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Locally Asplund Spaces of Holomorphic Functions

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1. Introduction

In [12] Defant introduced the local Radon–Nikodým property for duals of locally convex spaces and used it to understand the duality theory of injective and projective tensor products of locally convex spaces. This concept generalizes the concept of Radon–Nikodým property for Banach spaces in the sense that a Banach space has a dual with the local Radon–Nikodým property if and only if its dual has the Radon–Nikodým property. The class of locally convex spaces having duals with the local Radon–Nikodým property includes nuclear spaces, Schwartz spaces, quasi-normable semireflexive spaces, and (gDF)-spaces that have a separable strong dual. This class is stable with respect to subspaces, quotients, countable direct sums, arbitrary products, countable inductive limits, and arbitrary projective limits.

Let E be a (real or complex) locally convex space and let E'_b denote its strong dual. The polar V° of an absolutely convex closed neighbourhood V of 0 in E is equicontinuous and hence is bounded in E'_b. For a closed absolutely convex bounded subset B of E we use $[\![B]\!]$ to denote the normed space spanned by B and with closed unit ball B. Given E and F locally convex spaces, we let $L[\![E; F']\!]$ denote the space of all linear maps from E into F' transforming some neighbourhood of zero into an equicontinuous set. Hence $T \in L[\![E; F']\!]$ if and only if there exists an absolutely convex closed neighbourhood V of 0 in F such that T factors continuously through the Banach space $[\![V^\circ]\!]$.

Let (Ω, Σ, μ) be a finite measure space and let X be a Banach space. An operator $T: L^1(\mu) \to X$ is said to be *representable* [15] if there is a Bochner-integrable $f \in L^1(\mu; X)$ such that

$$T\phi = \int \phi f \, d\mu$$

for all $\phi \in L^1(\mu)$.

Given a locally convex space E, an operator $T \in L[\![L^1(\mu); E']\!]$ is said to be *locally representable* if there is a neighbourhood V of 0 in E and a representable operator $\hat{T} \in L(L^1(\mu); E'_{V^\circ})$ such that the following diagram commutes:

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$$\begin{tikzcd} L^1(\mu) \arrow[rr, "T"] \arrow[dr, "\hat{T}"'] & & E'. \\ & \llbracket V^\circ \rrbracket \arrow[ur, hook] & \end{tikzcd}$$

According to Defant [12], a locally convex space E has a dual with local Radon–Nikodým property if, for every finite measure space (Ω, Σ, μ), all operators in $L\llbracket L^1(\mu); E'\rrbracket$ are locally representable. We rename this property and say that E is *locally Asplund*. By [12], a locally convex space E is locally Asplund if and only if, for every absolutely convex neighbourhood U of 0 in E and every positive Radon measure ν on $(U^\circ, \sigma(E', E))$, there is an absolutely convex neighbourhood V of 0 in E, $V \subseteq U$, such that the embedding $(U^\circ, \sigma(E', E)) \hookrightarrow \llbracket V^\circ \rrbracket$ is ν-measurable.

In Section 2 we show that the ε-product of two locally Asplund locally convex spaces is locally Asplund. In Section 3, we prove that a continuous n-linear map u defined between locally convex spaces E and F is weakly (uniformly) continuous on bounded sets if and only if each of its associated maps $T^j : x \in E \mapsto T^j(x) \in \mathcal{L}(^{n-1}E; F)$, defined by $T^j(x)(z_1, \dots, z_{n-1}) = u(z_1, \dots, z_{j-1}, x, z_j, \dots, z_{n-1})$, maps bounded sets into precompact sets. This generalizes [3, Thm. 2.9]. In Section 4 we apply the preceding results to study local Asplundness of (a) the space $\mathcal{P}_w(^nE; F)$ of continuous n-homogeneous polynomials that are weakly continuous on bounded sets and (b) the space $\mathcal{P}_A(^nE; F)$ of approximable n-homogeneous polynomials. Both spaces are endowed with the topology of uniform convergence on bounded sets. Specifically, we show that when E'_b and F are locally Asplund then $\mathcal{P}_w(^nE; F)$ and $\mathcal{P}_A(^nE; F)$ are locally Asplund. In our final section, we examine local Asplundness of the spaces $(\mathcal{H}_{wu}(U; F), \tau_b)$ and $(\mathcal{H}(U; F), \tau_o)$.

2. Schwartz Products of Locally Asplund Spaces

Given a locally convex space E we let E'_c denote the dual of E, E', endowed with the topology of uniform convergence on all absolutely convex compact subsets of E. If E is quasi-complete then this topology coincides with the topology of uniform convergence on compact subsets of E. For locally convex spaces E and F, the ε-product (Schwartz ε-product) of E and F was introduced by L. Schwartz [27; 28] and is defined as the locally convex space $E\varepsilon F = \mathcal{L}_e(E'_c; F)$ of continuous linear operators from E'_c to F endowed with the topology of uniform convergence on equicontinuous subsets of E'. It is shown in [28] that $E\varepsilon F$ coincides with the space of all weak*–weakly continuous linear maps from E' into F that transform equicontinuous subsets of E' into relatively compact subsets of F. If both E and F are complete and if one of them has the approximation property, then $E\varepsilon F$ can be identified with $E\hat{\otimes}_\varepsilon F$ (see [23]). If $\mathcal{U}_E(0)$ is the collection of all absolutely convex closed neighbourhoods of 0 in E then a fundamental system of neighbourhoods of 0 in $E\varepsilon F$ is given by all sets of the form

$$N(U^\circ, V) := \{T \in E\varepsilon F : T(U^\circ) \subset V\},$$

where $U \in \mathcal{U}_E(0)$ and $V \in \mathcal{U}_F(0)$.

The space $\mathcal{C}(K; E)$ denotes all continuous maps from a compact Hausdorff space K into E. If E is the scalar field, we denote $\mathcal{C}(K; E)$ by $\mathcal{C}(K)$. We use $\mathcal{M}(K)$ to denote the space of all finite Radon measures on K.

LEMMA 1. *Let E, F, and G be locally convex spaces with E and F complete. For each $h \in (E\varepsilon F)\varepsilon G$, the expression $\hat{h}(x')(y') = h(x' \otimes y')$ for $x' \in E'$ and $y' \in F'$ defines an operator $\hat{h} \in E\varepsilon(F\varepsilon G)$.*

Note that $E' \otimes F'$ can be considered a subspace of $(E\varepsilon F)'$ by means of the identity $(x' \otimes y')(g) = y'(g(x'))$ for $x' \in E'$, $y' \in F'$, and $g \in E\varepsilon F$.

Proof. We start by proving that $\hat{h}(x') \in \mathcal{L}(F'_c; G)$ for a fixed $x' \in E'$. Clearly $\hat{h}(x')$ is linear. To prove that $\hat{h}(x')$ is continuous, fix $V \in \mathcal{U}_G(0)$. We have to check that there is a compact subset C of F such that $\hat{h}(x')(C^\circ)$ is contained in V.

By hypothesis there is a compact subset K of $E\varepsilon F$ such that $h(K^\circ) \subset V$. The set $C = \{g(x') : g \in K\}$ is a compact subset of F. Let $y' \in C^\circ$. Since $|(x' \otimes y')(g)| = |y'(g(x'))| \leq 1$ for all $g \in K$, we have $x' \otimes y' \in K^\circ$. Hence $\hat{h}(x')(y') = h(x' \otimes y') \in V$ and this proves $\hat{h}(x')(C^\circ) \subset V$.

We now show that $\hat{h} \in E\varepsilon(F\varepsilon G) = \mathcal{L}(E'_c; F\varepsilon G)$. Clearly $\hat{h}$ is linear. To prove that $\hat{h}$ is continuous, take $V \in \mathcal{U}_F(0)$ and $W \in \mathcal{U}_G(0)$. We must find a compact subset K of E such that $\hat{h}(K^\circ) \subset N(V^\circ, W)$; that is, $\hat{h}(x')(y') \in W$ for every $x' \in K^\circ$ and $y' \in V^\circ$.

Since $h \in (E\varepsilon F)\varepsilon G = \mathcal{L}((E\varepsilon F)'_c; G)$, there is a compact subset C in $E\varepsilon F$ such that $h(C^\circ) \subset W$. Set

$$C^* = \{g^* \colon F'_c \mapsto E : g \in C\},$$

where g^* is the transpose of g. Then $C^* \subset F\varepsilon E = \mathcal{L}(F'_c; E)$ and $K = C^*(V^\circ) = \{g^*(y') : g^* \in C^*, y' \in V^\circ\} \subset E$ is compact (see [23, Prop. 16.2.6]). Let $x' \in C^*(V^\circ)^\circ$ and $y' \in V^\circ$. Since

$$|(x' \otimes y')(g)| = |y'(g(x'))| = |x'(g^*(y'))| \leq 1$$

for all $g \in C$, it follows that $x' \otimes y' \in C^\circ$. Hence, $\hat{h}(x')(y') = h(x' \otimes y') \in W$. This proves that $\hat{h}(C^*(V^\circ)^\circ) \subset N(V^\circ, W)$. $\square$

LEMMA 2. *Let K be a compact Hausdorff space and let E and F be complete locally convex spaces. For each f in $\mathcal{C}(K; E\varepsilon F)$, the expression $T(y')(x') = (x' \otimes y') \circ f$ for $x' \in E'$ and $y' \in F'$ defines an operator $T \in F\varepsilon(E\varepsilon\mathcal{C}(K))$.*

Proof. Consider the canonical isomorphisms

$$\alpha \colon f \in \mathcal{C}(K; E\varepsilon F) \mapsto \alpha(f) \in (E\varepsilon F)\varepsilon\mathcal{C}(K),$$

where $\alpha(f)(\phi) = \phi \circ f$ for $\phi \in (E\varepsilon F)'$, and

$$\beta \colon g \in (E\varepsilon F)\varepsilon\mathcal{C}(K) \mapsto \beta(g) \in (F\varepsilon E)\varepsilon\mathcal{C}(K)$$

given by $\beta(g)(\phi) = g(\tilde{\phi})$, where $\tilde{\phi} \in (F\varepsilon E)'$ and $\tilde{\phi}(k) = \phi(k^*)$ for $k \in E\varepsilon F$. Note that, for $x' \in E'$ and $y' \in F'$, we have $\widetilde{x' \otimes y'} = y' \otimes x'$. Let

$$\gamma : h \in (F\varepsilon E)\varepsilon\mathcal{C}(K) \mapsto \hat{h} \in F\varepsilon(E\varepsilon\mathcal{C}(K))$$

be given as in Lemma 1. For $f \in \mathcal{C}(K; E\varepsilon F)$ we have $T = (\gamma \circ \beta \circ \alpha)(f)$, and this completes the proof. □

THEOREM 3. *Let E and F be locally Asplund locally convex spaces. Then $E\varepsilon F$ is locally Asplund.*

Proof. Without loss of generality, we may assume that E and F are complete. By [12, Cor. 5] we must prove that, for any compact Hausdorff space K and every ψ in $\mathcal{C}(K; E\varepsilon F)'$, there exist a bounded sequence of Radon measures $(\mu_i)_i$ in $\mathcal{M}(K)$ as well as an equicontinuous sequence $(y_i)_i$ in $(E\varepsilon F)'$ and $(\lambda_i)_i$ in ℓ_1 such that

$$\langle f, \psi\rangle = \sum_{i=1}^{\infty} \lambda_i \int_K y_i \circ f \, d\mu_i$$

for all f in $\mathcal{C}(K; E\varepsilon F)$.

Since F is locally Asplund, by [12, Thm. 5(b)] there exist $V \in \mathcal{U}_{\mathcal{C}(K;E)}(0)$, $W \in \mathcal{U}_F(0)$, and $z = \sum_i \lambda_i x_i' \otimes y_i' \in \llbracket V^\circ \rrbracket \hat{\otimes}_\pi \llbracket W^\circ \rrbracket$ with $(\lambda_i)_i \in \ell_1$, $(x_i)_i \subset W^\circ$, and $(y_i)_i \in V^\circ$ such that

$$\langle T, \psi\rangle = \sum_i \lambda_i \langle x_i', T(y_i')\rangle$$

for all T in $F\varepsilon\mathcal{C}(K; E)$.

By [12, Thm. 5(a)], for each i we can find $U \in \mathcal{U}_E(0)$ and $z_i' = \sum_{j=1}^{\infty} \gamma_{ij} u_{ij}' \otimes v_{ij}' \in \mathcal{M}(K) \hat{\otimes}_\pi \llbracket U^\circ \rrbracket$ with $\|u_{ij}'\| = \|v_{ij}'\| = 1$ and $\|z_i'\| = \inf \sum_{j=1}^{\infty} |\gamma_{ij}|$ so that

$$\langle T(y_i'), x_i'\rangle = \sum_{j=1}^{\infty} \gamma_{ij} \langle u_{ij}', T(y_i')(v_{ij}')\rangle$$

for all i. If $f \in \mathcal{C}(K; E\varepsilon F)$ and if $T \in F\varepsilon\mathcal{C}(K; E)$ is its associated operator given by Lemma 2, then

$$\begin{aligned}\langle f, \psi\rangle = \langle T, \psi\rangle &= \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \lambda_i \gamma_{ij} \langle u_{ij}', T(y_i')(v_{ij}')\rangle \\ &= \sum_{ij} \lambda_i \gamma_{ij} \langle u_{ij}', (v_{ij}' \otimes y_i') \circ f\rangle = \sum_{ij} \lambda_i \gamma_{ij} \int (v_{ij}' \otimes y_i') \circ f \, du_{ij}';\end{aligned}$$

this proves the theorem. □

Since local Asplundness is inherited by subspaces and since ℓ_1 is not Asplund, we have the following corollaries.

COROLLARY 4. *Let E and F be locally Asplund locally convex spaces. Then the injective tensor product $E \hat{\otimes}_\varepsilon F$ is locally Asplund.*

COROLLARY 5. *Let E and F be locally Asplund locally convex spaces. Then $E \hat{\otimes}_\varepsilon F$ does not contain a copy of ℓ_1.*

Corollary 4 was proved for Banach spaces by Ruess and Stegall [25, Thm. 1.9]. Samuel [26] showed that, if X is an Asplund Banach space and Y is a Banach space not containing copies of ℓ_1, then $X\hat{\otimes}_\varepsilon Y$ does not contain copies of ℓ_1. Corollary 5 is a weak version of Samuel's result for locally convex spaces.

3. Weakly Continuous Multilinear Mappings on Locally Convex Spaces

Given E and F locally convex spaces, let $\mathcal{L}(^nE; F)$ denote the space of all continuous n-linear mappings from E into F. We denote $\mathcal{L}(^nE; \mathbf{C})$ by $\mathcal{L}(^nE)$. Let $\mathcal{L}_w(^nE; F)$ (resp. $\mathcal{L}_w(^nE)$) denote the subspace of $\mathcal{L}(^nE; F)$ (resp. $\mathcal{L}(^nE)$) consisting of those mappings that are weakly continuous on bounded sets. We use $\mathcal{P}(^nE; F)$ to denote the space of all continuous n-homogeneous polynomials from E into F; that is, $P \in \mathcal{P}(^nE; F)$ if $P(x) = u(x, \dots, x)$ for some symmetric $u \in \mathcal{L}(^nE; F)$. We let $\mathcal{P}_w(^nE; F)$ denote the subspace of $\mathcal{P}(^nE; F)$ consisting of those polynomials that are weakly continuous on bounded sets. A polynomial $P \in \mathcal{P}(^nE; F)$ is said to be *of finite type* if there exist finite subsets $\{\phi_i\}_{i=1}^l$ in E' and $\{y_i\}_{i=1}^l$ in F such that $P(x) = \sum_{i=1}^l \phi_i^n(x) y_i$ for all $x \in E$. An n-homogeneous polynomial is said to be *approximable* if it can be uniformly approximated on bounded sets by polynomials of finite type. We denote by $\mathcal{P}_A(^nE; F)$ the space of all approximable polynomials. We consider all these spaces endowed with the topology of uniform convergence on bounded sets of E or E^n.

Given $u \in \mathcal{L}(^nE; F)$, for each $j = 1, \dots, n$ let $T^j \colon x \in E \mapsto T^j(x) \in \mathcal{L}(^{n-1}E; F)$ be defined by $T^j(x)(z_1, \dots, z_{n-1}) = u(z_1, \dots, z_{j-1}, x, z_j, \dots, z_{n-1})$.

The following result is essentially in [20] for locally convex spaces and generalizes Banach space results in [3, Thm. 2.9] (see also [17, Sec. 2.1] and [30, Cor. 3]). A similar result for symmetric n-linear mappings also holds.

PROPOSITION 6. *Let E and F be locally convex spaces, and let u be in $\mathcal{L}(^nE; F)$. The following statements are equivalent*:

(a) *u is weakly continuous on bounded sets*;
(b) *u is weakly uniformly continuous on bounded sets;*
(c) *T^j is weakly continuous on bounded sets for all $j = 1, \dots, n$*;
(d) *T^j maps bounded sets into precompact sets for all $j = 1, \dots, n$.*

Proof. The equivalence between (a) and (b) has been proved in [20, Cor. 1.7].

To prove that (b) implies (c), let $(x_d)_{d\in D}$ be a bounded net that converges weakly to $x \in E$. Suppose that $(T^j(x_d - x))_{d\in D}$ does not converge to zero for some j. Then there exists a bounded subset B of E, a continuous seminorm p of F, a cofinal subset D_0 of D, and an $\varepsilon_0 > 0$ such that

$$\sup_{y_i \in B} p(T^j(x_d - x)(y_1, y_2, \dots, y_{j-1}, y_{j+1}, \dots, y_n)) \geq \varepsilon_0 \quad \text{for all } d \in D_0.$$

Hence, for each $d \in D_0$ and each $i \neq j$, there exist $y_d^i \in B$ such that

$$p\big(T^j(x_d - x)(y_d^1, \dots, y_d^{j-1}, y_d^{j+1}, \dots, y_d^n)\big) \geq \varepsilon_0/2. \tag{1}$$

Since $(y_d^i)_{d\in D_0}$ is bounded, by taking subnets if necessary we can assume without loss of generality that $(y_d^i)_{d\in D_0}$ is weakly Cauchy for all $i \neq j$. By [20, Thm. 3],

$$p\big(u(y_d^1, \dots, y_d^{j-1}, x_d - x, y_d^{j+1}, \dots, y_d^n)\big) = p\big(T^j(x_d - x)(y_d^1, \dots, y_d^{j-1}, y_d^{j+1}, \dots, y_d^n)\big)$$

tends to zero as $d \to \infty$, and this contradicts (1).

Statement (c) implies (d) because every bounded set in E is weakly precompact and its image by T^j, which is weakly uniformly continuous on bounded sets [20, Cor. 7], is precompact. Statement (d) implies (c) by arguments in [17, Lemma 2.3].

To prove that (c) implies (a), let $(y_d^1)_{d\in D}, \dots, (y_d^n)_{d\in D}$ be n bounded weakly Cauchy nets in E such that the $(y_d^j)_{d\in D}$ converge weakly to zero for some j. Let B be a bounded subset of E with $(y_d^i)_{d\in D} \subset B$ for all i.

Since T^j is weakly continuous on bounded sets, given $\varepsilon > 0$ and a continuous seminorm p of F, there exists a $d_0 \in D$ such that

$$\sup_{y_i \in B} p(T^j(y_d^j)(y_1, y_2, \dots, y_{j-1}, y_{j+1}, \dots, y_n)) \le \varepsilon \quad \text{for all } d \ge d_0.$$

Hence

$$p(u(y_d^1, \dots, y_d^n)) \le \sup_{z_i \in B, i \neq j} p(u(z_1, \dots, z_{j-1}, y_d^j, z_{j+1}, \dots, z_n)) =: \|T^j(y_d^j)\|_{B^{n-1},p} < \varepsilon.$$

By [20, Thm. 3], this proves that u is weakly continuous on bounded sets. □

The preceding result can be combined with an earlier result due to Aron and Prolla [4] for Banach spaces to obtain the following proposition (see also [17, Prop. 2.6] and [30, Cor. 4]).

PROPOSITION 7. *Let E and F be locally convex spaces and let $P \in \mathcal{P}(^nE; F)$. The following statements are equivalent*:

(a) *P is weakly continuous on bounded sets*;
(b) *P is weakly uniformly continuous on bounded sets*;
(c) *for each k $(0 \le k \le n)$, $(\hat{d}^k)P/k!$ is weakly continuous on bounded sets*;
(d) *for each k $(0 \le k \le n)$, $(\hat{d}^k)P/k!$ maps bounded sets into precompact sets*;
(e) *$(\hat{d}^{n-1})P/(n-1)!$ maps bounded sets into precompact sets.*

4. Local Asplundness for Spaces of Homogeneous Polynomials

For X and Y locally convex spaces, $\mathcal{K}^p(X; Y)$ denotes the space of continuous operators from X into Y that map bounded sets into precompact sets endowed with the topology of uniform convergence on bounded sets.

THEOREM 8. *Let E and F be locally convex spaces with F quasi-complete. If E_b' and F are locally Asplund, then $\mathcal{L}_w(^nE; F)$ is locally Asplund for all $n \in \mathbf{N}$.*

Proof. We proceed by induction on $n \in \mathbf{N}$. By Proposition 6 and Theorem 3, the result is true for $n = 1$.

Assume that $\mathcal{L}_w({}^{n-1}E; F)$ is locally Asplund. Consider the mapping $u \in \mathcal{L}_w({}^nE; F) \mapsto T^n \in \mathcal{L}(E; \mathcal{L}_w({}^{n-1}E; F))$ defined by $T^n(x)(x_1, \ldots, x_{n-1}) = u(x_1, \ldots, x_{n-1}, x)$. Since

$$\sup_{x \in B_n} \sup_{x_i \in B_i, 1 \le i \le n-1} p(T^n(x)(x_1, \ldots, x_{n-1})) = \sup_{x_i \in B_i, 1 \le i \le n} p(u(x_1, \ldots, x_n))$$

for every continuous seminorm p on F, this mapping is continuous and open. Furthermore, Proposition 6 implies that $T^n \in \mathcal{K}^p(E; \mathcal{L}_w({}^{n-1}E; F))$. We may therefore identify $\mathcal{L}_w({}^nE; F)$ with a subspace of $\mathcal{K}^p(E; \mathcal{L}_w({}^{n-1}E; F))$. We can identify $\mathcal{K}^p(E; \mathcal{L}_w({}^{n-1}E; F))$ with a subspace of $E'_b \varepsilon \overline{\mathcal{L}_w({}^{n-1}E; F)}$, where $\overline{\mathcal{L}_w({}^{n-1}E; F)}$ is the completion of $\mathcal{L}_w({}^{n-1}E; F)$. Since a locally convex space is locally Asplund if and only if its completion is locally Asplund, Theorem 3 shows that $E'_b \varepsilon \overline{\mathcal{L}_w({}^{n-1}E; F)}$ is locally Asplund. Since (by [12]) the class of locally convex spaces that are locally Asplund is stable under the formation of subspaces, we conclude that $\mathcal{L}_w({}^nE; F)$ is locally Asplund. □

Corollary 9. *Let E and F be locally convex spaces. If E'_b and F are locally Asplund, then $\mathcal{P}_w({}^nE; F)$ is locally Asplund for all $n \in \mathbf{N}$.*

Corollary 9 generalizes [2, Thm. 5(a)] and [29, Cor. 1.1] to locally convex spaces (note that $\mathcal{P}_{w^*}({}^nX'')$ is isometrically isomorphic to $\mathcal{P}_w({}^nX)$).

Let E and F be locally convex spaces. An n-homogeneous polynomial P from E into F is said to be *nuclear* if there exist $(\lambda_i)_{i=1}^{\infty} \in \ell_1$, $\{\phi_i\}_{i=1}^{\infty}$ equicontinuous in E', and $\{y_i\}_{i=1}^{\infty}$ bounded in $\hat{F}$ such that

$$P(x) = \sum_{i=1}^{\infty} \lambda_i \phi_i^n(x) y_i$$

for all $x \in E$. The space of n-homogeneous nuclear polynomials from E into F is denoted by $\mathcal{P}_N({}^nE; F)$.

Given a locally convex space E and a Banach space F, let $\mathcal{P}_a({}^nE; F)$ denote the space of all (algebraic) polynomials from E into F. A locally convex space E is *polynomially bornological* if, for every locally convex space F, every $P \in \mathcal{P}_a({}^nE, F)$ that is bounded on compact subsets is continuous.

Proposition 10. *Let E be a polynomially bornological locally convex space such that E'_b is locally Asplund and has the approximation property, and let F be a Banach space. If either $\mathcal{P}_N({}^nE'_b)$ or F'_b has the approximation property, then $(\mathcal{P}_w({}^nE; F), \tau_b)' = \mathcal{P}_N({}^nE'_b; F'_b)$.*

Proof. By modifying [9, Thm. 3 and Rem. (1)], we see that $\mathcal{P}_w({}^nE; F)$ is isomorphic to $\mathcal{P}_w({}^nE)\varepsilon F$. Since E'_b is locally Asplund, Corollary 9 implies that $\mathcal{P}_w({}^nE)$ is locally Asplund and, by [7, Prop. 7 and Thm. 3], $\mathcal{P}_w({}^nE)' = \mathcal{P}_N({}^nE'_b)$. Applying [12, Thm. 5] and the fact that $\mathcal{P}_N({}^nE'_b)$ or F'_b has the approximation property, we conclude that each element of $\mathcal{P}_w({}^nE; F)'$ has a representation of the form

$\sum_{k=1}^{\infty} \lambda_i \phi_k^n(\cdot) y_i$, where $(\phi_i)_{i=1}^{\infty}$ is equicontinuous in $(E_b')'$, $(y_i)_{i=1}^{\infty}$ is bounded in F_b', and $(\lambda_i)_{i=1}^{\infty} \in \ell_1$. This is the space $\mathcal{P}_N({}^nE_b'; F_b')$. $\square$

Let E be a Banach space. A complex-valued n-homogeneous polynomial on E is said to be *integral* if there is a finite Borel regular measure μ on $(B_E', \sigma(E', E))$ such that

$$P(x) = \int_{B_{E'}} \phi(x)^n \, d\mu(\phi)$$

for all $x \in E$.

The Banach space ℓ_2 is (locally) Asplund. Since $\mathcal{P}_N({}^2\ell_2) = \mathcal{P}_I({}^2\ell_2) = \ell_2 \hat{\otimes}_{s,\pi} \ell_2$ contains a copy of ℓ_1, it follows that $\mathcal{P}_N({}^2\ell_2) = \mathcal{P}_I({}^2\ell_2)$ is not locally Asplund. This shows that Corollary 9 does not extend to spaces of nuclear or integral homogeneous polynomials.

Alencar [1] and Valdivia [30] show that, if E is a Banach space such that E'' has the Radon–Nikodým property and the approximation property, then $(\mathcal{P}_w({}^nE), \|\cdot\|)''$ and $(\mathcal{P}({}^nE''), \|\cdot\|)$ are isomorphic. This was extended to vector-valued holomorphic functions by Jaramillo and Moraes in [22] (see also [19]). The first author [7, Prop. 9], in extending Alencar's result to Fréchet spaces, noted that the Radon–Nikodým property on E'' needed to be replaced by local Asplundness on E_b' and that strong duals needed to be replaced by inductive duals. We have the following extension to vector-valued polynomials on Fréchet spaces.

THEOREM 11. *Let E be a Fréchet space and F a Banach space such that E_b' is locally Asplund and $(E_b')_b'$ has the approximation property. Then it follows that $((\mathcal{P}_w({}^nE; F), \tau_b)_i')_i'$ is isomorphic to $(\mathcal{P}({}^n((E_b')_b'; (F_b')_b'), \tau_\omega)$ for each integer n.*

Proof. Since E_b' is locally Asplund, Corollary 9 implies that $\mathcal{P}_w({}^nE)$ is locally Asplund for each integer n. By [13, Prop. 4.2(3)], E_b' has the approximation property. It now follows from [9, Rem. (2)], [13, Ex. 3.2], and [7, Prop. 2 and Thm. 3] that $(\mathcal{P}_w({}^nE; F), \tau_b)' = ((\mathcal{P}_w({}^nE), \tau_b)\varepsilon F)' = ((\mathcal{P}_A({}^nE), \tau_b) \hat{\otimes}_\varepsilon F)'$ is (algebraically) isomorphic to $\mathcal{P}_I({}^nE_b') \hat{\otimes}_\pi F_b' = \mathcal{P}_N({}^nE_b') \hat{\otimes}_\pi F_b'$. By the definition of the inductive dual, this implies that $(\mathcal{P}_w({}^nE; F), \tau_b)_i' = \mathcal{P}_N({}^nE_b') \hat{\otimes}_\pi F_b'$. Since $(E_b')_b'$ has the approximation property, [14, Thm. 1.4] implies that this space is isomorphic to $\left(\widehat{\bigotimes}_{s,n,\pi}(E_b')_b'\right) \hat{\otimes}_\pi F_b'$. By [8, Thm. 3], the inductive dual of this space is $(\mathcal{P}({}^n(E_b')_b'; (F_b')_b'), \tau_\omega)$. $\square$

5. Locally Asplund Spaces of Weakly Uniformly Continuous Holomorphic Functions

Let U be an open subset of a locally convex space E, and let F be a Banach space. We denote by $\mathcal{H}(U; F)$ the space of all holomorphic functions from U into F and by $\mathcal{H}_{wu}(U, F)$ the subspace of $\mathcal{H}(U; F)$ of all functions that are weakly uniformly continuous on bounded sets. We use $\mathcal{H}(U)$ and $\mathcal{H}_{wu}(U)$ for $\mathcal{H}(U; \mathbf{C})$ and $\mathcal{H}_{wu}(U; \mathbf{C})$. The subset of $\mathcal{H}(E, F)$ of holomorphic functions that map bounded sets to bounded sets is denoted by $\mathcal{H}_b(E; F)$. Again, we use τ_b to denote the topology on $\mathcal{H}_{wu}(U; F)$ (resp. $\mathcal{H}_b(E; F)$) of uniform convergence on bounded sets of U (resp. E).

In [21, I, pp. 95, 108], Grothendieck (see also [11; 12]) gives a representation theorem for continuous linear functionals on ε-products of locally convex spaces. In order to use this result to study local Asplundness of spaces of holomorphic functions, we require a more quantitative analysis of the neighbourhoods involved. We therefore reproduce Defant's proof [12, Thm. 5], keeping track of how the neighbourhoods involved are related to each other.

THEOREM 12 [12; 21]. *Let E and F be locally convex spaces with E locally Asplund. If $\psi \in (E\varepsilon F)'$ then there exist absolutely convex neighbourhoods of 0, V in E and W in F, and*

$$z = \sum_{i=1}^{\infty} \lambda_i x_i' \otimes y_i' \in \llbracket V^\circ \rrbracket \hat{\otimes}_\pi \llbracket W^\circ \rrbracket,$$

so that

$$\langle T, \psi \rangle = \sum_{i=1}^{\infty} \lambda_i \langle T(x_i'), y_i' \rangle$$

for all $T \in E\varepsilon F$. Furthermore, if $\psi \in (N(U^\circ, W))^\circ$ then V may be chosen so that the inclusion

$$i\colon (U^\circ, \sigma(E', E)) \hookrightarrow \llbracket V^\circ \rrbracket$$

is ν-measurable for some positive Radon measure ν on $(U^\circ, \sigma(E', E))$.

Proof. Fix $\varepsilon > 0$. By scaling U and W we may assume that $\|\psi\|_{(N(U^\circ, W))^\circ} = 1$. Let $H = (U^\circ \times W^\circ, \sigma(E', E) \times \sigma(F', F))$ and let $S\colon \mathcal{M}(H) \to (E\varepsilon F)'$ be defined by

$$\mu \rightsquigarrow \left(T \rightsquigarrow \int_H \langle T(x'), y' \rangle \, d\mu(x', y') \right).$$

It follows as in [12, Thm. 5] that $(N(U^\circ, W))^\circ \subset A := S(\mu \in \mathcal{M}(H) : \mu(H) = 1)$ and there exists a $\mu \in \mathcal{M}(H)$ such that

$$\langle T, \psi \rangle = \int_H \langle T(x'), y' \rangle \, d\mu(x', y')$$

for all $T \in E\varepsilon F$. We have $\|\psi\|_{(N(U^\circ, W))^\circ} \leq \|\mu\|_H$. Since $\|\psi\|_{(N(U^\circ, W))^\circ} = 1$, this implies

$$\|\psi\|_{(N(U^\circ, W))^\circ} = \|\mu\|_H = 1.$$

Let $p\colon H \to (U^\circ, \sigma(E', E))$ be defined by $p(x', y') \to x'$, and let $\nu = p(\mu)$. Because E is locally Asplund, we can choose $V \subseteq U$ so that

$$i\colon (U^\circ, \sigma(E', E)) \hookrightarrow \llbracket V^\circ \rrbracket$$

is ν-measurable. Let $f = i \circ p$. Since $V \subseteq U$, we have $f(x', y') \subseteq V^\circ$ for all $(x', y') \in H$ and hence $\|f\| \leq 1$. Since $L^1(\mu, \llbracket V^\circ \rrbracket)$ is isometrically isomorphic to $L^1(\mu) \hat{\otimes}_\pi \llbracket V^\circ \rrbracket$, we can write f in the form

$$f = \sum_{i=1}^{\infty} \lambda_i \phi_i \otimes x_i$$

with $\|\phi_i\|_{L^1(\mu)} \le 1$ and $\|x_i\|_{V^\circ} \le 1$ for all i and where $\sum_{i=1}^{\infty}|\lambda_i| \le 1+\varepsilon$. Define $R\colon L^1(\mu) \to \llbracket W^\circ \rrbracket$ by letting

$$\phi \rightsquigarrow \left(y \rightsquigarrow \int_H \phi(x', y')\langle y, y'\rangle \, d\mu(x', y') \right).$$

Then we have

$$\begin{aligned}
\langle T, \psi\rangle &= \int_H \langle x', T^t(y')\rangle \, d\mu(x', y') \\
&= \int_H \langle f(x', y'), T^t(y')\rangle \, d\mu(x', y') \\
&= \sum_{i=1}^{\infty} \lambda_i \int_H \phi_i(x', y')\langle x_i, T^t(y')\rangle \, d\mu(x', y') \\
&= \sum_{i=1}^{\infty} \lambda_i \langle T(x_i) R(\phi_i)\rangle.
\end{aligned}$$

Since $\|R\| \le 1$, this implies that

$$z = \sum_{i=1}^{\infty} \lambda_i x_i \otimes R(\phi_i),$$

where $\|x_i\|_{V^\circ} \le 1$ and $\|R(\phi_i)\|_{W^\circ} \le 1$ for all i and where $\sum_{i=1}^{\infty}|\lambda_i| \le 1+\varepsilon$. □

THEOREM 13. *Let E be a holomorphically bornological locally convex space. Then $\mathcal{H}_{wu}(E)$ is locally Asplund if and only if E'_b is locally Asplund.*

Proof. Clearly E'_b is locally Asplund whenever $\mathcal{H}_{wu}(E)$ is locally Asplund.

Now suppose that E'_b is locally Asplund. Then the space $(\mathcal{P}_w({}^nE), \tau_b)$ is a complemented subspace of $(\mathcal{L}_w({}^nE), \tau_b)$, which in turn is a subspace of $E'_b\varepsilon(\mathcal{L}_w({}^{n-1}E), \tau_b)$. Furthermore, if B is bounded in E then it follows that $\{P \in \mathcal{P}_w({}^2E) : \|P\|_B \le 1\}$ is identified with the intersection of $\mathcal{P}_w({}^2E)$ and $N(B^{\circ\circ}, B^\circ)$ in $E'_b\varepsilon E'_b$. Let $\tilde{B}^2 = N(B^{\circ\circ}, B^\circ)$, and inductively define $\tilde{B}^n$ in $E'_b\varepsilon(E'_b\varepsilon E'_b\varepsilon \dots \varepsilon E'_b)$ by

$$\tilde{B}^n = N(B^{\circ\circ}, (\tilde{B}^{n-1})^\circ).$$

Then we can identify $\{P \in P_w({}^nE) : \|P\|_B \le 1\}$ with the intersection of $\mathcal{P}_n({}^nE)$ and $\tilde{B}^n$ in $E'_b\varepsilon(E'_b\varepsilon E'_b\varepsilon \dots \varepsilon E'_b)$.

Let K be a compact Hausdorff set and suppose $\psi \in \mathcal{C}(K; \mathcal{H}_{wu}(E))'$. By [23, Cor. 16.6.30] and [9, Thm. 3],

$$\mathcal{C}(K; \mathcal{H}_{wu}(E)) \cong \mathcal{C}(K)\varepsilon\mathcal{H}_{wu}(E) \cong \mathcal{H}_{wu}(E)\varepsilon\mathcal{C}(K) \cong \mathcal{H}_{wu}(E; \mathcal{C}(K)).$$

Similarly,

$$\mathcal{C}(K; \mathcal{P}_w({}^nE)) \cong \mathcal{P}_w({}^nE; \mathcal{C}(K)).$$

We denote this isomorphism and its inverse by $f \mapsto f^s$, where $f^s(x)(k) = f(k)(x)$ and $f^s(k)(x) = f(x)(k)$ for $x \in E$ and $k \in K$.

By using [17, (3.42)], we see that $\{\mathcal{P}_w({}^nE;\mathcal{C}(K))\}_n$ is an $\mathcal{S}$-absolute decomposition for $\mathcal{H}_{wu}(E;\mathcal{C}(K))$. Thus, for each $f \in \mathcal{C}(K;\mathcal{H}_{wu}(E))$ we have that

$$\langle f, \psi \rangle = \sum_{n=0}^{\infty} \left\langle \left(\frac{\hat{d}^n f^s(0)}{n!} \right)^s, \psi \right\rangle.$$

Since ψ is continuous and linear on $\mathcal{C}(K;\mathcal{H}_{wu}(E))$ and since the decomposition is $\mathcal{S}$-absolute, there exists a bounded set B in E and $C > 0$ such that

$$|\langle f, \psi \rangle| \leq C \sum_{n=0}^{\infty} \left\| \left(\frac{\hat{d}^n f^s(0)}{n!} \right)^s \right\|_B$$

for all $f \in \mathcal{C}(K;\mathcal{H}_{wu}(E))$. For $n \in \mathbf{N}$, let $\psi_n = \psi|_{\mathcal{P}_w({}^nE;\mathcal{C}(K))}$. Let A be bounded in E and let $B \subset A$ be such that

$$(B^{\circ\circ}, \sigma(E'', E')) \hookrightarrow \| A^{\circ\circ} \|$$

is measurable for the measure associated with ψ on $(B^{\circ\circ}, \sigma(E'', E'))$ (see [12, Thm. 5]). Let $\varepsilon > 0$. It follows from the above and induction that we can choose our representations in Theorem 12 so that, for every $n \in \mathbf{N}$, there exist a sequence of measures $\{\mu_{i,n}\}_{i=1}^{\infty}$ on K with $\|\mu_{i,n}\|_K \leq 1$ for all i, a sequence $\{y'_{i,n}\}_{i=1}^{\infty} \subset (E'_b \varepsilon E'_b \varepsilon \ldots \varepsilon E'_b)'$ with $\|y'_{i,n}\|_{(\tilde{A}^n)^\circ} \leq C$ for all i, and a sequence $\{\lambda_{i,n}\}_{i=1}^{\infty}$ so that $\sum_{i=1}^{\infty} |\lambda_{i,n}| \leq 1 + \varepsilon$ which together satisfy

$$\left\langle \left(\frac{\hat{d}^n f^s(0)}{n!} \right)^s, \psi \right\rangle = \sum_{i=1}^{\infty} \lambda_{i,n} \int_K y'_{i,n} \circ \left(\frac{\hat{d}^n f^s(0)}{n!} \right)^s d\mu_{i,n}$$

for every $f \in \mathcal{C}(K;\mathcal{H}_{wu}(E))$.

Then, for every $f \in \mathcal{C}(K;\mathcal{H}_{wu}(E))$, we have

$$\begin{aligned} \langle \psi, f \rangle &= \sum_{n=0}^{\infty} \left\langle \left(\frac{\hat{d}^n f^s(0)}{n!} \right)^s, \psi \right\rangle \\ &= \sum_{n=0}^{\infty} \sum_{i=1}^{\infty} \lambda_{i,n} \int_K y'_{i,n} \circ (\pi_n(f))\, d\mu_{i,n}, \end{aligned}$$

where π_n is the (continuous) projection of $\mathcal{H}_{wu}(E)$ onto $\mathcal{P}_w({}^nE)$, $g \to \hat{d}^n g(0)/n!$. Thus

$$\langle \psi, f \rangle = \sum_{n=0}^{\infty} \sum_{i=1}^{\infty} \frac{\lambda_{i,n}}{n^2} \int_K (n^2 y'_{i,n} \circ \pi_n)(f)\, d\mu_{i,n}$$

with $\|\mu_{i,n}\|_K \leq 1$ for all i and all n. Therefore,

$$\sum_{n=0}^{\infty} \sum_{i=1}^{\infty} \frac{\lambda_{i,n}}{n^2} = \sum_{n=0}^{\infty} \frac{1}{n^2} \sum_{i=1}^{\infty} |\lambda_{i,n}| \leq \sum_{n=0}^{\infty} \frac{1}{n^2} (1+\varepsilon) < \infty$$

and

$$\|n^2 y'_{i,n} \circ \pi_n(g)\|_A = \left\| y'_{i,n} \left(n^2 \frac{\hat{d}^n g(0)}{n!} \right) \right\|_A < \infty$$

for all g in $\mathcal{H}_{wu}(E)$ satisfying

$$\sum_{n=0}^{\infty} n^2 \left\| \frac{\hat{d}^n g(0)}{n!} \right\|_A \leq 1.$$

Since the decomposition is $\mathcal{S}$-absolute, it follows that this is a neighbourhood of 0 in $\mathcal{H}_{wu}(E)$ and hence $\{n^2 y'_{i,n} \circ \pi_n\}_{i,n}$ is equicontinuous. By [12, Cor. 5], this implies that $\mathcal{H}_{wu}(E)$ is locally Asplund. □

This result is easily extended to the vector-valued case by using ε-products, and it can be used to give an alternative proof of [22, Cor. 2.3].

A locally convex space E is a *k-space* if continuity on compact subsets of E implies continuity on E. If E is a k-space then we can identify $\{P \in \mathcal{P}(^nE) : \|P\|_K \leq 1\}$, K compact in E, with the intersection of $\mathcal{P}(^nE)$ and the set $\tilde{K}^n := N(K^{\circ\circ}, (\tilde{K}^{n-1})^\circ)$ $(\tilde{K}^2 = N(K^{\circ\circ}, K^\circ))$ in $E'_c \varepsilon E'_c \varepsilon \ldots \varepsilon E'_c$. The preceding proof is easily modified to yield the next theorem.

THEOREM 14. *Let E be a locally Asplund locally convex k-space E. Then $(\mathcal{H}(E), \tau_o)$ is locally Asplund if and only if E'_c is locally Asplund.*

Motivated by Theorem 12, Defant [12] introduced the following notation. Given locally convex spaces E and F, we define $\sum(E \hat{\otimes}_\pi F)$ to be the dense subspace of $E \hat{\otimes}_\pi F$ given by

$$\Big\{ z \in E \hat{\otimes}_\pi F : z = \sum_{i=1}^{\infty} \lambda_i x_i \otimes y_i : (\lambda_i)_{i=1}^{\infty} \in \ell_1, \\ \{x_i\}_{i=1}^{\infty} \text{ and } \{y_i\}_{i=1}^{\infty} \text{ are equicontinuous} \Big\}.$$

For U an open subset of a locally convex space E and for F a Banach space, we denote by τ_o the topology on $\mathcal{H}(U; F)$ of uniform convergence on compact subsets of U. A seminorm p on $\mathcal{H}(U; F)$ is τ_δ-continuous if, for each increasing countable open cover $\{V_n\}_{n=1}^{\infty}$ of U, there is a positive integer n_o and a $C > 0$ such that

$$p(f) \leq C\|f\|_{V_{n_o}} \quad \text{for all } f \in \mathcal{H}(U; F).$$

The τ_δ topology on $\mathcal{H}(U; F)$ is the topology generated by all τ_δ-continuous seminorms.

Let U be an open subset of a locally convex space E. Mujica and Nachbin [24] show that there is a complete locally convex space $G(U)$ and a $\delta_U \in \mathcal{H}(U, G(U))$ with the following universal property: Given any complete locally convex space F and any $f \in \mathcal{H}(U; F)$, there exists a unique continuous linear map $T_f : G(U) \to F$ such that $T_f \circ \delta_U(x) = f(x)$ for all x in U. Furthermore, by [24, Prop. 2.3], if F is a Banach space then

$$(\mathcal{H}(U, F), \tau_\delta) = \underset{\alpha \in \text{c.s.}(G(U))}{\overset{\text{ind}}{\longrightarrow}} \mathcal{L}(G(U)_\alpha; F).$$

We finish with the following application of the concept of local Asplundness.

THEOREM 15. *Let U be a balanced open subset of a Fréchet space E, and let F be a Banach space such that F'_b has the approximation property. Then $((\mathcal{H}(U; F), \tau_o)'_i)'_i = ((\mathcal{H}(U; F), \tau_o)'_b)'_i$ is isomorphic to $(\mathcal{H}(U; (F'_b)'_b), \tau_\delta)$.*

Proof. By [16, Thm. 1.8], $(\mathcal{H}(U), \tau_o)$ is a Schwartz space and hence is locally Asplund. Therefore, [13, Thm. 3] shows that the dual of $(\mathcal{H}(U; F), \tau_o) = (\mathcal{H}(U), \tau_o)\varepsilon F$ is (algebraically) isomorphic to $\sum((\mathcal{H}(U), \tau_o)'_b \hat{\otimes}_\pi F'_b)$. Applying [23, Cor. 15.5.4], [16, Thm. 1.8], and [5], we obtain

$$(\mathcal{H}(U; F), \tau_o)'_i = \sum((\mathcal{H}(U), \tau_o)'_i \hat{\otimes}_\pi F'_b) = \sum((\mathcal{H}(U), \tau_o)'_b \hat{\otimes}_\pi F'_b).$$

By [13, 1.1], the strong dual of $(\mathcal{H}(U; F), \tau_o)$ also equals $\sum((\mathcal{H}(U), \tau_o)'_b \hat{\otimes}_\pi F'_b)$ with the topology induced by $(\mathcal{H}(U), \tau_o)'_b \hat{\otimes}_\pi F'_b$. Since the completion of $(\mathcal{H}(U), \tau_o)'_b$ is $G(U)$ (see [6, Lemma 8]), the completion of $(\mathcal{H}(U; F), \tau_o)'_i$ is $G(U) \hat{\otimes}_\pi F'_b$. Hence, by [24, Prop. 2.3], $((\mathcal{H}(U; F), \tau_o)'_i)'_i = ((\mathcal{H}(U; F), \tau_o)'_b)'_i$ is isomorphic to $(\mathcal{H}(U; (F'_b)'_b), \tau_\delta)$. $\square$

For a related scalar-valued result, see [18] and [6]; see also [8].

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