

Book Review

Colin McLarty. *Elementary Categories, Elementary Toposes*. Oxford University Press, Oxford, 1991, xiii + 265 pages.

1 Introduction Writing an introductory book on category theory and topos theory is a serious challenge, particularly if one wants to reach logicians, philosophical logicians, and philosophers. The reason is simple: students in those fields are not usually exposed to the vast sample of mathematical structures like monoids, groups, vector spaces, rings, fields, topological spaces, manifolds, and so on, which are used as examples and motivation for categorical structures and definitions. This is a serious problem since examples play a crucial role in the understanding and learning of a discipline. One therefore has to find a way to circumvent this difficulty, either by finding simple examples, if this is possible, which are nevertheless interesting and relevant enough or presenting the field in such a way that the reader can, maybe with the help of an instructor, construct her own examples as she goes along. Colin McLarty, in his book *Elementary Categories, Elementary Toposes*, has opted for the latter approach. His sole presupposition is that the reader is skilled at abstraction and the only real prerequisite is a familiarity with logic and formal systems in general. The bet is to present category theory and topos theory “as if” they were easy and almost mere formal languages. Learning category theory then becomes, more or less, learning the “language” of commutative diagrams and diagram chasing, and learning topos theory amounts to, again approximating roughly, learning a certain higher-order type theory which is related to this diagrammatic language in a certain systematic manner.

Anyone with the required skill will certainly benefit immensely from McLarty’s well-organized and clear book. All the basic definitions are given and whenever a new concept is introduced, the author takes the time to motivate it and indicates how one can think about it informally. The book covers the fundamental notions of category theory and topos theory. The choice of some topics might surprise some readers, but it is clear that it is not innocent and that it follows a definitive point of view of the subject, though it is never mentioned nor discussed explicitly. McLarty, who has published many papers on technical as well as philosophical aspects of topos theory and categorical logic, is faithful to the *spirit* of those theories and certainly wants the reader to understand what this spirit is by putting it in practice. By learning and doing category theory and topos theory the way it is presented in this book, the reader will

assimilate, probably without even noticing it, categorical ways of thinking about formal matters. By covering the topics chosen, the reader will be led to reflect about the point of category theory and topos theory, again probably without noticing it, in a way that is quite close to the way the practitioners of the field think about it themselves. It is probably one of the very few books in the field, and certainly the only introductory book I know of, which achieves this result in an effective, sober, and economic style.

We will first look at the structure of the book, then at the content of the various sections. We will afterward say a few words about how the book compares to other books with more or less the same goal and end with a few suggestions to the reader.

2 *The structure of the book* The book opens up with a very nice introduction which will probably be more appreciated after a first reading of the entire work. It ends with a short section with suggestions on further reading and a good bibliography. In between, the book can be decomposed into two conceptual parts: the first one on basic category theory, which covers roughly the first half, and the second one on topos theory, which covers roughly the second half of the book. More precisely, it is divided into four parts. Part I is entitled “Categories” and has 59 pages, Part II is entitled “The Category of Categories” and has 43 pages, Part III is entitled “Toposes” and is 93 pages long, and Part IV, which is called “Some Toposes”, is 44 pages long. Thus the first two parts on categories make for 102 pages of the book and the last two parts on toposes account for 137 pages. Each part is then cut further into short sections, each of which ends with a series of exercises. As McLarty himself emphasizes, the exercises constitute an intrinsic and inescapable part of the exposition, for they sometimes contain additional information or results which are used later on. The difficulty of the exercises ranges from the elementary verification to the more involved demonstration of new results. There are some exercises which presuppose more advanced mathematical knowledge but they are not required for the rest of the book.

3 *Part I: Categories* This part introduces the basic definitions and concepts for an individual category. In the first section, categories are defined and fundamental arrow-theoretic concepts like monics, isomorphisms, terminal, and initial objects are introduced. One idiosyncratic element of the approach is also introduced, the so-called generalized elements. This is not used by all category theorists and some even wonder whether this way of thinking about categories will be passed on to future generations. Be that as it may, it is not a necessary part of the theory and it is a pedagogical choice on the part of the author. It is certainly justified by the fact that most philosophers and philosophical logicians (and even mathematicians to some extent) are familiar with the notation of set theory and think “more easily” in terms of elements than in terms of arrows only. But it might also be a bit confusing, for generalized elements are not elements in the set-theoretical sense and have properties which are incompatible with set-theoretical elements. Indeed, a generalized element T of an object A is simply an arrow $x : T \longrightarrow A$. Then one writes $x \in_T A$ and says that x is a generalized element of A . Thus, it is merely a notational device. The advantage is that, written in this notation, certain results and definitions look like the familiar set-theoretical results and definitions. But, the notation has its limits and drawbacks. For instance, since arrows always compose in a category, if $x \in_T A$ and $y \in_A B$, then

$yx \in_T B$, where ‘ yx ’ is simply the composition of x and y , which is a form of transitivity which does not make sense from a set-theoretical point of view. Be that as it may, if the reader understands the notation and does not confuse it with the set-theoretical notation, then it is, at times, heuristically valuable, particularly when one starts using the so-called external semantics of toposes. But it is really a matter of taste and some students might find it more confusing than helpful.

Section 2 introduces products, equalizers, and their dual and proves some of their fundamental properties. Section 3 can be skipped altogether but it shows how the language of diagrams can be used to define well-known mathematical structures in a category, in this case, groups. The author is here just one step away from an interesting development of categorical logic, though again not an indispensable one nor a universally accepted one, namely, the notion of sketch. However, it might have been interesting to point out the existence of sketches as an adequate syntactic framework for category theory, although while the book was written, sketches had not been resurrected yet to their present form and use. The interested reader can consult Barr and Wells [1] or Makkai [14] for the definition of a sketch and their relevance in logic. The next two sections introduce subobjects, pullbacks, the general notion of limit, and the categorical treatment of relations and proves the usual results about them.

Section 6 introduces Cartesian closed categories and constitutes a conceptual, if not a technical, leap. I know by experience that this is the section where most students start having serious problems. But this is not due to any weakness in the exposition, it is just intrinsically more difficult than the previous sections.

Section 7 is very short, two pages, and offers a simple syntactical solution to an interesting aspect of category theory. Indeed, the objects defined by categorical means are only defined up to isomorphism. For instance, given two objects A and B of a category C , a product for A and B is an object P of C together with arrows $p_A : P \longrightarrow A$ and $p_B : P \longrightarrow B$ such that, for any object T with arrows $f : T \longrightarrow A$ and $g : T \longrightarrow B$, there is a unique arrow $h : T \longrightarrow P$ such that $p_A h = f$ and $p_B h = g$. Now P is any object of C with the appropriate pair of arrows. It can easily be shown that P is defined up to an (unique) isomorphism thus: for any object Q with arrows q_A, q_B satisfying the foregoing condition, Q can be shown to be isomorphic to P and any object Q isomorphic to P is also a product for A and B . But, in practice, one has to fix such a product for objects A and B . When this has to be done for all pairs A, B of a category, then one has to use the axiom of choice. The question is whether the axiom of choice can be avoided and, if so, how. McLarty’s solution is to extend the usual axiomatization of category theory with new operators in such a way that an arbitrary choice will be made once and for all. Since the nature of that choice is irrelevant, we can work without thinking about it. There is an alternative, developed in Makkai [13], which consists in redefining the notion of functor and this allows one to avoid the axiom of choice altogether. Moreover, the latter approach might have some benefits when it is applied to higher-order category theory. It is too early to tell, but the issue is important since it is connected with the foundations of category theory, and thus, from a certain perspective, with the foundations of mathematics period. But questions related to the foundations of mathematics are treated in Part II of McLarty’s book.

4 Part II: The category of categories Whereas the first part treated solely of individual categories and some of their “internal” properties, the second part considers how categories are related to one another.

Functors and certain fundamental constructions on categories which are used later on in the book are introduced and followed by a presentation of natural transformations and the notion of equivalence of categories. Then comes the fundamental notion of adjunction. This section is particularly interesting, for here the author has to comply by his rule of using exclusively first-order language concepts. This means that the notion of adjoint functors has to be treated in a somewhat different fashion than in most textbooks. Nonetheless, the presentation is convincing, clear and very instructive. However, it might not be as “intuitive” as the standard presentation in terms of hom-sets, at least for those readers who think in terms of sets. Again, this is a pedagogical choice, but also a consequence of the decision to present “elementary” category theory. Indeed, elementary means that second-order variables are avoided and hom-sets are second-order entities. Consequently, the notion of representable functor, which played an important part historically, at least in the Grothendieck school, is avoided too. Of course, the later is equivalent to the notion of universal arrow and Section 10 opens with the latter notion and the usual presentation in terms of isomorphism of hom-sets is “recovered” via an isomorphism of categories instead. Thus, the student who might afterward go on and read more advanced textbooks in which hom-sets, representable functors and other similar notions are introduced and used should not have difficulties seeing how they relate to McLarty’s elegant presentation.

Section 11 presents slice categories and is clearly written with later chapters in mind. Section 12 treats the delicate question of the foundations of mathematics and the foundations of category theory. This is seldom presented in an introductory book, but it certainly should be. Two technical problems naturally show up in this regard:

1. How can a set theoretical framework encompass large categories?
2. Is it possible to axiomatize directly the category of categories and proceed, from then on, by ignoring a set theoretical framework altogether?

There are a few standard answers to the first question, though most authors adopt the point of view of the so-called Grothendieck’s universes. McLarty introduces those and also presents MacLane’s simplification to one universe. The second question is considerably more interesting, given the fact that it is not settled yet. Well aware of this fact, the author briefly sketches a few axioms for the category of categories to illustrate what they might look like. Needless to say, this is an important issue, both from philosophical and technical points of view and one which is very much alive and in which important developments are in fact taking place while this review is being written. Ironically though, the latter developments take into account the higher-order nature of the category of categories and it seems that any discussion of this point will have to take this additional structure into consideration.

One methodological remark about the first two parts has to be made. There is some flexibility in the organization of the material presented. Indeed, one could very well start with Section 1 and then jump immediately to Sections 8, 9, and 10, then come back to Sections 2, 4, and 6, look at those as special cases of the previous sec-

tions, and finish with Sections 11 and 12. This would be a completely legitimate way to present the material and would have some advantages. But then again, one might start with Sections 1, 2, 4, and 6 and then jump immediately to Chapter 13 and come back to the other sections only when necessary. It all depends on one's goals, but it is clear that the book allows these possibilities.

5 Part III: Toposes Section 13 introduces the basic definition of a topos and some of the elementary properties following from the properties of the subobject classifier. This is in fact the set up for the next important section. Section 14 introduces the internal language of a topos. From this section on, the author uses the internal language to prove various results about toposes. It is one of the striking features of toposes that such a language can be introduced and effectively used in the proofs of theorems. This is certainly one of the most original features of the book, at least from the pedagogical point of view, and certainly one of the most interesting. Very often, authors introduce the internal language but never really use it in the subsequent proofs or use it in such a way that the categorical aspects of toposes are lost or hidden. The present book teaches the student how to work with the language by going from the language to the topos and back. This interplay shows how to write up formulas in the internal language, thus how to express mathematical facts in this language, how to deduce certain facts in the theory, then how to interpret them in the topos and see what they amount to from a categorical point of view.

Section 15, which is optional, proves the soundness of the internal language and briefly discusses completeness. Section 16 uses the internal language to prove various important facts about toposes. This is a very useful section, both conceptually and pedagogically. In Section 17, the author proves the so-called fundamental theorem of topos theory, which says that every slice of a topos is itself a topos. The reason why this theorem is deemed fundamental is that many important facts about the structure of toposes follow directly from this property. Following the fundamental theorem, the author discusses the case of Boolean toposes and the nature of the axiom of choice in the topos theoretical context. He proves the important theorem of Diaconescu, which shows that if a topos satisfies the axiom of choice, then it is necessarily Boolean.

After a short section, Section 18, on the external semantics, also called the Kripke-Joyal semantics, the author defines natural number objects in a topos and develops the fundamentals of arithmetic and its extensions, namely, the construction of the rationals and the reals in a topos. Section 20 treats categories in a topos. This section will probably look very mysterious at first, for the treatment is very formal and strange for the uninitiated. However, it is of the utmost importance, for it leads to toposes of presheaves and sheaves, which is the topic of the next section entitled "Topologies," in which Grothendieck topologies are defined, their basic properties proved, and the importance of this concept discussed.

6 Part IV: Some toposes In this section, the author introduces some examples of toposes from various fields. Clearly, the motivation is to show that specific toposes can be chosen for specific needs and that when one makes the appropriate choice, one can then develop mathematics within these toposes in an interesting way. The toposes chosen are: the topos of set, which could be described as a nontrivial, well-pointed

topos with a natural number object, the topos of spaces, for the purpose of differential geometry, the effective topos, for the purpose of recursion theory, and lambda calculus and results on regular categories, which are needed for the presentation of the effective topos. These sections are not exhaustive and are not meant to be. The author wants to introduce the reader to the basic definitions and exhibits the fundamental features and interesting aspects of these toposes. It should be pointed out that these constructions, except for the topos of set, which is, in a sense, “well known”, are not trivial and are demanding for the reader. They presuppose some knowledge of recursion theory, calculus, and general set theory, that is, equivalence relations and quotients. But they do have the advantage of showing that topos theory is not merely a generalized set theory and is not done merely for the sake of generalization. This is consistent with the history of topos theory and its use in mathematics. For toposes were invented by Grothendieck in the sixties for specific purposes in algebraic geometry and algebraic topology. Later on, Lawvere and Tierney [11] axiomatized the notion of an elementary topos with the goal of providing an appropriate foundation for differential geometry. As MacLane pointed out, the move toward topos theory in the seventies and eighties in the community of category theorists reflected a move toward concrete applications and not a move toward more abstraction. Toposes are used by algebraic geometers, mostly via Grothendieck topologies and very often implicitly, on a day to day basis. As McLarty himself has pointed out in [16], this leads to a certain view about toposes and the foundations of mathematics, a view which is quite different from what set theorists have in mind when thinking about foundational issues.

Indeed, the goal here is not to construct an all-encompassing foundation for mathematics, a theory which would cover in a stroke all mathematical constructs and fields. Rather, one works, so to speak, in two steps. First, in order to understand a particular field, for example, differential geometry, one tries to axiomatize the given field within the language of topos theory, that is, to characterize a class of toposes which are well suited for this type of mathematics. Then, once this is done, one studies the properties of these toposes and the relationships of these toposes with other well-known toposes, for example, the topos of set. This constitutes the second step. It should be emphasized that this way of thinking of the foundational relevance of topos theory is in fact the direct extension of the foundational relevance of category theory itself. Indeed, it can be argued that category theory became foundationally relevant when Grothendieck axiomatized, improving MacLane and Bushbaum earlier attempts, the notion of an abelian category and used it in an essential way to extend homological algebra to algebraic topology. This showed how a whole field, in this case, homological algebra, could be based on an abstract category. Before, mathematicians were using particular *concrete* categories, such as the category of topological spaces and the category of abelian groups, and *concrete* functors between them, such as the homology functors. After Grothendieck, one could envisage the possibility of axiomatizing in a categorical fashion, that is, abstractly or purely in terms of structural features of a category or categories, whole areas of mathematics, including, in particular, set theory. Lawvere quickly tried to provide a categorical axiomatization of the category of sets. However, his first attempts were too specific. He then realized that Grothendieck’s notion of topos was a first step in the right direction. What was left to be done was

to provide an elementary axiomatization of this notion, which he did with Tierney. It is correct to say that the notion of an elementary topos constitutes an adequate axiomatization of “the” category of sets and, in this sense, one can claim that a topos is a generalization of a universe of sets. Indeed, working within a topos, that is, with its internal language, is like working within a universe of sets from a purely abstract point of view. (This later claim has to be qualified somewhat, for there are numerous logical details one has to be sensitive to in this context, which are usually taken for granted in a classical set theoretical framework. Reading McLarty’s book is probably the best way to see and understand what these qualifications are.) The search for specific axioms which extend the axioms of a topos and are suited for particular mathematical fields becomes a genuine foundational enterprise, one with concrete and relevant results. As we have already pointed out, McLarty’s book contains some examples. Each one of his examples is still undergoing development. To mention but a few instances of these developments, Joyal and Moerdijk [9] provide an analysis of models of ZF set theory, thus the cumulative hierarchy, within the universe of toposes; what is now called Synthetic Domain Theory, which is the extension of the ideas presented in the chapter on the effective topos, is a very active industry with competing approaches, developments which one can find in Hyland [7], Taylor [19], Fiore [3], Fiore and Rosolini [4], Fiore [5], and Rosolini [18], to mention but a few.

Of course, there is also a more general point of view, a search for an *overall* foundation for mathematics. For toposes are categories and one can certainly consider the category of all (small) categories and try to provide an axiomatization for that field, considered as “abstract mathematics” in general. As we have already mentioned, work is presently being done in this area. One of its potential fruits, from the philosophical point of view, is that it would provide flesh to the “structuralist” approach to mathematical knowledge. It is, however, still too early to say what the result will look like.

7 McLarty’s book and other books in the field It is only natural to compare McLarty’s book with other books which aim more or less at the same audience, namely, introductory books which might be used by mathematicians and nonmathematicians alike. We will here consider Goldblatt’s [6] and Bell’s [2].

The most striking differences between McLarty’s book and these two books are the choice of topics, their organization, and their styles. Both [6] and [2] cover topics which are close to *standard* logical and set-theoretical concerns, for instance, Kripke models or Boolean extensions, whereas McLarty’s book explores specific toposes with a motivation which is not necessarily related to set-theoretical considerations. Indeed, Goldblatt’s [6] exposition relies on the fact that the reader is already familiar with set theory and introduces toposes as a generalization of the latter. This might be pedagogically sound, but as was pointed out by numerous people, it obscures one of its important aspects. In a way, Bell proceeds in a similar fashion, though less explicitly. Indeed, in [2], even though the contrast with set theory is not used as a pedagogical device, the way Bell uses what he calls ‘local set theory’, that is, the internal language, is very close to the way someone might use the language of classical set theory and, thus, one loses the arrow theoretic viewpoint. McLarty stays closer to the categorical nature of toposes, both in the way he introduces the various concepts and

in the way he proves various results. This difference shows also in the way category theory is covered in these books. Whereas Goldblatt introduces categorical notions as he goes along—and in this way ends up introducing adjoint functors at the very end, even though adjunction is the central notion of category theory—and Bell offers a survey of category theory which might be used more or less as a reference to which one turns whenever necessary, McLarty develops categorical notions thoroughly and provides the student with many elementary proofs as a guide and many exercises and hints to help her assimilate the various abstract notions. This is to say that the styles of the books are very different from one another. I want to emphasize this latter point, for, in a way, these books are more complementary than it might at first appear. It all depends on one's background and one's goals. For those who feel comfortable with classical logic and set theory and want an introduction which uses this knowledge as a springboard, Goldblatt's book is certainly indicated. For those with a strong background in logic and set theory and who are used to thinking in terms of syntactical properties of theories, Bell's book might be appropriate. For those who want to get into the categorical way of thinking right from the start and stay close to that way of thinking, then, clearly, McLarty's book is adequate. If one wants to understand what topos theory is about, then one should definitely read McLarty's book at some point. However, the reader should be warned that McLarty's book suffers from one drawback, apart from the examples of toposes at the end, very few examples are worked out and this might make the reading quite demanding. But it is now quite easy to circumvent this difficulty and we will now sketch a recipe.

8 *A few examples to carry along while reading any book on topos theory* Having used McLarty's book in a graduate seminar on category theory and categorical logic with philosophy students with no other prerequisite than the standard curriculum in formal logic, I can attest that at least my group of students felt that McLarty's approach was too abstract at moments. They did complain about the absence of a few different, clear, and simple examples. Thus, anyone who intends to use the book in a philosophy seminar should develop right from the start a spectrum of good examples to help the students master this "language game." For that purpose, a short introduction to the category of sets, the category of directed graphs, and categories of evolutive sets would certainly be more than useful and this can certainly be done with the help of Lawvere and Schanuel's excellent book *Conceptual Mathematics* [10]. With these examples at hand, the student will not only be capable of learning the abstract definitions and results presented in McLarty's book, but will also be capable of calculating particular important cases. This will allow her to see how these abstract notions are implemented and, therefore, see how they can be used and developed. Thus, we do recommend the use of the category of sets, but as a category, right from the start, together with the category of directed graphs and categories of evolutive sets. These are quite easy to describe directly, that is, without any mathematical prerequisites, and are all interesting examples of toposes. In each case, calculating products, coproducts, equalizers, and pullbacks are more or less straightforward and one can also discover and compute what the subobject classifier ought to be in each case. This is very instructive, for students learn how to find and compute these important concepts in particular cases and they can develop visual representations of

these abstract constructions. Moreover, these three categories are sufficiently different from one another to provide students with a real sample of the variety of toposes and their relationships. For there are “natural” or “obvious” functors between these simple toposes and again, in each case, the student can compute the various constructions and see what they “look like.” Moreover, the topos of graphs has been used by Reyes and Zolfaghari [17] to develop modal logic in a topos theoretical framework. (See also Makkai and Reyes [15] for completeness proofs of modal logic in a categorical setting.) Thus, one can also cover topics in modal logic if the students have already done some in their curriculum.

Armed with these examples, reading McLarty’s book can become not only instructive and useful, but also a very pleasant and enjoyable experience which prepares the reader for more advanced books such as MacLane and Moerdijk [12] and Johnstone [8].

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