produce properly intersecting atomic significance ranges. In this case, it would be more of a property of the predicates than a property of the world and its classification into sorts of thing.

Sommers' Principle does not have the same intuitive appeal for atomic significance ranges consisting of n-tuples as it does for those consisting of 1-tuples. Sommers' Principle can fail for n-tuples even though it holds for 1-tuples. For example, let $F \subseteq V^2$ and $\sim F \subseteq V^3$ and let $F = G_1 \cup G_2$, where F, G_1 and G_2 are atomic and $G_1 \cap G_2 \neq \emptyset$, $F - G_1 \neq \emptyset$, and $F - G_2 \neq \emptyset$. Also let $\mathfrak{D}(F) = \mathfrak{D}(G_1)$, $\mathfrak{R}(F) = \mathfrak{D}(G_2)$, $\mathfrak{D}(G_2) \subset \mathfrak{D}(G_1)$ and $\mathfrak{R}(G_1) \subset \mathfrak{R}(G_2)$, where $\mathfrak{D}(G_1)$, $\mathfrak{D}(G_2)$, $\mathfrak{R}(G_1)$ and $\mathfrak{R}(G_2)$ are all atomic and all consisting of 1-tuples. Then Sommers' Principle fails for F without contradicting it for the domains and ranges, which consist of 1-tuples.

7.3 *The inverted tree structure of atomic significance ranges* As in Sommers' theory, the Principle yields an inverted tree structure for atomic significance ranges, consisting of 1-tuples. Two such atomic significance ranges can be identical, one can be properly contained in the other, and they can be disjoint. Hence the class of atomic significance ranges containing a given one is totally ordered by the relation of proper containment. The tree structure is of the type as shown:



The dots represent atomic significance ranges. If a dot X is higher than a dot Y and connected by a line going upwards only then the significance range represented by X properly contains the significance range represented by Y. Note that there is no null atomic significance range contained in all atomic significance ranges, also that there is no universal atomic significance range containing all atomic significance ranges. I think it is unlikely that the universal significance range is atomic because each thing in the universe can probably be classified in some way, as in the Aristotelian categories, so that it is contained in an atomic significance range which is not universal. Sommers' Principle has the effect of preventing anything of the form  appearing in the tree.

Sommers' Principle provides a structure result which completes the structure of homogeneous and heterogeneous set significance ranges F, consisting of n-tuples, such that $\mathfrak{D}_i(F) \cap V^2 = \emptyset$, for all $i = 1, \ldots, n$, since these significance ranges can be expressed as a union (disjoint or otherwise) of Cartesian products of atomic significance ranges consisting of 1-tuples, and since Sommers' Principle provides the structure for atomic significance ranges consisting of 1-tuples.

7.4 *Consequences of Sommers' Principle*

Theorem 21 *Given Sommers' Principle, if F is an atomic significance range which is finite*[6] *union of atomic significance ranges G_i, consisting of 1-tuples, for $i = 1, \ldots, k$, then F is equal to one of these atomic significance ranges G_i.*

Proof: The proof is by induction on k.
(a) If $k = 1$, the theorem holds.
(b) Let the theorem hold for k atomic significance ranges G_i. Let F be a finite union of $(k + 1)$ atomic significance ranges G_i, $i = 1, \ldots, k + 1$. Consider $F' = \bigcup_i G_i$, for $i = 1, \ldots, k$. Hence by the induction hypothesis, $F' = G_j$, where G_j is one of $G_1, \ldots, G_k$. Therefore, $F = G_j \cup G_{k+1}$. By Sommers' Principle $G_j \cup G_{k+1} = G_j$ or G_{k+1}, since F is atomic. Hence $F = G_j$ or G_{k+1} and the theorem is proved.

Theorem 22 *Given Sommers' Principle, if f is an atomic set significance range, such that $f \cap V^2 = \emptyset$ and such that there are only finitely many significance ranges contained in f, then there is a member z' of f such that $(\mathbf{A}H)(z' \in H \supset f \subseteq H)$.*

Proof: By Theorem 9, for each member y' of f, there is an atomic significance range $g_{y'}$ such that $(\mathbf{A}H)(y' \in H \supset g_{y'} \subseteq H)$. As in the proof of Theorem 11, form the union h of all these atomic significance ranges, $g_{y'}$. Also, as in the proof of Theorem 11, $f = h$. By the condition of the theorem, there are only finitely many $g_{y'}$'s and hence, by Theorem 21, $f = g_{z'}$, for some $z' \in f$. Also, $(\mathbf{A}H)(z' \in H \supset f \subseteq H)$.

NOTES

1. Here I am assuming that the significance ranges consist of 1-tuples, i.e., are not contained in V^2. [V is the class of all sets and individuals.]

2. *Cf.* Goddard and Routley's definition of "minimal category" in [4].

3. The relation "owns" is discussed in [3], p. 162.

4. Note that every $(m + k)$-tuple, for $k \geqslant 1$, is an m-tuple.

5. To take an example, if a union G of two significance ranges, F_1 and F_2, is being formed such that $\langle x_1', \ldots, x_m' \rangle \in G \equiv \langle x_1', \ldots, x_m' \rangle \in F_1 \vee \langle x_1', \ldots, x_m' \rangle \in F_2$, where F_1 consists of m-tuples and F_2 consists of $(m + k)$-tuples $(k \geqslant 1)$, then there may not be a predicate ψ such that $\langle x_1', \ldots, x_m' \rangle \in F_2 \equiv S\psi(x_1', \ldots, x_m')$.

6. Theorem 21 will not follow for a transfinite union of atomic significance ranges because of the following counterexample in the case of the limit ordinal, ω. Let the atomic significance ranges G_i, for $i \in \omega$, be such that $G_i \subset G_{i+1}$. F, being the union of these G_i's will be atomic. But F is not equal to any one of the G_i's.

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