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Linear Diagrams for Syllogisms (with Relationals)

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Abstract A system for diagramming syllogisms is developed here. Unlike Venn, and other planar diagrams, these diagrams are linear. This allows one to diagram inferences which exceed the virtual four term limit on nonlinear systems. It also can be extended (by the use of vectors) to inferences involving all kinds of relational expressions.

. . . be he a Triangle, Square, Pentagon, Hexagon, Circle, what you will—a straight Line he looks and nothing else.

E. A. Abbott, *Flatland*

1 Euler and Venn diagrams are simple and effective devices for illustrating syllogistic validity. Their potential is limited, however, since they cannot apply to arguments with more than four terms. Attempts at extending the scope of plane figure diagrams (e.g., by Lewis Carroll) have been only marginally successful. (For a survey of such attempts, see Gardner [2].) Aristotle probably used some sort of diagram method in teaching the syllogistic. And many ancient commentaries made use of linear diagrams, though our understanding of just how they worked is sketchy (see Ross [13], pp. 301-302 and Kneale [5], pp. 71-72). If the ancient syllogists used linear diagrams rather than planar ones, and if they diagrammed not only simple syllogisms but sorites, polysyllogisms, and compound syllogisms, then it is likely that there is a satisfactory linear method for logical diagrams which can readily go beyond the virtual four term limit on plane diagrams. (In the eighteenth century, Lambert attempted linear diagrams for syllogistic [5] (pp. 111-150). See the critiques of his systems by Venn [15] (pp. 504-527), Peirce [7], and Keynes [4] (pp. 243-247).) Here, I will describe such a linear diagram method, illustrate some of its uses, and do what no syllogist has done before—extend the method to relations. I will conclude with a brief polemical remark in favor of syllogistic in general.

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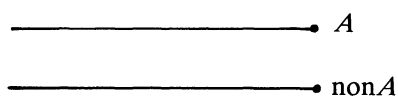
2 Rather than follow the nineteenth century practice of representing each term of an inference as a set of points constituting a plane figure, let us follow the ancient suggestion of representing such terms as points on a straight line segment. (Topologically, we might think of the line as a covering space on the Venn circle.) Thus a term like '*A*' (for 'animal', say) will be represented as a straight line segment, the extent of which is undetermined. Each such line segment will be labeled at its right terminus.

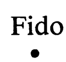
Terms may be negated or unnegated. But in either case their diagrammatic representation is a straight line segment. Thus 'non*A*' will be diagrammed as

Since nothing could ever satisfy both a term and its negation their linear representations can have no point in common. In other words, the two lines representing such terms must be parallel.



This diagrams the logical truth that no *A* is non*A*

A limiting case of a line segment is a single point. Singular terms will be represented quite naturally by such lines (point-lines). Thus a term like 'Fido' will be diagrammed as a single point.

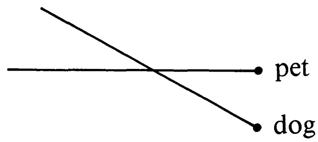


If Fido is a dog then we want the point-line representing Fido to be one of the points constituting the line representing dogs. If

represents the term 'dog' we will place the point representing 'Fido' at the left terminus of this line since we have agreed to label each line at its right terminus and a point-line has no other point to its left.

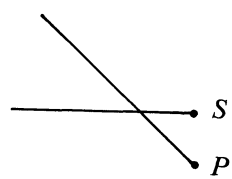
We will now show how categorical propositions in general are represented by linear diagrams. But first a preliminary condition. A line consisting of no points is no line. So no terms are empty. Every term is represented as a line of one or more points.

We have seen how to diagram a proposition like 'Fido is a dog'. Suppose however that we want to diagram 'Some pets are dogs'. Here what needs to be illustrated is the claim that there is at least one thing common to both pets and dogs. Thus the lines for 'pet' and 'dog' must have at least one common point—they must intersect.



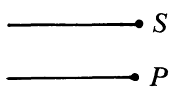
Notice that we represent 'pet' and 'dog' as having a single common point. Yet, for all we know, they may have many points in common. Nonetheless, from a logical point of view, the truth-claim made by 'Some pets are dogs' is just that at least one thing is both a pet and a dog. This is what we have diagrammed. Generally, then, an I categorical ('Some *S* is *P*') will be diagrammed as

(I)



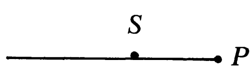
If two lines do not intersect then they must have no common point, i.e. they must be parallel. The contradictory of an I proposition, therefore, must be represented by parallel lines. An E categorical ('No *S* is *P*') will be diagrammed as

(E)



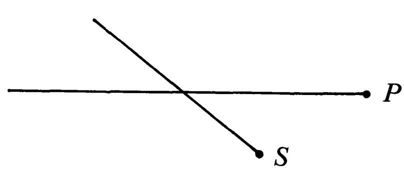
Universal affirmations claim that whatever satisfies the subject-term satisfies the predicate-term. So the subject-term line must be represented as a (possibly proper) part of the predicate-term line. We will represent, then, an A categorical ('Every *S* is *P*') as

(A)

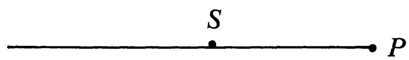


Notice that if every *S* is *P and* every *P* is *S* then the number of points between the right terminus of *S* and the right terminus of *P* will be zero.

To be very clear, then, our diagrams for universal and particular affirmatives are stipulated to be understood in such a way that



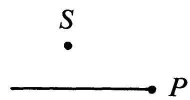
permits interpretation (or reading) wherein more than one *S* is *P* (more than one point shared by lines *S* and *P* and possibly all points on *S* on *P*, and even *vice versa*) and



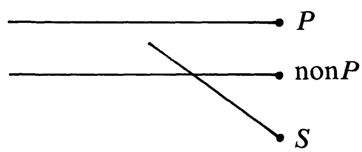
permits interpretation (or reading) wherein all the points on *P* are points on *S* as well. So, one line crossing another at a single point is to be interpreted to mean that *at least one* point is shared by the two lines. (This parallels exactly a single 'x' on a Venn Diagram interpreted as "at least one".) And one line (*S*) partially coinciding with another (*P*) is to be interpreted to mean that all points on *S* are points on *P* and possibly no points on *P* are left over.

The contradictory of an A proposition claims that at least one thing satisfies the subject-term but not the predicate term. So an O categorical ('Some *S* is not *P*') must be diagrammed as an *S*-point outside the *P*-line.

(O)



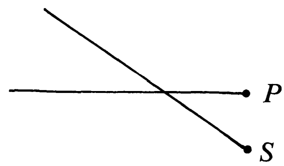
Note that '*S*' is represented as a point-line here. But, for all we know, there may be more than one *S*, and the line representing them may or may not be parallel to the *P* line. Indeed, to say that some *S* is not *P* is to say that some *S* is non*P*, which can be diagrammed as



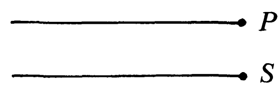
The simpler diagram for O consists just of the *P*-line and the point of intersection of the *S*- and non*P*-lines.

3 Line diagrams represent inferences in the usual way. First the premises are diagrammed. Then, either the conclusion has already been diagrammed or it has not. If it has, the inference is valid. If it has not, the inference is invalid.

Equivalences are immediate inferences in which each of a pair of propositions can be validly inferred from the other. Thus, a linear diagram for one must be the diagram for the other. For example, simple conversion equates 'Some *S* is *P*' and 'Some *P* is *S*'. Our diagram method illustrates this by representing both propositions by a pair of intersecting *S*- and *P*-lines.

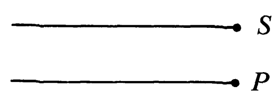


Universal negatives are likewise simply convertible. Both ‘No *S* is *P*’ and ‘No *P* is *S*’ are diagrammed by parallel *S*- and *P*-lines.



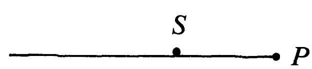
Subalternation is an example of an immediate inference between two nonequivalent propositions. Any universal proposition will validly entail its corresponding particular just because no term is empty. Diagramatically, whenever there is a line there must be at least one point in that line. For example, we can infer ‘Some *S* is *P*’ from ‘Every *S* is *P*’ since the premise is diagrammed as

Since every part of a line intersects (at least once) that same line, it follows that at least one point in *S* is in *P*. Also, from ‘No *S* is *P*’ we can validly infer ‘Some *S* is not *P*’ since if lines *S* and *P* are parallel (given the premise), i.e.

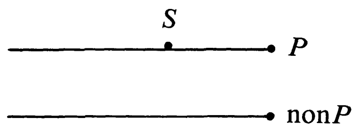


and every term is nonempty (every line consists of at least one point), there must be at least one point in line *S* outside of line *P*.

Obversion is an example of immediate inference relying on the fact that a term and its negation satisfy nothing in common (so that their line representations must be parallel). Consider, for example, ‘Every *S* is *P*’. It is diagrammed as

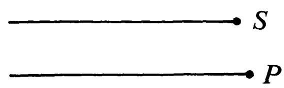


But we also know that ‘non*P*’ is contrary to ‘*P*’, so that a non*P*-line is parallel to the *P*-line. By adding this we have



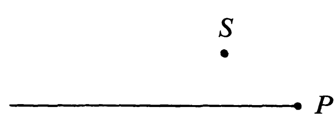
Since every point on *P* is outside non*P*, and since every point on *S* is on *P*, it follows that every point on *S* is outside non*P*. In other words, lines *S* and non*P* are parallel (i.e. 'No *S* is non*P*').

But the true importance of obversion is seen when applied to E and O forms. 'No *S* is *P*' has been diagrammed thus far as

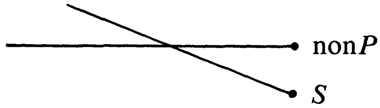


By obversion, 'No *S* is *P*' is equivalent to 'Every *S* is non*P*', i.e.

Likewise, 'Some *S* is not *P*' is equivalent to 'Some *S* is non*P*'. Thus, both



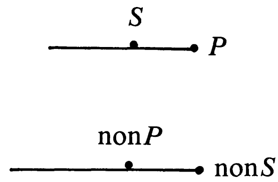
and



can be used to diagram an O proposition.

Now an obverted A proposition can be converted. The resulting proposition can then be obverted to yield the contrapositive of the original. Thus the contrapositive of 'Every *S* is *P*' is 'Every non*P* is non*S*'. Diagramatically, then, the *full* representation of 'Every *S* is *P*' must be

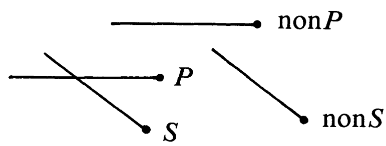
(A.1)



(A.1) represents such equivalent propositions as 'Every *S* is *P*', 'Every nonP is nonS', 'No *S* is non*P*', 'No non*P* is *S*', 'No non*S* is *P*', 'No *P* is non*S*', 'No *S* is non*S*', 'No non*S* is *S*', 'No *P* is non*P*', and 'No non*P* is *P*'. These last four are tautological and are instances of the law of noncontradiction. A *full* representation of any proposition will necessarily represent the law of noncontradiction as well.

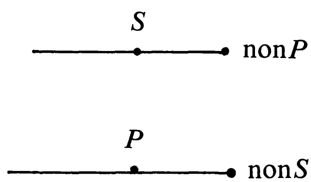
Consider the I proposition 'Some *S* is *P*'. Conversion and obversion on I demand that a *full* representation must exhibit such equivalent propositions as 'Some *P* is *S*', 'Some *S* is not non*P*' and 'Some *P* is not non*S*'. Thus

(I.1)

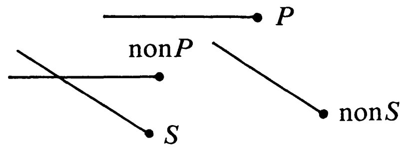


Universal and particular negations can also be given *full* representations in order to exhibit logical equivalences.

(E.1)



(O.1)



Notice that the law of noncontradiction is also represented (twice) by each *full* diagram.

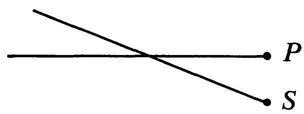
The *full* representation of a categorical will always be a diagram consisting of two pairs of parallel lines. However, for most purposes of logical reckoning, the simple (A), (E), (I), and (O) diagrams are sufficient. These are the results of "minimizing" (see Gardner [2], p. 72) the *full* diagrams, which will usually represent far more information than we need.

Minimized diagrams:

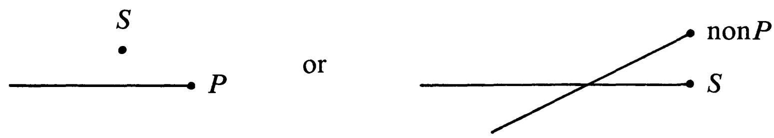
(A)

(E)

(I)



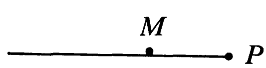
(O)



4 The classical simple syllogisms are easily diagrammed by our linear method using just the simple categorical diagrams. The premise diagrams for the first figure are as follows (in each case the conclusion can be seen to be already diagrammed—the mark of validity).

Barbara: Every *M* is *P*
Every *S* is *M*
So every *S* is *P*

Here the major is diagrammed first as



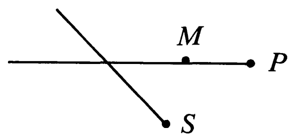
The minor is then added to get

From this the conclusion can be read directly.

Darii: Every *M* is *P*
Some *S* is *M*
So some *S* is *P*

We diagram the major as

Next the minor is added.

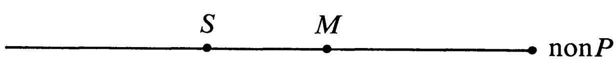


The conclusion again is already diagrammed since the *S*- and *P*-lines intersect.

Celarent: No *M* is *P*
Every *S* is *M*
So no *S* is *P*

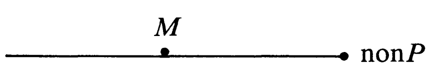
The major is diagrammed as

Adding the minor we get

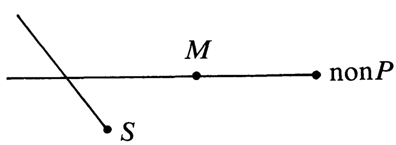


Ferio: No *M* is *P*
Some *S* is *M*
So some *S* is not *P*

The major is

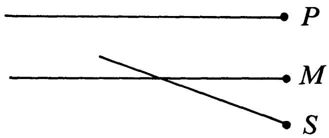


Adding the minor,

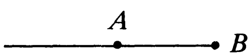


Similar diagrams can easily be constructed for all 24 valid classical syllogisms from AAA-1 to EIO-4. So, the method is complete. It is sound as well. An exhaustive (and exhausting) check of each of the 232 classical invalid forms shows that none is valid by our method.

The diagram method (whether planar or linear) is most effective in determining validity and in discovering missing premises or conclusions. Consider the premise pair 'No *P* is *M*' and 'Some *M* is *S*'. In this case these are diagrammed linearly simply as



We can readily see that the inference of 'Every *P* is *S*' from this is invalid. But we can also see what the missing conclusion is – 'Some *S* is non*P*'. Enthymemes with missing premises are most easily resolved by diagramming the explicit premise along with the contradictory of the conclusion. What follows, then, will be the contradictory of the missing premise. For example, let the explicit premise be 'Every *A* is *B*' and the conclusion be 'Some *C* is not *A*'. We first diagram the premise as



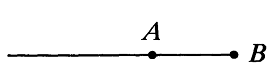
We next add the contradictory of the conclusion (viz. 'Every *C* is *A*').

From this we conclude 'Every *C* is *B*', which is the contradictory of the tacit premise (viz. 'Some *C* is not *B*').

5 Thus far we have seen that linear diagrams can do more or less what planar diagrams can do. One minor advantage they may have is due to the fact that they are faster to construct (since lines and points are easier to draw than circles, squares, ellipses, etc.). But the major advantage of line diagrams is their ability to represent inferences involving relatively large numbers of terms (viz. more than four) without destroying the original simplicity of the diagrams. Here is an example of a relatively elementary valid inference which Venn diagrams are powerless to represent in a simple, perspicuous manner.

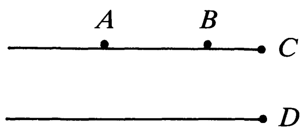
Every *A* is *B*
Every *B* is *C*
No *C* is *D*
Some *D* is *E*
So some *E* is not *A*

Diagramming the first premise we have

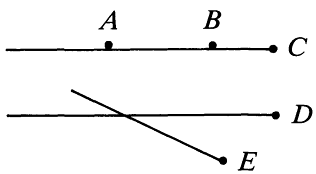


Adding the second gives us

Adding the third gives us



Finally, adding the fourth, we have



That's all—three lines, five labels. The conclusion is already diagrammed. Sorites of any number of terms can be diagrammed using the linear method. The geometric restrictions on closed plane figures which prevent perspicuous representations involving more than four terms using simple continuous figures do not apply to the still simpler linear figures.

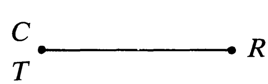
Finally, identity statements are a special kind of proposition easily diagrammed by our method. A proposition of the form '*A* is (identical to) *B*' claims that the *A*-point is on the *B*-line. Since the *B*-line is a point-line, this means that '*A*' and '*B*' label the same point, i.e.

A • *B*

An argument like the following

Tully is Cicero
Cicero is Roman
So Tully is Roman

would be diagrammed as



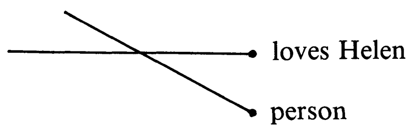
where, by the first premise, '*C*' and '*T*' both label the same point-line.

6 The simple diagrams outlined in Section 3 can be extended to represent relational propositions. In this section I make some tentative suggestions as to how this can be done. (Peirce made an attempt at this as well. See [7], pp. 353ff.) The key idea here is to see relational expressions as terms. It is the Leibnizian notion that relational (indeed, all) propositions are logically categorical, so that the logic of relationals is syllogistic. That this can be shown at all should be surprising to those who insist that syllogistic methods are powerless beyond the range of simple categoricals consisting of monadic terms.

Consider a simple relational proposition such as 'Paris loves Helen'. The claim here is that Paris is among the things which love Helen. Let us diagram these by a line labeled 'loves Helen'.

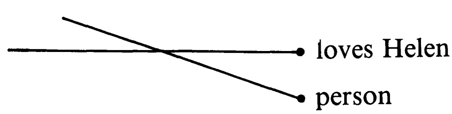
Paris is one of them. So:

'Some person loves Helen' would be diagrammed as an I proposition.

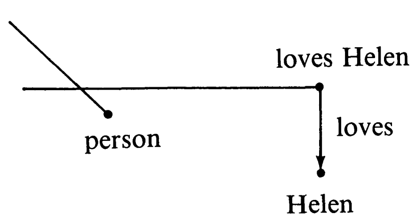


(Notice the converse here is 'Something which loves Helen is a person'.)

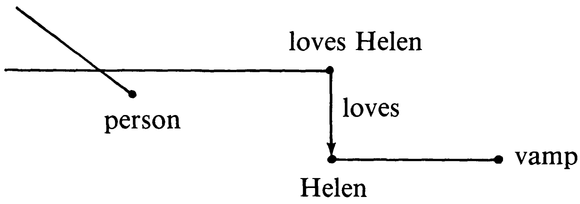
Thus far 'loves Helen' is treated just like any other term. It is represented by a single line segment representing things which love Helen. Often in inferences, however, it is necessary to analyze such relational terms, abstracting from them one or more of their constituents for independent treatment. Consider the argument 'Someone loves Helen, but Helen is a vamp. So someone loves a vamp'. The first premise can be diagrammed as



But in order to diagram the second premise we need a way to represent the relative term 'loves' and the object term 'Helen' separately. Let us represent relative terms by arrows (indicating the "direction" of the relation) connecting their relata. Then 'loves, in this case, would be represented by an arrow from lovers of Helen to Helen.



'Helen' is now extracted from the relational 'loves Helen' and we are free to diagram our second premise, 'Helen is a vamp'.



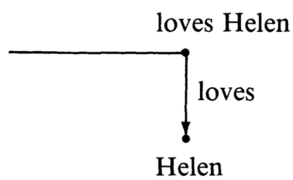
Given that from this we can see that every lover of Helen loves Helen, the conclusion, 'Someone loves a vamp', can be read directly.

Look once more at our diagram for 'Paris loves Helen'.

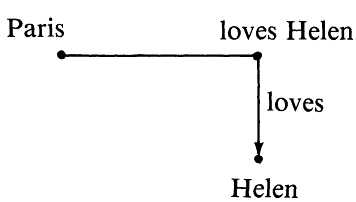
We know that whatever loves Helen loves Helen. In fact, we can say generally:

Whatever is *r* to some/every *X* is *r* to some/every *X*, i.e.

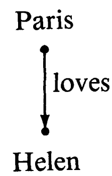
It is this tautology which permits the tautological



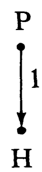
so that 'Paris loves Helen' can be rendered as



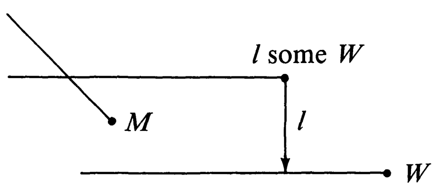
This *full* representation diagrams more information than we often need in computing logical inferences. But it can be simplified (by suppressing tautological information as with categoricals in general) to the more natural looking



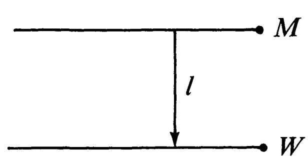
or simply



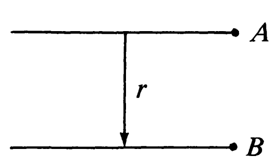
If we agree to read arrows in reverse direction as representing the converses of the relations they represent when read in their indicated direction, we can take the preceding diagram to represent both ‘Paris loves Helen’ and its equivalent passive transform, ‘Helen is loved by Paris’. This same process can be used to simplify diagrams for propositions such as ‘Some man loves some woman’. Its *full* representation is



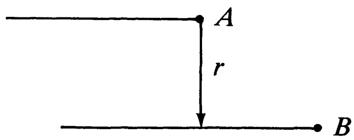
Simplifying, by suppressing tautological information, we get



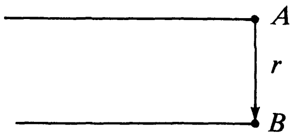
Notice that the locations of the end points of a relational arrow indicate the quantities of the relata. The quantity is universal when the relational arrow meets the term-line at the line’s right terminus; otherwise the term is particular in quantity. For example, ‘Some A is r to some B’ is diagrammed as



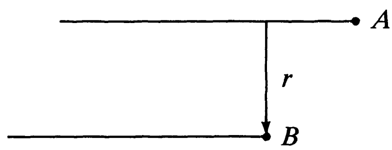
'Every A is r to some B' is diagrammed as



'Every A is r to every B' is diagrammed as

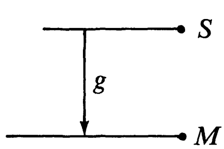


And 'Some A is r to every B' is diagrammed as

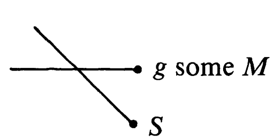


In spite of our simplifications, it is important to remember that in the context of a logical inference it may be necessary to restore some or all of the *full* relational expressions. This is especially so when those relational expressions occur as logical subjects in subsequent premises or conclusions. For example, a proposition like 'Some senator gives away some money' could be diagrammed in one of three ways.

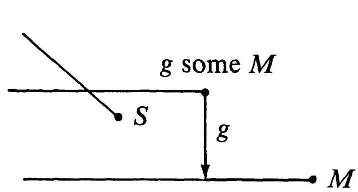
(i)



(ii)

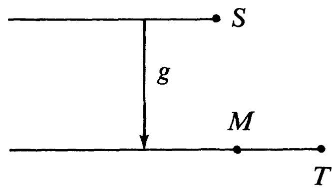


(iii)



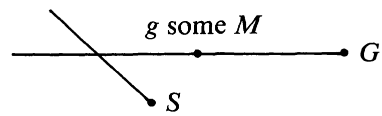
We can use (i) if the relative term 'gives away' occurs elsewhere without the logical object 'money'. Thus, suppose the second premise is 'All money is tainted'. Our premises are diagrammed then as

(i.1)



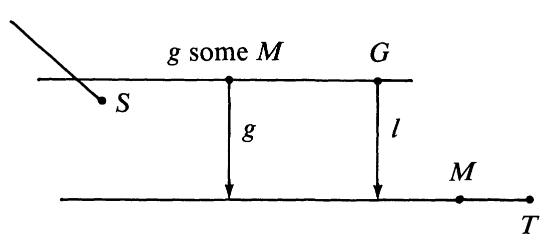
from which we might conclude 'Some senator gives away something tainted'. We can use (ii), however, whenever the analysis of the relational expression, 'gives away some money', is not demanded by any subsequent premise or conclusion. Suppose our second premise is 'Whoever gives away some money is generous'. Then we can diagram our premises as

(ii.1)



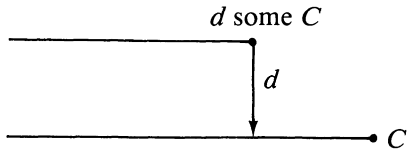
And from this we could read the conclusion 'Some senator is generous'. Finally, we are in need of the *full* representation, (iii), when the relational expression occurs subsequently both analyzed and unanalyzed. For example, suppose our second premise is 'All money is tainted', our third premise is 'Whoever gives away some money is generous', and our fourth premise is 'Whoever is generous loses some money'. The conclusion, 'Some senator loses something tainted', is diagrammed by diagramming the premises thusly:

(iii.1)

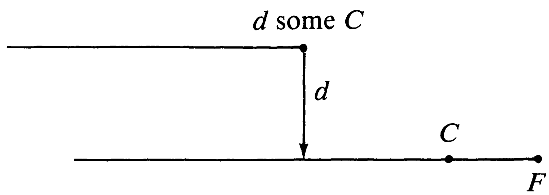


Consider next the following famous inference (see Sommers [14], pp. 42ff). 'Every circle is a figure. So whoever draws a circle draws a figure.' The premise is easy enough.

We also know that whoever draws a circle draws a circle. Thus



Together these give is



And here we can see 'Whoever draws a circle draws a figure'. We can develop a useful general rule out of this. Call it "Rule *R*", based at bottom on four kinds of cases. These cases are argument forms, each of which has the premise 'Every *X* is *Y*'. Then, two kinds of conclusions occur with four variations in the remaining premise—as follows:

Case 1. Every *X* is *Y*
Some *S* is *R* to some *X*/ so, Some *S* is *R* to some *Y*

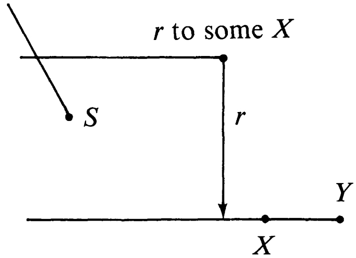
Case 2. Every *X* is *Y*
Some *S* is *R* to every *X*/ so, Some *S* is *R* to some *Y*

Case 3. Every *X* is *Y*
Every *S* is *R* to some *X* /so, Every *S* is *R* to some *Y*

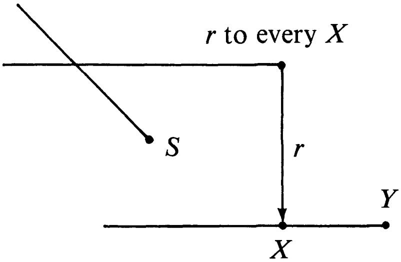
Case 4. Every *X* is *Y*
Every *S* is *R* to every *X* /so, Every *S* is *R* to some *Y*

So, in general, the conclusion has one and the same grammatical predicate, '. . . is *R* to *some Y*', even when the same predicate in the second premise has an 'every' in it. All of these cases seem valid intuitively. Each is confirmed to be valid when placed on linear diagrams—as follows:

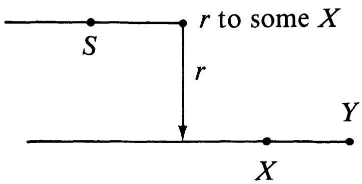
Case 1



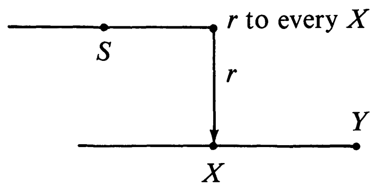
Case 2



Case 3



Case 4



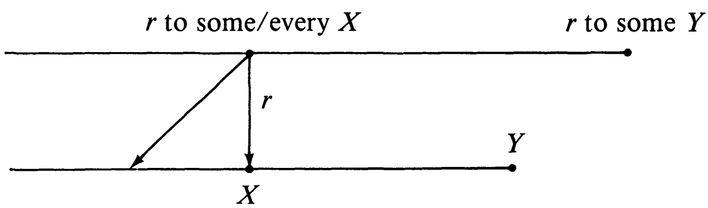
It is important to note that obviously valid as Cases 2 and 4 appear in our diagrams, they are not valid in the predicate calculus. To prove them valid in the predicate calculus requires adding a premise tantamount to the existential import embedded in our diagram method—the premise (for these schemata) 'There are *Xs*' or 'Some *Xs* exist' (for Cases 2 and 4).

Rule R is simply the obvious generalization from these cases—to wit:

Rule R: If every *X* is *Y*, then whatever is *R* to some/every *X* is *R* to some *Y*.

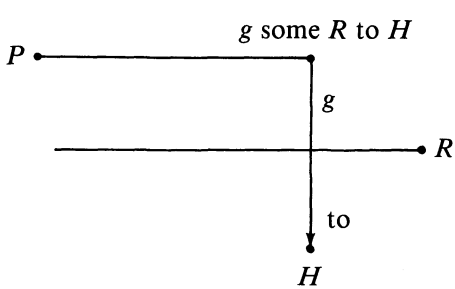
Diagrammatically, we state it as follows:

Rule R:



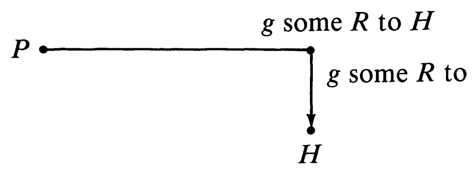
Thus, Rule R permits extending the line for 'is *r* to some/every *X*' so that it is a subportion of 'is *r* to some *Y*' so that the top line (in the statement of Rule R) can be read 'Everything that is *r* to some/every *X* is *r* to some *Y*'.

Now let us diagram the proposition 'Paris gave a rose to Helen' as

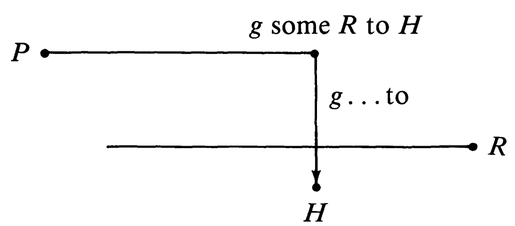


We have arrived at this representation in the following way. First diagram the proposition simply as

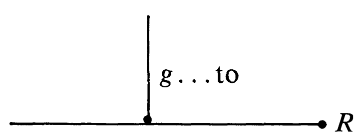
Next, analyze the relational expression 'gave a rose to Helen' as a relative term 'gave a rose to ' and its object term 'Helen', i.e.,



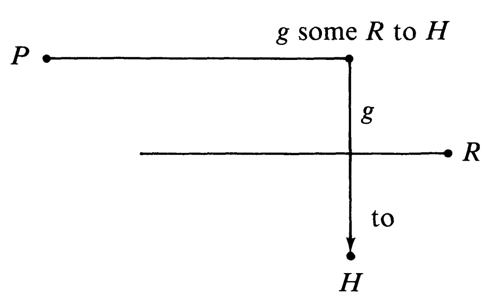
Now the relative term 'gave a rose to' is itself a relational expression. We analyze it as a relative term 'gave . . . to' and its object term 'a rose'. Thus



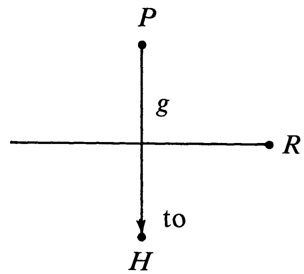
Here the arrow labeled 'gave a rose to' has been replaced by



Since the vertical line here is now only a part of the arrow from 'gave a rose to Helen' to 'Helen' we can relabel the arrow segments to give us

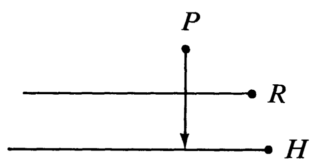


As before, we can often, unless the context demands otherwise, simplify this as

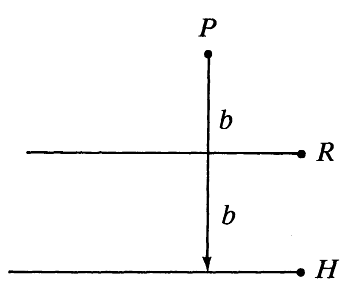


And this also diagrams such equivalent converse relationals as 'Helen was given a rose by Paris', 'A rose was given to Helen by Paris', 'Paris gave Helen a rose', 'Helen was given by Paris a rose', and 'A rose was given by Paris to Helen'.

In diagramming relational expressions it must be kept in mind that ultimately the entire arrow represents the relation. This is especially important for relations which are not usually expressed in natural language by multi-world terms. Consider, for example, 'Paris is between a rock and a hardplace'. This can be simply diagrammed as



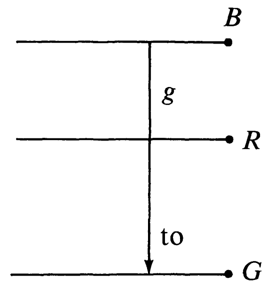
but there is no way to independently label the arrow-segments. The entire arrow represents the 'between' relation. Perhaps the most perspicuous diagram would thus be one which labels all such arrow segments by 'between', viz.



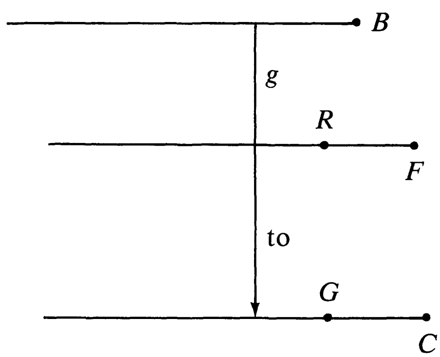
Our analysis of relationals with more than one object is useful for diagramming such inferences as

Some boy gave a rose to a girl
Every rose is a flower
Every girl is a child
So some boy gave a child a rose.

The first premise is diagrammed (simply) as



Adding the next two premises gives us

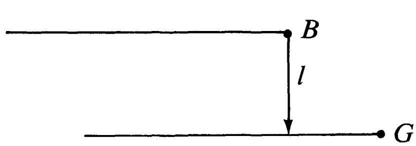


The conclusion is read directly.

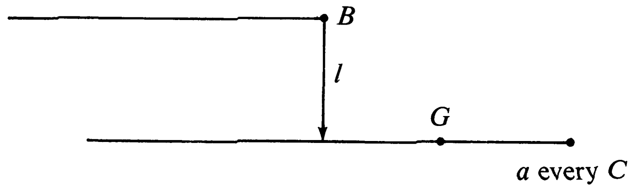
Of course, when entire relational expressions occur more than once in an inference we are often required to use their unanalyzed representations. Consider

Every boy loves some girl
Every girl adores every cat
Whoever adores every cat is a fool
So every boy loves a fool.

Here 'loves some girl' requires analysis while 'adores every cat' does not. So we diagram the first premise as

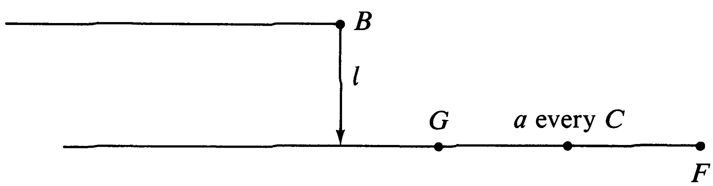


Adding the second premise we get



(We could add the analysis of 'adores every cat', viz.

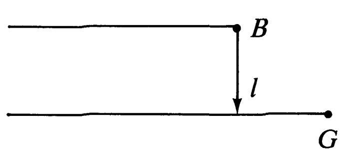
but it is unnecessary in this context.) Finally, we add the last premise.



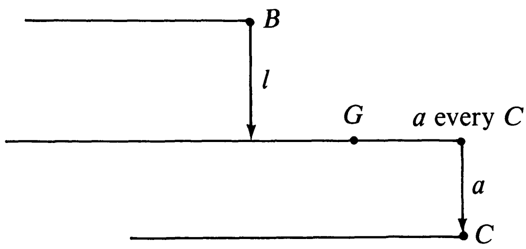
Let us consider one last example.

Every boy loves some girl
Every girl adores every cat
Every cat is mangy
Whoever adores something mangy is a fool
So every boy loves some fool.

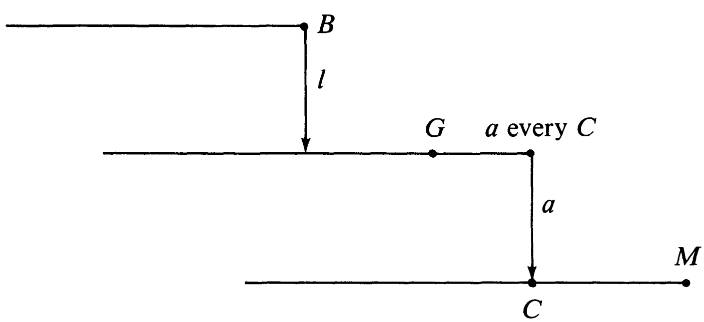
In this case, unlike the preceding one, both relational expressions must be analyzed. But, as we will see, 'adores every cat' must have an unanalyzed representation as well. The first premise is diagrammed as



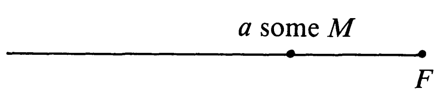
Adding the second premise we have



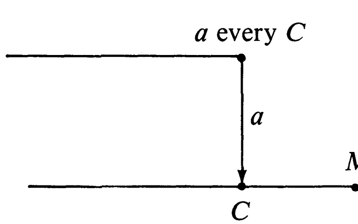
The third premise gives us



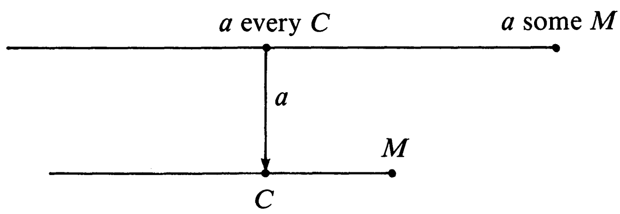
Now the fourth premise is diagrammed as



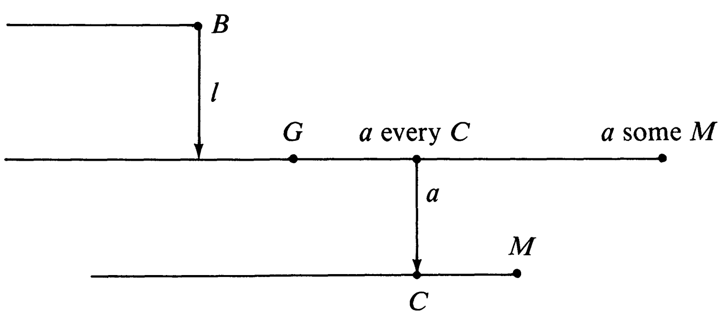
Clearly what is missing to connect the two diagrams is a representation of 'Whatever adores a cat adores something mangy'. And, in fact, this does hold, given the third premise and our Rule R, for our Rule R explicitly sanctions changing a diagram like



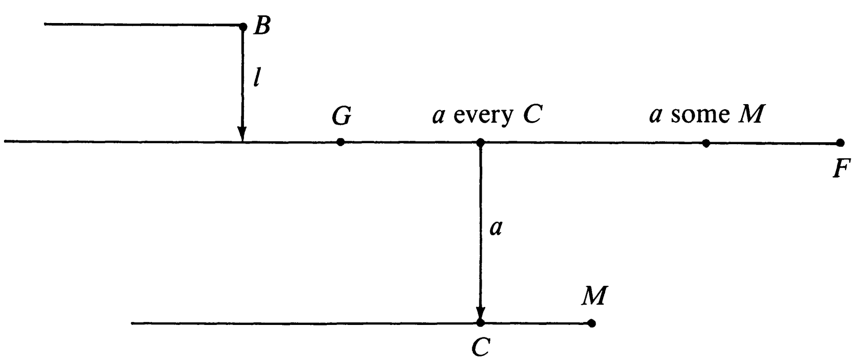
into



So instead of trying to add an *F*-line to the *M*-line (a mistake, since "Every *M* is an *F*" is not a premise), we change the last diagram for this argument into the following (which does not illustrate more information and is merely a rearrangement):

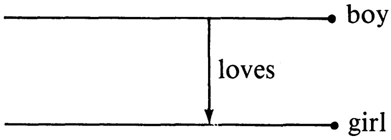


For then we can add a representation of "Every adorer of some *M* (mangy-thing) is an *F* (fool)" correctly and thereby show how the conclusion "every *B* (boy) loves some *F* (fool)" is already represented:

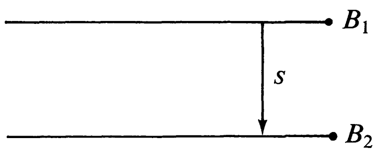


And notice that this final diagram also contains a very clear representation of the intermediate conclusion that many might think is the most important one for this argument—viz., "Every *G* (girl) adores some *M* (mangy-thing)".

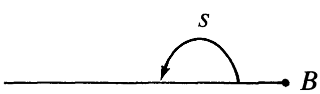
7 We have adopted the convention that any arrow which touches the right terminus of a line touches every point on that line; an arrow which touches a line only at a point left of the right terminus touches that line at no other point. Yet the presence of repeated references raises the possibility of counterexamples to the second part of our convention. Consider the proposition 'Some barber shaves some barber'. If we diagram this using 'Some boy loves some girl' as a model, i.e.



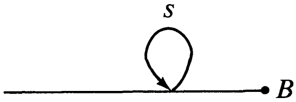
we get this:



where B_1 represents shaving barbers and B_2 represents shaved barbers. But clearly B_1 and B_2 must represent the same set of objects. After all, if just one barber shaves himself then the proposition is true. In fact B_1 and B_2 must be identical lines. Let us represent barbers, then, by a single line (as we have always done). Then our proposition can be diagrammed by drawing an arrow from one nonterminal point to another.



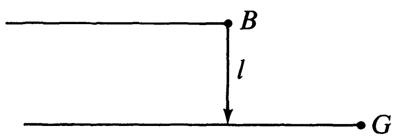
Here the distance between the base and head of the arrow may be equal to or greater than zero (equal to zero if some barber shaves himself). This last suggests that we have a preliminary way of diagramming the proposition 'Some barber shaves himself', viz.



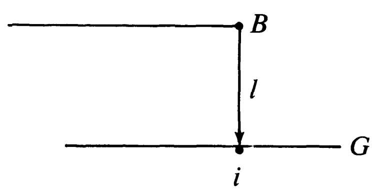
and the proposition 'Every barber shaves himself', viz.

But how, given our right terminus convention, do we diagram 'Every barber shaves every barber'?

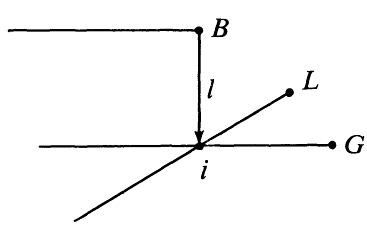
What is required for us to diagrammatically distinguish between propositions of the form 'Every *X* is *r* to every *X*' and 'Every *X* is *r* to itself' is an additional convention. Our new convention will involve the representation of pronominal expressions. Consider the inference 'Some girl is loved by every boy. She is lucky. So some boy loves something which is lucky'. Here specific reference has been made to some girl (i.e., to a certain girl rather than to some girl or other). Then a pronoun is used to make subsequent reference to that very girl. We don't know her name, but we can give her an arbitrary one (that is just what we use the pronoun 'she' for in this example). Let us agree to label every unnamed individual to which specific reference is made with a small Roman numeral. That label can then represent the pronoun in subsequent pronominal references. For our sample inference we could diagram the first premise initially as



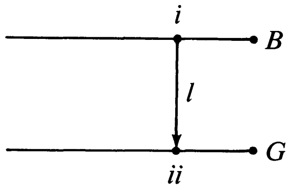
But, since the reference to some girl is specific, we will label it.



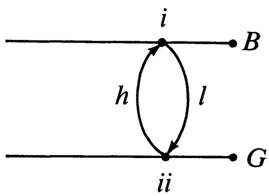
The second premise says that she (*i*, the girl loved by every boy) is lucky. Adding this yields



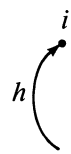
Consider next the argument 'Some boy loves a girl. She hates him. So he loves a hater of him'. We diagram the first premise, adding the pronoun labels for subsequent use.



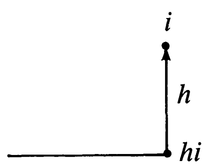
Adding the second premise, we get



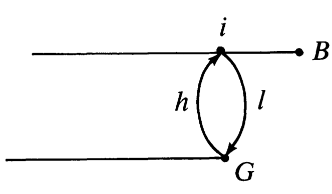
The conclusion is diagrammed here once we recognize that 'hates him', i.e.



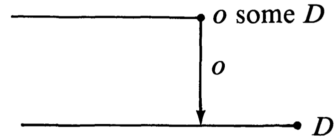
is a simplification of 'is a hater of him', i.e.



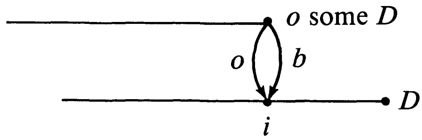
This same simplification allows us to diagram 'Some boy loves every girl. They hate him. So some boy loves a hater of him'.



Next consider the proposition 'Every owner of a donkey beats it' (see Geach's important discussion of such sentences in [3], pp. 116ff). Analytically, every owner of a donkey owns a donkey, i.e.



It, the donkey so owned, is beaten by its owner. So:

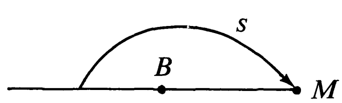


We are adopting the convention of labeling individuals to which specific reference is made by small Roman numerals, which are then used to represent subsequent pronominal references. This convention permits us to diagram, now, 'Every barber shaves every barber' and 'Every barber shaves himself' respectively as follows:

Now since 'Every barber shaves every barber' implies 'Every barber shaves himself' we might justify this by a rule (called "*i*-insertion" by an anonymous reader) analogous to existential instantiation in the predicate calculus. Such a rule allows us to pronominalize at will by marking any point on a given diagram with a Roman numeral. It must be noted that once a pronoun is so marked it cannot subsequently be ignored (otherwise one might derive, e.g., 'Every barber shaves every barber' from 'Every barber shaves himself'). We would diagram the valid argument 'Some barber shaves every man. Every barber is a man. So some barber shaves himself' as follows:

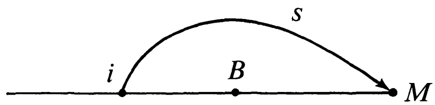
Second premise:

Adding the first premise:

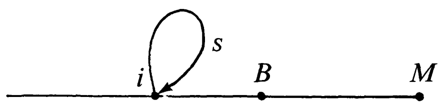


(i.e., 'Some barber shaves every Man').

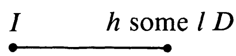
Given our pronominal convention (*i*-insertion) and our right terminus convention we can read the conclusion directly from:



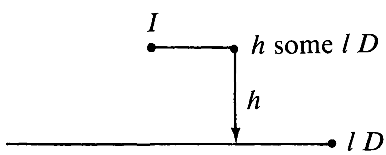
In particular, the latter convention permits movement at the arrowhead to the left (with the *i*-point as its leftmost limit). In other words,



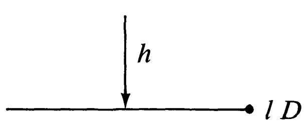
Let us conclude this section with some brief remarks concerning relative products. Consider the proposition 'Iago is a hater of a lover of Desdemona'. Leaving 'hater of a lover of Desdemona' unanalyzed, we have



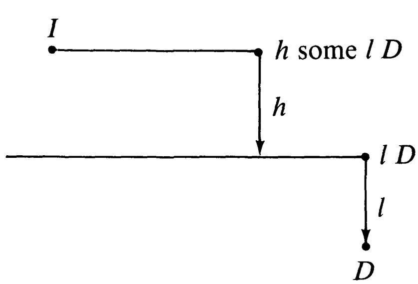
Here Iago (*I*) is an individual member of the set of things which hate (*h*) some lover (*l*) of Desdemona (*D*). We could analyze the relational here first as



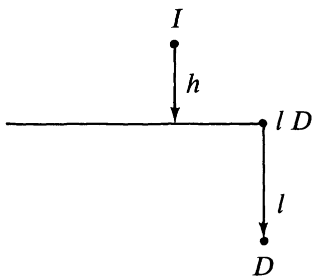
where



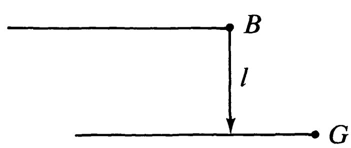
represents 'hates a lover of Desdemona'. Further analysis yields



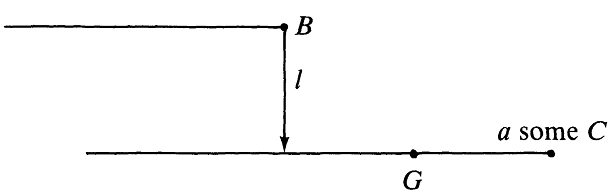
By suppressing some of the analytic content here we could simplify this as



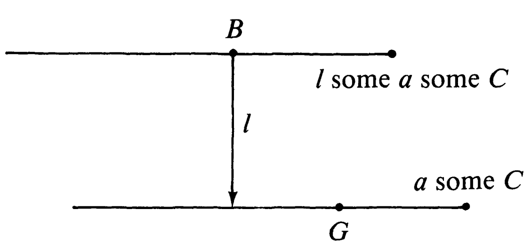
Next, consider the argument 'Every lover of an adorer of a cat is a fool. Every boy loves some girl. Every girl adores some cat. So every boy is a fool'. We diagram the second premise simply as



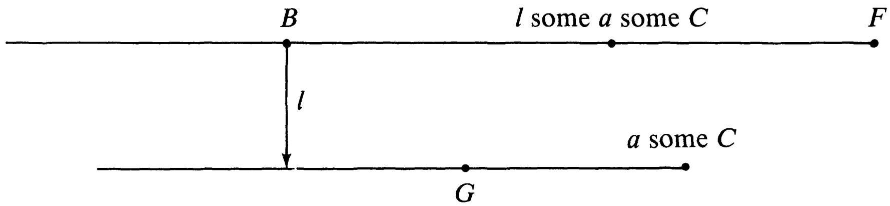
There is no need to analyze 'adores some cat', so we can add the representation of the third premise to give us



Now, by Rule R, we can add

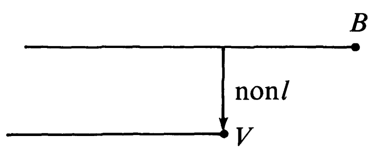


Finally, we add the first premise to get

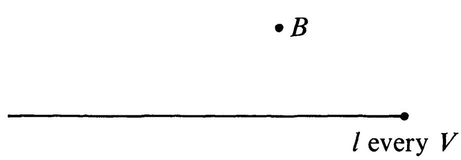


from which we can read the conclusion directly.

Finally, let us note (under the prompting of our anonymous but friendly reader) that negative relationals can be diagrammed by our method. First we must recognize that, as with nonrelational terms, relational terms have corresponding negatives. Consider the sentence 'Some boy does not like every vegetable'. The sentence is ambiguous (in at least two ways), given there are no contextual clues, between (1) 'Some boy dislikes every vegetable' and (2) 'Some boy fails to like every vegetable'. The difference here is due to the scope of the negation. In (1) only the relational term 'likes' is negated (i.e. 'Some boy does not-like every vegetable'). In (2) the entire relational predicate 'like every vegetable' is negated (i.e. 'Some boy does not-(like every vegetable)'), an O form. We can think of 'dislike' as the logical contrary of 'like' and 'fails to like' as the contradictory of 'like'. Such relationals are diagrammed just as nonnegative relationals are diagrammed. Thus, (1) can be diagrammed as:



(2) can be diagrammed as:

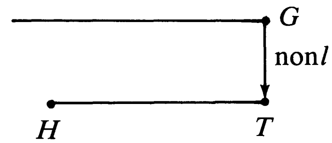


The real value of recognizing negative relationals is seen when we approach inferences in which such expressions play logically effective roles. Consider the argument 'Paris loves Helen. Every Greek fails to love every Trojan. Helen is a Trojan. So Paris is not a Greek'. The third premise is diagrammed as:

(i) 

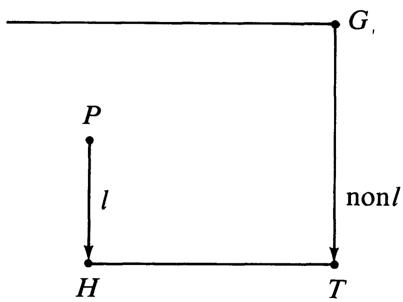
Now (without accounting for the relations between contrary and contradictory negative relations) we can diagram the second premise (along with the third) as:

(ii)



Adding the first premise, we have:

(iii)



At this point the question naturally arises concerning how we know that the *P*-point is *not* on the *G*-line. For if it is not then our conclusion is diagrammed. First, note that the *l*-line and non*l*-line are (indeed, must be) parallel. Now, we could not diagram the *P*-point on the *G*-line without contradicting the first premise. So the only way to diagram all three premises consistently is by keeping the *P*-point off the *G*-line. Diagram (iii) represents the simplest and most perspicuous way of doing this, and from it the conclusion is easily read. Of course there is much more to be said concerning negative relationals and, indeed, negation in general (see [14]).

8 I have offered these suggestions about linear diagrams because I believe syllogistic is alive and well—indeed, stronger than ever. Quine's predicate-functor algebra (see [9] through [12]) is one version of it. Also see Peterson [8]. But by far the best version is Sommer's (see especially [14] and the essays in Englebretsen [1]). His new syllogistic is simple, natural, effective, and more powerful than the standard first-order predicate calculus. With an easy diagram method to accompany it, who could ask for anything more?

Acknowledgment—Conversations with Fred Sommers, Mary Rhodes, and Thomas Coats have helped me to get clearer about the linear diagrams. I especially want to thank Professor Sommers, in spite of the fact that I have probably ignored far too many of his good suggestions, and an anonymous reader for this journal, who provided many invaluable suggestions, criticisms and, indeed, additions. Finally, my thanks to my mathematician wife, Genevieve Boulet, who kept me in line during the long period of work on these ideas.

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