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A SIMPLE DEFENSE OF MATERIAL IMPLICATION

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If any truth-values are consistently assigned to a conditional of the form 'If p then q' (for which *modus ponens* and *modus tollens* are valid argument-forms and affirming the consequent is a fallacy), then 'If p then q' and '$p \supset q$' may be shown to be logically equivalent. The argument is straightforward, but before proceeding further it would be well to state the assumptions explicitly.

Ordinarily, an argument is said to be invalid if and only if the form of that argument has at least one substitution instance with true premisses and a false conclusion. Also, logicians agree that 'If p then q' is false when p is true and q is false. As both of these conditions are not controversial, they will be presupposed without comment. Two remaining assumptions seem initially plausible, and, for the sake of this defense, will be assumed as well.

First, a valid form of ordinary language argumentation has at least one substitution instance with true premisses and a true conclusion. A good argument for such a necessary condition for validity has been given by George A. Clark: "A valid argument establishes the truth of its conclusion if its premisses are true; it cannot establish its conclusion on the basis of premisses asserted to be false or on the basis of premisses asserted to be true and false."[1] Let this be called a necessary condition for validity.

Second, 'If p then q' is assumed to have its truth-values consistently but arbitrarily assigned; in other words, 'If p then q' is assumed to be truth-functional without prejudice toward the assignment of a particular truth value for the whole expression. For convenience, let the value *1* be assigned in case both the antecedent and consequent are true, the value F in case the antecedent is true and the consequent is false (according to an assumption given above), the value *3* in case the antecedent is false and the consequent is true, and the value *4* in case both antecedent and consequent are false. In this manner, the values *1*, *3*, and *4* taken separately are not presupposed to be true or presupposed to be false, and 'If p then q' seems to be initially only partially truth-functional. From the above conditions, this defense of material implication will take three short steps.

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Step 1 An ordinary language argument-form in terms of *modus ponens* is arrayed in Table 1.

TABLE 1

	Guide Columns		First Premiss	Second Premiss	Conclusion
	p	q	If p then q	p	q
1	T	T	*1*	T	T
2	T	F	F	T	F
3	F	T	*3*	F	T
4	F	F	*4*	F	F

Lines 2, 3, and 4 of Table 1 all have a false premiss. By Clark's necessary condition for validity, line 1 then must have true premisses. Therefore, the value *1* is 'true'.

Step 2 Modus tollens as an ordinary-language argument form is given in Table 2.

TABLE 2

	Guide Columns		First Premiss	Second Premiss	Conclusion
	p	q	If p then q	not-q	not-p
1	T	T	*1*	F	F
2	T	F	F	T	F
3	F	T	*3*	F	T
4	F	F	*4*	T	T

Lines 1, 2, and 3 of Table 2 all have a false premiss. Again, by the necessary condition for validity line 4 must have true premisses; thus, *4* answers to 'true'.

Step 3 An ordinary language argument-form for the fallacy of affirming the consequent is displayed in Table 3.

TABLE 3

	Guide Columns		First Premiss	Second Premiss	Conclusion
	p	q	If p then q	q	p
1	T	T	*1*	T	T
2	T	F	F	F	T
3	F	T	*3*	T	F
4	F	F	*4*	F	F

Since this argument-form is invalid, at least one substitution instance has true premisses and a false conclusion. Line 3 of Table 3 meets this requirement if and only if the value of *3* is taken as 'true'.

Clearly then on our assumptions the derived truth values for 'If p then *q*' are identical to '$p \supset q$'. The resultant truth-values (when considered in the order of Step 1, the sufficiency condition for the falsity of 'If *p* then *q*', Step 3, and Step 2) correspond exactly to the four respective lines of the truth-table for '$p \supset q$'.

This simple defense of material implication helps to clarify the debate between the orthodox logicians, who claim that 'If *p* then *q*' and '$p \supset q$' are not interderivable, and the nonorthodox logicians, who claim that the two expressions are interderivable. In order to maintain the orthodox position, 'If *p* then *q*' cannot be assumed to be truth-functional without prejudice toward the truth-values of the various substitution instances, for if *modus ponens* and *modus tollens* are valid forms and affirming the consequent is invalid, then conditionals in these arguments are wholly truth-functional cases of material implication. Consequently, the orthodox logician must deny that this class of conditionals is not truth-functional in *any* consistent sense on three lines of the truth-table.

NOTES

1. George A. Clark, "Note on False Premises and True Conclusions," *Journal of Philosophy*, vol. LV (1958), p. 1148. If, however, one objects to this stringent condition, it can be dropped in favor of less rigorous conditions without impugning the validity of the rest of the argument. For example, W. S. Thomblison has pointed out that instead of the stated necessary condition for validity, one need assume only that *modus ponens* and *modus tollens* have sound instances.

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