Free Duals and Regular Sequences

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Introduction

The first part of this paper is devoted to a study of the following question. Let A be a local Noetherian ring, let I be an ideal of A of finite projective dimension ($\mathrm{pd}_A(I)<\infty$), and let H_1 be the first Koszul homology module associated to a system of generators of I. If the dual module $(H_1)^*=\mathrm{Hom}_{A/I}(H_1, A/I)$ is A/I-free, then is I generated by a regular sequence? We obtain an affirmative answer in some cases, for example, when I/I^2 is torsion-free as an A/I-module.

In the second part we consider an analogous problem for the conormal module. Let K be a field, let R be a smooth K-algebra of essentially finite type, and let $B=R/J$. If $(J/J^2)^*=\mathrm{Hom}_B(J/J^2, B)$ is a projective B-module, what consequences are brought on B? A particular case of a conjecture of Vasconcelos [12] asserts that B must be a locally complete intersection. Under some additional hypotheses, we construct an exact sequence

$$0 \to H_1(K,B,B) \to J/J^2 \to (J/J^2)^{**} \to \Omega_{B|K} \to (\Omega_{B|K})^{**} \to \mathrm{Ext}^2_B(H^1(K,B,B),B) \to 0,$$

and we use it to obtain some evidence for the conjecture.

Finally we give a condition, in terms of Hochschild cohomology, for a locally complete intersection algebra to be regular.

We will use some properties of André–Quillen (co)homology $H(A,B,-)$ (see [1], [5], [8]).

1. On the Freeness of the Dual of the First Koszul Homology

Let A be a local Noetherian ring, let I be an ideal of A, and let H_1 be the first Koszul homology module associated to a system of n generators of I. Let $\alpha: H_1 \to (H_1)^{**}$ and $\beta: I/I^2 \to (I/I^2)^{**}$ be the canonical homomorphisms into the bidual module. We start by obtaining an exact sequence which relates the homomorphisms α and β.

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PROPOSITION 1.1. *Assume that* $\operatorname{Hom}_{A/I}(H^2(A,A/I,A/I),A/I)=0$ *and* $\operatorname{Ext}^1_{A/I}(H^2(A,A/I,A/I),A/I)=0$. *Then there exists an exact sequence*

$$0 \to H_2(A,A/I,A/I) \to H_1 \xrightarrow{\alpha} (H_1)^{**} \to I/I^2 \xrightarrow{\beta} (I/I^2)^{**}.$$

Moreover, we have

(1) $\operatorname{Coker}\beta \simeq \operatorname{Ext}^1_{A/I}((H_1)^*,A/I)$ *if* $\operatorname{Ext}^2_{A/I}(H^2(A,A/I,A/I),A/I)=0$, *and*

(2) $\operatorname{Coker}\beta \simeq \operatorname{Ext}^2_{A/I}(H^2(A,A/I,A/I),A/I)$ *if* $\operatorname{Ext}^i_{A/I}((H_1)^*,A/I)=0$, $i=1,2$.

Proof. It is known [5] that there exist exact sequences of A/I-modules

$$0 \to H_2(A,A/I,A/I) \to H_1 \to F/IF \to I/I^2 \to 0 \quad \text{and}$$
$$0 \to (I/I^2)^* \to (F/IF)^* \to (H_1)^* \to H^2(A,A/I,A/I) \to 0,$$

where F is a free A-module of rank n. Let

$$T=\operatorname{Ker}((H_1)^* \to H^2(A,A/I,A/I)).$$

The exact sequence

$$0 \to T \to (H_1)^* \to H^2(A,A/I,A/I) \to 0$$

yields an exact sequence

$$\begin{aligned} 0 &\to H^2(A,A/I,A/I)^* \to (H_1)^{**} \to T^* \to \operatorname{Ext}^1_{A/I}(H^2(A,A/I,A/I),A/I) \\ &\to \operatorname{Ext}^1_{A/I}((H_1)^*,A/I) \to \operatorname{Ext}^1_{A/I}(T,A/I) \\ &\to \operatorname{Ext}^2_{A/I}(H^2(A,A/I,A/I),A/I) \to \operatorname{Ext}^2_{A/I}((H_1)^*,A/I). \end{aligned}$$

We obtain $(H_1)^{**} \simeq T^*$. Moreover, $\operatorname{Ext}^1_{A/I}(T,A/I) \simeq \operatorname{Ext}^1_{A/I}((H_1)^*,A/I)$ with the hypothesis of (1), and $\operatorname{Ext}^1_{A/I}(T,A/I) \simeq \operatorname{Ext}^2_{A/I}(H^2(A,A/I,A/I),A/I)$ with the hypothesis of (2).

On the other hand, the exact sequence

$$0 \to (I/I^2)^* \to (F/IF)^* \to T \to 0$$

induces

$$0 \to T^* \to (F/IF)^{**} \to (I/I^2)^{**} \to \operatorname{Ext}^1_{A/I}(T,A/I) \to 0.$$

We thus obtain a commutative diagram of exact sequences

$$\begin{array}{ccccccccc} 0 \to H_2(A,A/I,A/I) \to & H_1 & \to & F/IF & \to & I/I^2 & \to 0 & & \\ & \alpha\downarrow & & \| & & \beta\downarrow & & & \\ 0 \to & (H_1)^{**} & \to & (F/IF)^{**} & \to & (I/I^2)^{**} & \to & \operatorname{Ext}^1_{A/I}(T,A/I) & \to 0 \end{array}$$

Application of the Ker–Coker lemma yields the result. □

The next lemma generalizes Lemma 2.3 of [12].

LEMMA 1.2. *Let A be a local Noetherian ring of depth at most* 1 *and let M be a finitely generated A-module. If* $\mathrm{pd}_A(M^*) < \infty$, *then the natural homomorphism $M \to M^{**}$ is surjective.*

Proof. Let $L \to F \to M \to 0$ be an exact sequence of A-modules such that L and F are free of finite type. Dualizing, we obtain an exact sequence

$$0 \to M^* \to F^* \to L^* \to D(M) \to 0,$$

from which we deduce $\mathrm{pd}_A(D(M)) < \infty$. Therefore $\mathrm{pd}_A(D(M)) \leq \mathrm{depth}(A) \leq 1$. Hence $\mathrm{Ext}_A^2(D(M), A) = 0$. The result follows [2] from the exact sequence

$$0 \to \mathrm{Ext}_A^1(D(M), A) \to M \to M^{**} \to \mathrm{Ext}_A^2(D(M), A) \to 0. \qquad \square$$

Let (A, m) be a local Noetherian ring, let I be an ideal of A with $\mathrm{pd}_A(I) < \infty$, and let H_1 be the first Koszul homology over a system of n generators of I. Assume that $(H_1)^*$ is A/I-free. (Observe that the freeness of $(H_1)^*$ is independent of the system of generators of I [3, pp. 30–31].)

LEMMA 1.3. *If* $n = \mu(I)$ (= minimum number of generators of I) *and* $H_1 \to (H_1)^{**}$ *is surjective, then I is generated by a regular sequence.*

Proof. Since $n = \mu(I)$, for the trace ideal of the A/I-module H_1 we have [12, p. 371] $\mathrm{Tr}_{A/I}(H_1) \subseteq m/I$. Therefore $(H_1)^* = 0$. Hence the exact sequence

$$0 \to H_2(A, A/I, A/I) \to H_1 \to F/IF \to I/I^2 \to 0$$

shows that I/I^2 is A/I-free; that is, I is generated by a regular sequence (see [11, Proposition]). $\square$

LEMMA 1.4. *In the above hypothesis, we have* $H^2(A, A/I, A/I)^* = 0$ *and* $\mathrm{Ext}_{A/I}^1(H^2(A, A/I, A/I), A/I) = 0$. *Moreover, the rank of* $(H_1)^*$ *is* $n - \mathrm{ht}(I)$ ($\mathrm{ht}(I)$ = height of I) *and I_p is generated by a regular sequence for every prime ideal p of A such that* $\mathrm{depth}(A/I)_p \leq 1$.

Proof. Let p be a prime ideal of A such that $\mathrm{depth}(A/I)_p \leq 1$. Let $H_1(I_p)$ denote the first Koszul homology associated to a minimal system of generators of I_p; $(H_1)_p$ is the homology associated to a system of n generators of I_p. Since $((H_1)^*)_p \simeq ((H_1)_p)^*$ is free, we obtain that $(H_1(I_p))^*$ is free. Application of Lemma 1.2 yields that $H_1(I_p) \to (H_1(I_p))^{**}$ is surjective. It follows from Lemma 1.3 that I_p is generated by a regular sequence. In particular, $H^2(A, A/I, A/I)_p \simeq H^2(A_p, A_p/I_p, A_p/I_p) = 0$ [1, Thm. 6.25]. Hence $\mathrm{grade}(H^2(A, A/I, A/I)) \geq 2$ and so $\mathrm{Ext}_{A/I}^i(H^2(A, A/I, A/I), A/I) = 0$, $i = 0, 1$ [7, p. 103, Proposition].

On the other hand, since I_p is generated by a regular sequence, $(H_1)_p$ is free of rank $n - \mathrm{ht}(I_p)$ [3, pp. 30–31]. Taking p such that $\mathrm{depth}(A/I)_p = 0$, we obtain that the rank of $(H_1)^*$ is $n - \mathrm{ht}(I)$. $\square$

Recall that a finitely generated module M over a Noetherian ring A is said to be torsion-free if $\mathrm{Ass}(M) \subseteq \mathrm{Ass}(A)$; that is, if each nonzero divisor on A is a nonzero divisor on M.

THEOREM 1.5. *Let A be a local Noetherian ring, let I be an ideal of A with $\mathrm{pd}_A(I) < \infty$, and let H_1 be the first Koszul homology over a system of generators of I. Assume that $(H_1)^*$ is A/I-free. Then the following conditions are equivalent:*

(1) *I is generated by a regular sequence;*
(2) *I/I^2 is torsion-free as an A/I-module;*
(3) *$\mathrm{Ext}_A^2(A/I, A/I)$ is torsion-free as an A/I-module;*
(4) *$H^2(A, A/I, A/I) = 0$.*

Proof. It is well known that condition (1) implies (2), (3), and (4). We shall prove that each one of conditions (2), (3), and (4) implies (1). We may assume that H_1 is associated to a minimal system of generators.

By Lemma 1.4 and Proposition 1.1, there is an exact sequence

$$0 \to H_2(A, A/I, A/I) \to H_1 \to (H_1)^{**} \to I/I^2 \\ \to (I/I^2)^{**} \to \mathrm{Ext}^2_{A/I}(H^2(A, A/I, A/I), A/I) \to 0.$$

Moreover, I_p is generated by a regular sequence if $\mathrm{depth}(A/I)_p \leq 1$ and $(H_1)^*$ has rank $\mu(I) - \mathrm{ht}(I)$. In particular, $(I/I^2)_p$ is free for $p \in \mathrm{Ass}(A/I)$. Hence, I/I^2 is torsion-free if and only if $I/I^2 \to (I/I^2)^{**}$ is injective.

Thus condition (2) implies that $H_1 \to (H_1)^{**}$ is surjective. From Lemma 1.3 we deduce that I is generated by a regular sequence.

Assume now that $H^2(A, A/I, A/I) = 0$. We have an exact sequence $0 \to (I/I^2)^* \to (F/IF)^* \to (H_1)^* \to 0$, where F is a free A-module of rank $\mu(I)$. Since $(H_1)^*$ is free of rank $\mu(I) - \mathrm{ht}(I)$, it follows that $(I/I^2)^*$ is free of rank $\mathrm{ht}(I)$. Moreover $I/I^2 \to (I/I^2)^{**}$ is surjective. Therefore I is generated by a regular sequence [11, Proposition].

Assume finally that $\mathrm{Ext}_A^2(A/I, A/I)$ is torsion-free. Since

$$H^2(A, A/I, A/I)_p = 0 \quad \text{for } p \in \mathrm{Ass}(A/I),$$

we deduce from the following lemma that $H^2(A, A/I, A/I) = 0$. □

LEMMA 1.6. *Let A be a ring, let I be an ideal of A, and let M be an A/I-module. Then there exists an exact and natural sequence of A/I-modules*

$$0 \to H^2(A, A/I, M) \to \mathrm{Ext}_A^2(A/I, M) \to \mathrm{Hom}_{A/I}(I/I^2 \wedge I/I^2, M).$$

Proof. Let F be a free A-module such that there exists an exact sequence of A-modules

$$0 \to U \to F \xrightarrow{j} I \to 0.$$

Let U_0 be the image of the homomorphism of A-modules $\phi: F \otimes_A F \to F$, $\phi(x \otimes y) = j(x)y - j(y)x$. We have a commutative diagram of exact sequences [5]:

$$\begin{array}{ccccccccc}
 & & & & & & 0 & & \\
 & & & & & & \downarrow & & \\
0 \to & \operatorname{Hom}_{A/I}(I/I^2, M) & \to & \operatorname{Hom}_{A/I}(F/IF, M) & \to & \operatorname{Hom}_{A/I}(U/U_0, M) & \to & H^2(A, A/I, M) & \to 0 \\
 & \| & & \| & & \downarrow & & & \\
0 \to & \operatorname{Hom}_{A/I}(I/I^2, M) & \to & \operatorname{Hom}_{A/I}(F/IF, M) & \to & \operatorname{Hom}_{A/I}(U/IU, M) & \to & \operatorname{Ext}^2_A(A/I, M) & \to 0 \\
 & & & & & \downarrow & & & \\
 & & & & & \operatorname{Hom}_{A/I}(U_0/IU, M). & & &
\end{array}$$

Therefore we have an exact sequence

$$0 \to H^2(A, A/I, M) \to \operatorname{Ext}^2_A(A/I, M) \to \operatorname{Hom}_{A/I}(U_0/IU, M).$$

It is sufficient to show that there exists an epimorphism

$$I/I^2 \wedge I/I^2 \to U_0/IU.$$

Consider the canonical exact sequence

$$U \otimes_A F \oplus F \otimes_A U \xrightarrow{\psi} F \otimes_A F \to I \otimes_A I \to 0.$$

Since $\operatorname{Im}(\phi\psi) \subseteq IU$, it follows that ϕ induces an epimorphism $I \otimes_A I \to U_0/IU$ which factors through $I/I^2 \wedge I/I^2$. □

2. Projective Normal Modules

Let K be a field. A K-algebra B of essentially finite type (e.f.t.) is a ring of fractions of a finitely generated K-algebra. If R is a K-algebra of e.f.t. and J is an ideal of R, then R/J is also of e.f.t. Moreover, if B is a K-algebra of e.f.t., then there exists a smooth K-algebra of e.f.t. and an ideal J of R such that $R/J \simeq B$.

Assume that $B = R/J$, where R is a smooth K-algebra of e.f.t. Consider the normal B-module $(J/J^2)^* = \operatorname{Hom}_B(J/J^2, B)$. Since R is a regular ring, a special case of a conjecture of Vasconcelos [12, p. 373] asserts that B is a locally complete intersection (l.c.i.) if $(J/J^2)^*$ is B-projective. In this section we shall obtain results that provide some evidence for this conjecture. We start by obtaining an analogue to Proposition 1.1.

If $B = R/J$, where R is a smooth K-algebra of e.f.t., then there are exact sequences [5]

$$0 \to H_1(K, B, B) \to J/J^2 \to \Omega_{R|K} \otimes_R B \to \Omega_{B|K} \to 0 \quad \text{and}$$

$$0 \to (\Omega_{B|K})^* \to (\Omega_{R|K} \otimes_R B)^* \to (J/J^2)^* \to H^1(K, B, B) \to 0.$$

$\Omega_{R|K} \otimes_R B$ is a finitely generated projective B-module, and therefore it is reflexive. Then the following proposition can be proved similarly to Proposition 1.1.

PROPOSITION 2.1. *Let K be a field, let R be a smooth K-algebra of e.f.t., let J be an ideal of R, and let $B = R/J$. Assume that* $\operatorname{Hom}_B(H^1(K, B, B), B) = 0$ *and* $\operatorname{Ext}^1_B(H^1(K, B, B), B) = 0$. *Then there exists an exact sequence*

$$0 \to H_1(K,B,B) \to J/J^2 \xrightarrow{\alpha} (J/J^2)^{**} \to \Omega_{B|K} \xrightarrow{\beta} (\Omega_{B|K})^{**}.$$

Moreover, we have

(1) $\operatorname{Coker}\beta \simeq \operatorname{Ext}^1_B((J/J^2)^*, B)$ *if* $\operatorname{Ext}^2_B(H^1(K,B,B),B)=0$, *and*
(2) $\operatorname{Coker}\beta \simeq \operatorname{Ext}^2_B(H^1(K,B,B),B)$ *if* $\operatorname{Ext}^i_B((J/J^2)^*,B)=0$, $i=1,2$.

If K is a perfect field and B is a regular K-algebra of e.f.t., then B is a smooth K-algebra and therefore $H^1(K,B,B)=0$. Hence, in analogy with Lemma 1.4, we obtain the next lemma.

LEMMA 2.2. *Let K be a perfect field and let B be a K-algebra of e.f.t. If B has properties (R_n) and (S_{n+1}), then*

$$\operatorname{Ext}^i_B(H^1(K,B,B),B)=0, \quad 0 \le i \le n.$$

Let K be a field, R a smooth K-algebra of e.f.t., and let $B=R/J$. It is known that B is l.c.i. if and only if J/J^2 is B-projective. Moreover, if K is perfect and B is reduced, then B is l.c.i. if and only if $\operatorname{pd}_B(\Omega_{B|K}) \le 1$.

For the rest of this section, K is a perfect field and $B=R/J$ where R is a smooth K-algebra of e.f.t.

If B is reduced, then $\Omega_{B|K}$ is torsion-free if and only if $\beta: \Omega_{B|K} \to (\Omega_{B|K})^{**}$ is injective. Therefore, it follows from Lemma 2.2 and Proposition 2.1 that $\Omega_{B|K}$ is torsion-free if B is l.c.i. and normal. This result has already been obtained in another way by Suzuki [10].

PROPOSITION 2.3. *Assume that B is normal, $\Omega_{B|K}$ is torsion-free, and $(J/J^2)^*$ is B-projective. Then B is l.c.i.*

Proof. The preceding results show that $\alpha: J/J^2 \to (J/J^2)^{**}$ is an epimorphism. We have to prove that J is locally generated by a regular sequence. Localizing in each prime ideal containing J, we see that it is sufficient to show the following.

LEMMA 2.4. *Let A be a local Noetherian ring and let I be an ideal of A such that $\operatorname{pd}_A(I)<\infty$. Assume that*

(1) *$(I/I^2)^*$ is A/I-free,*
(2) *$I/I^2 \to (I/I^2)^{**}$ is surjective, and*
(3) *I_p is generated by a regular sequence for every $p \in \operatorname{Ass}(A/I)$.*

Then I is generated by a regular sequence.

Proof. The lemma follows easily from [11, Proposition]. □

PROPOSITION 2.5. *Assume that K has characteristic zero and B is reduced. Then B is regular if and only if $H^1(K,B,B)=0$ and $(J/J^2)^*$ is B-projective.*

Proof. We shall prove the "if" part. We have an exact sequence

$$0 \to (\Omega_{B|K})^* \to (\Omega_{R|K} \otimes_R B)^* \to (J/J^2)^* \to 0,$$

from which we deduce that $(\Omega_{B|K})^*$ is B-projective. Moreover, by Proposition 2.1, $\beta: \Omega_{B|K} \to (\Omega_{B|K})^{**}$ is surjective. Since K has characteristic zero, this implies that B is regular [6, §§3–4]. □

Assume now that B is l.c.i. and (R_2). Then $\beta: \Omega_{B|K} \to (\Omega_{B|K})^{**}$ is an isomorphism (this result has already been obtained by Lipman [6, Prop. 8.1]). Therefore we have the following.

COROLLARY 2.6. *Assume that B is l.c.i. and (R_2). Then B is regular if and only if* $\mathrm{Der}_K(B, B)$ *is a projective B-module.*

This is a particular case of the Zariski–Lipman conjecture. Let us recall the conjecture. Let K be a field of characterisic zero, let B be a finitely generated reduced K-algebra, and let q be a prime ideal of B. If $\mathrm{Der}_K(B_q, B_q)$ is a free B_q-module, then B_q is regular.

Corollary 2.6 resolves affirmatively the conjecture for complete intersections with property (R_2). Moreover, it implies that the conjecture is true for all complete intersections if it is true for complete intersections of dimension 2; that is, if B_q is a complete intersection, $\dim(B_q) = 2$, and $\mathrm{Der}_K(B_q, B_q)$ is B_q-free, then B_q is regular.

3. Regular Rings and Hochschild Cohomology

In this section we shall use Lemma 1.6 to give a condition, in terms of Hochschild cohomology, for a l.c.i. to be regular.

Let A be a ring and let I be an ideal of A. There exist homomorphisms of graded A/I-modules (see [4, p. 389] and [1, Def. 14.20])

$$\gamma: \mathrm{Ext}_A(A/I, A/I) \to \mathrm{Hom}_{A/I}(\mathrm{Tor}^A(A/I, A/I), A/I),$$
$$\phi: \wedge I/I^2 \to \mathrm{Tor}^A(A/I, A/I), \quad \textit{and}$$
$$\psi: \mathrm{Ext}_A(A/I, A/I) \to \mathrm{Hom}_{A/I}(\wedge I/I^2, A/I),$$

where $\psi = \phi^* \gamma$.

It is easy to check that $\psi_2: \mathrm{Ext}_A^2(A/I, A/I) \to \mathrm{Hom}_{A/I}(I/I^2 \wedge I/I^2, A/I)$ is the homomorphism of Lemma 1.6.

LEMMA 3.1. *Assume that I/I^2 is a projective A/I-module and*

$$H_n(A, A/I, A/I) = 0 \quad \textit{for } n \geq 2.$$

Then γ, ϕ, ψ are isomorphisms.

Proof. ϕ is an isomorphism by [1, Thm. 14.22]. Therefore $\mathrm{Tor}_s^A(A/I, A/I)$ is A/I-projective for all s. Hence γ is an isomorphism [9, Prop. 4.1]. □

Let K be a field, let B be a K-algebra, and let I be the kernel of the homomorphism $B\otimes_K B \to B$, $x\otimes y \to xy$. Then $\Omega_{B|K}=I/I^2$. Moreover,

$$H_n(K,B,B)\simeq H_{n+1}(B\otimes_K B,B,B) \quad\text{and}\quad H^n(K,B,B)\simeq H^{n+1}(B\otimes_K B,B,B)$$

for $n\geq 0$ [9, Prop. 4.1].

We have the homomorphisms

$$\gamma: \operatorname{Ext}_{B\otimes_K B}(B,B) \to \operatorname{Hom}_B(\operatorname{Tor}^{B\otimes_K B}(B,B),B);$$

$$\psi: \operatorname{Ext}_{B\otimes_K B}(B,B) \to \operatorname{Hom}_B(\wedge\,\Omega_{B|K},B).$$

Moreover, $\operatorname{Ker}\psi_2\simeq H^2(B\otimes_K B,B,B)\simeq H^1(K,B,B)$.

PROPOSITION 3.2. *Let K be a perfect field and B be a reduced K-algebra of e.f.t. Assume that B is l.c.i. The following conditions are equivalent:*

(1) *B is regular;*
(2) *γ is an isomorphism;*
(3) *ψ is an isomorphism.*

Proof. If B is regular, then B is a smooth K-algebra since K is perfect. Therefore $\Omega_{B|K}$ is B-projective and $H_n(K,B,B)=0$ for $n\geq 1$. It follows from Lemma 3.1 that γ and ψ are isomorphisms.

If ψ is an isomorphism, then

$$H^1(K,B,B)\simeq \operatorname{Ker}\psi_2=0.$$

Hence $\operatorname{Ext}^1_B(\Omega_{B|K},B)=0$ [9, p. 495]. On the other hand, $\operatorname{pd}_B(\Omega_{B|K})\leq 1$ since B is l.c.i. It follows that $\Omega_{B|K}$ is B-projective. Therefore B is regular.

Assume that γ is an isomorphism. To prove that B is regular, we shall show that ψ_2 is a monomorphism. There exists an exact sequence [8, p. 77]

$$H_2(K,B,B) \to \Omega_{B|K}\wedge\Omega_{B|K} \to \operatorname{Tor}_2^{B\otimes_K B}(B,B) \to H_1(K,B,B) \to 0.$$

Since K is perfect and B is reduced, we have

$$H_2(K,B,B)^*=0=H_1(K,B,B)^*.$$

Therefore ϕ_2^* is a monomorphism. Hence $\psi_2=\phi_2^*\gamma_2$ is a monomorphism. □

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