REMARKS ON ABSOLUTELY SUMMING TRANSLATION INVARIANT OPERATORS FROM THE DISC ALGEBRA AND ITS DUAL INTO A HILBERT SPACE

S. Kwapien and A. Pełczyński

In this note among other results we prove the following

THEOREM 1. *Let* $f_j \in L^1$ *for* $j = 1, 2, \ldots$. *Assume that*

$$(1) \qquad \sum_{j=1}^{\infty} \left| \int_0^{2\pi} f_j(t)\, h(t)\, dt \right| < +\infty \qquad \textit{for every } h \in H^\infty .$$

Then for every scalar sequence (m_k) *with* $\sum_{k=0}^{\infty} |m_k|^2 < +\infty$,

$$(2) \qquad \sum_{j=1}^{\infty} \sqrt{\sum_{k=0}^{\infty} |m_k \hat{f}_j(-k)|^2} < +\infty ,$$

where $\hat{f}(k) = \frac{1}{2\pi} \int_0^{2\pi} f(t)\, e^{-ikt}\, dt$ *for* $k = 0, \pm 1, \pm 2, \ldots$.

By L^p $(0 < p \le \infty)$ we denote the space of equivalence classes of p-absolutely integrable with respect to the Lebesgue measure complex-valued measurable functions on $[0, 2\pi]$, and by $C_{2\pi}$ the space of 2π-periodic continuous complex-valued functions on $[0, 2\pi]$. For $f \in L^p$ we put $\|f\|_p = \left(\frac{1}{2\pi} \int_0^{2\pi} |f(t)|^p\, dt \right)^{1/p}$ for $p \ge 1$ and $\|f\|_p = \frac{1}{2\pi} \int_0^{2\pi} |f(t)|^p\, dt$ for $0 < p < 1$. The Hardy spaces H^p $(1 \le p \le \infty)$ and the Disc Algebra A are defined by

$$H^p = \{f \in L^p : \hat{f}(k) = 0 \text{ for } k < 0\}, \qquad A = \{f \in C_{2\pi} : \hat{f}(k) = 0 \text{ for } k < 0\}.$$

In the language of absolutely summing operators Theorem 1 means that the adjoint of every translation invariant operator from H^2 into A is 1-absolutely summing. It is an open question whether every bounded linear operator from H^2 into A has 1-absolutely summing adjoint.

Our proof of Theorem 1 is indirect. Our argument uses the duality between nuclear and bounded operators and Theorem 2 below which asserts that a translation invariant operator $M : A \to H^2$ is nuclear if and only if it is p-absolutely summing for some p with $1 > p > 0$.

Received February 28, 1977.

Michigan Math. J. 25 (1978)

PRELIMINARIES

Let $T: X \to Y$ be a linear operator (X, Y, Banach spaces). Recall that T is nuclear if and only if it has a nuclear representation, say $\sum_j x_j^* \otimes y_j$; i.e., there are sequences $(x_j^*) \subset X^*$ and $(y_j) \subset Y$ such that $\sum \|x_j^*\| \|y_j\| < +\infty$ and $T(x) = \sum_j x_j^*(x) y_j$ for every $x \in X$. We put $n(T) = \inf \sum_j \|x_j^*\| \|y_j\|$ where the infimum is extended over all the nuclear representations of T. T is L^1-factorable if there is an L^1-factorization of T, say (U, V), i.e. there are an $L^1(\mu)$ space and operators $U: X \to L^1(\mu)$, $V: L^1(\mu) \to Y^{**}$ with $VU = \kappa T$ where $\kappa: Y \to Y^{**}$ is the canonical embedding. We put $\gamma_1(T) = \inf \|U\| \|V\|$ where the infimum is extended over all the L^1-factorizations of T. Let $0 < p < \infty$. An operator T is p-absolutely summing if there is a constant $C > 0$ such that

$$\sum_{x \in F} \|T(x)\|^p \leq C^p \sup_{\|x^*\| \leq 1} \sum_{x \in F} |x^*(x)|^p \qquad \text{for every finite } F \subset X. \tag{3}$$

We put $\pi_p(T) = \inf\{C : C \text{ satisfies (3)}\}$. It can be easily seen that if $T: X \to Y$ is p-absolutely summing then

$$\int_\Omega \|T(\phi(\omega))\|^p m(d\omega) \leq [\pi_p(T)]^p \sup_{\|x^*\| \leq 1} \int_\Omega |x^*(\phi(\omega))|^p m(d\omega) \tag{4}$$

for every probability space (m, Σ, Ω) and every weakly measurable function $\phi: \Omega \to X$.

We shall deal with translation invariant function spaces on the circle group which is represented as the interval $[0, 2\pi]$ with addition mod 2π as the group operation. An operator M acting between those spaces is translation invariant if and only if it commutes with all the translations T_α for $0 \leq \alpha < 2\pi$ (where $(T_\alpha f)(t) = f(t + \alpha)$ for $t \in [0, 2\pi]$). If M is a translation invariant operator, then $M(e^{int}) = m_n e^{int}$ whenever the exponent e^{int} belongs to the domain of M; we put $\hat{M} = \{m_n : e^{int} \in \text{domain of } M\}$.

We end this section with the following well known fact:

LEMMA 1. *Let $0 < p < 1$. Then there is an absolute constant K_p such that, for every complex Borel measure ν on $[0, 2\pi]$ with $\nu(\{0\}) = \nu(\{2\pi\})$,*

$$\frac{1}{2\pi} \int_0^{2\pi} \left| \sum_{j=0}^{n} \hat{\nu}(j) e^{ijt} \right|^p dt \leq K_p^p \|\nu\|^p \qquad \textit{for } n = 1, 2, \ldots.$$

Here $\|\nu\|$ denotes the total variation of ν and

$$\hat{\nu}(j) = \int_0^{2\pi} e^{-ijt} \nu(dt) \qquad \textit{for } j = 0, \pm 1, \pm 2, \ldots.$$

Proof: The unit ball of L^1 is dense in the unit ball of the dual $(C_{2\pi})^*$ in the $\sigma((C_{2\pi})^*, C_{2\pi})$ – topology. Hence, given a measure ν as above, a positive integer n and an $\varepsilon > 0$, there exists an $h \in L^1$ such that $\|h\|_1 = \|\nu\|$ and

$$|\hat{h}(j) - \hat{\nu}(j)| < \frac{\varepsilon}{n+1} \qquad \text{for } j = 0, 1, \ldots, n.$$

Let $(Rh)(t) = \lim_{r \uparrow 1} \sum_{j=0}^{\infty} \hat{h}(j) e^{ijt} r^j$. By the Kolmogorov Theorem (cf. [1]), the limit exists t-almost everywhere and the function Rh belongs to L^p for every p with $0 < p < 1$. Moreover there is an absolute constant $K_p > 0$ such that

$$\|Rh\|_p = \frac{1}{2\pi} \int_0^{2\pi} |(Rh)(t)|^p \, dt \leq \frac{K_p^p}{2} \|h\|_1^p.$$

Since $\sum_{j=0}^{n} \hat{h}(j) e^{ijt} = (Rh) - R(he^{-i(n+1)t})$, we get

$$\frac{1}{2\pi} \int_0^{2\pi} \left| \sum_{j=0}^{n} \hat{h}(j) e^{ijt} \right|^p dt \leq \|Rh\|_p + \|R(he^{i(n+1)t})\|_p$$
$$\leq K_p^p \|h\|_1^p = K_p^p \|\nu\|^p$$

Thus

$$\frac{1}{2\pi} \int_0^{2\pi} \left| \sum_{j=0}^{n} \hat{\nu}(j) e^{ijt} \right|^p dt \leq \frac{1}{2\pi} \int_0^{2\pi} \left| \sum_{j=0}^{n} \hat{h}(j) e^{ijt} \right|^p dt + \varepsilon$$
$$\leq K_p^p \|\nu\| + \varepsilon .$$

Letting ε tend to 0 we get the desired conclusion.

RESULTS AND PROOFS

We begin with

THEOREM 2. *Let* $M : A \to H^2$ *be a translation invariant operator with* $\hat{M} = (m_j)_{0 \leq j < \infty}$. *Then the following conditions are equivalent:*

(i) M *is nuclear,*

(ii) M *is* L^1*-factorable,*

(iii) M *is* p*-absolutely summing for every* $p > 0$,

(iv) M *is* p*-absolutely summing for some* p *with* $0 < p < 1$,

(v) $\hat{M} \in \ell^2$.

Proof: The implications (i) ⇒ (ii) and (iii) ⇒ (iv) are trivial; (ii) ⇒ (iii) follows from a result of Maurey [6, Théorème 94] which says that every bounded operator from an L^1-space into a Hilbert space is p-absolutely summing for every $p > 0$.

(iv) $\Rightarrow$ *(v)*. Let us fix a positive integer n and put

$$f_\alpha(t) = \sum_{j=0}^{n} e^{ijt} \cdot e^{ij\alpha}, \qquad 0 \le t \le 2\pi, \quad 0 \le \alpha \le 2\pi .$$

Consider the map $\alpha \to f_\alpha$ from $[0, 2\pi]$ into A. It follows from (iv) and (4)

$$(5) \qquad \frac{1}{2\pi} \int_0^{2\pi} \|M(f_\alpha)\|_2^p \, d\alpha \le [\pi_p(M)]^p \sup_{\|x^*\| \le 1} \frac{1}{2\pi} \int_0^{2\pi} |x^*(f_\alpha)|^p \, d\alpha .$$

Clearly, for $0 \le \alpha < 2\pi$,

$$\|M(f_\alpha)\|_2 = \left(\frac{1}{2\pi}\int_0^{2\pi} |M(f_\alpha)(t)|^2 \, dt\right)^{1/2} = \left(\sum_{j=0}^{n} |m_j e^{ij\alpha}|^2\right)^{1/2} = \left(\sum_{j=0}^{n} |m_j|^2\right)^{1/2} .$$

Hence

$$(6) \qquad \frac{1}{2\pi} \int_0^{2\pi} \|M(f_\alpha)\|_2^p \, d\alpha = \left(\sum_{j=0}^{n} |m_j|^2\right)^{p/2} .$$

Now fix an $x^* \in A^*$. By the Hahn-Banach and the Riesz Representation Theorems, there exists a complex Borel measure $\nu_{x^*} \in (C_{2\pi})^*$ such that $\|\nu_{x^*}\| = \|x^*\|$ and $\int_0^{2\pi} g(-t)\, \nu_{x^*}(dt) = x^*(g)$ for $g \in A$. In particular we have

$$x^*(f_\alpha) = \sum_{j=0}^{n} \hat{\nu}_{x^*}(j)\, e^{ij\alpha} \qquad \text{for } 0 \le \alpha < 2\pi .$$

Thus, by Lemma 1,

$$(7) \qquad \frac{1}{2\pi} \int_0^{2\pi} |x^*(f_\alpha)|^p \, d\alpha \le K_p^p \|\nu_{x^*}\|^p = K_p^p \|x^*\|^p .$$

Combining (5), (6) and (7) we get $\left(\sum_{j=0}^{n} |m_j|^2\right)^{1/2} \le K_p \pi_p(M)$. This completes the proof of the implication (iv) $\Rightarrow$ (v) .

(v) $\Rightarrow$ *(i)*. Consider the commutative diagram

$$\begin{array}{ccccc} C_{2\pi} & \xrightarrow{\quad I_2 \quad} & L^2 & \xrightarrow{\quad I_{2,1} \quad} & L^1 \\ \uparrow J & & & & \downarrow \tilde{M} \\ A & & \xrightarrow{\qquad\qquad M \qquad\qquad} & & H^2 \end{array}$$

where $J : A \hookrightarrow C_{2\pi}$ is the isometric inclusion, $I_2 : C_{2\pi} \to L^2$ and $I_{2,1} : L^2 \to L^1$ are natural injections, ($I_2(f)$ (resp. $I_{2,1}(f)$) is the equivalence class of f regarded as the element of L^2 (resp. of L^1)), $\tilde{M}(f)(s) = \frac{1}{2\pi}\int_0^{2\pi} f(t)\,g(s-t)\,dt$ where

$$g = \sum_{j=0}^{\infty} m_j e^{ijt}.$$

Clearly, by (v), $g \in H^2$. Hence $\tilde{M}(f) \in H^2$ for $f \in L^1$ and

$$\|\tilde{M}\| \leq \|g\|_2 = \left(\sum_{j=0}^{\infty} |m_j|^2\right)^{1/2} = \|\hat{M}\|_{\ell^2}$$

(by the Young inequality). Thus $M = \tilde{M} I_{2,1} I_2 J$. Clearly $\pi_2(I_2 J) \leq \pi_2(I_2) \leq 1$ and $\pi_2(\tilde{M} I_{2,1}) \leq \|\tilde{M}\|$ because $\tilde{M} I_{2,1}$ is a Hilbert Schmidt operator with the Hilbert Schmidt norm less than or equal to $\|\tilde{M}\|$ (cf. [2]). Thus, by a result of [8], $M = \tilde{M} I_{2,1} I_2 J$ is nuclear and $n(M) \leq \pi_2(\tilde{M} I_{2,1})\,\pi_2(I_2 J) \leq \|\tilde{M}\| \leq \|\hat{M}\|_{\ell^2}$ This completes the proof.

Remark 1. Theorem 2 can be restated as follows:

For every translation invariant operator $M : A \to H^2$ and for $0 < p < 1$ we have the following chain of inequalities

$$\|\hat{M}\|_{\ell^2} \geq n(M) \geq \gamma_1(M) \geq C_p \pi_p(M) \geq C_p K_p^{-1} \|\hat{M}\|_{\ell^2}, \tag{8}$$

where C_p is the constant appearing in the Maurey Theorem [6] quoted above and K_p is the constant appearing in Lemma 1.

Remark 2. It is interesting to compare Theorem 2 to what is known about the spaces $\Pi_p^{inv}(A, H^2)$ of all the p-absolutely summing translation invariant operators from A into H^2 for $p \geq 1$. We have (folklore):

There is a natural isometric isomorphism between the space $\Pi_p^{inv}(A, H^2)$ with the norm $\pi_p(\cdot)$ and the space $B^{inv}(H^p, H^2)$ of all the bounded translation invariant operators from H^p into H^2 $(p \geq 1)$.

Proof: If $I_p : A \to H^p$ is the natural injection and if $\tilde{M} \in B^{inv}(H^p, H^2)$, then $M = \tilde{M} I_p \in \Pi_p^{inv}(A, H^2)$ and $\pi_p(M) \leq \pi_p(I_p)\|\tilde{M}\| = \|\tilde{M}\|$. Conversely, by the Grothendieck-Pietsch Theorem (for $p \geq 1$) (cf. [5], [7], [8]), given $M \in \Pi_p^{inv}(A, H^2)$ there is a finite positive Borel measure on $[0, 2\pi]$, say μ, which p-dominates M; i.e., $\|M(f)\|_2^p \leq \int_0^{2\pi} |f(t)|^p \mu(dt)$ for $f \in A$. Now using the standard averaging technique and taking into account that M is translation invariant we infer that M is p-dominated by a multiple of the Haar measure (the normalized Lebesgue measure on $[0, 2\pi]$). Thus $M = \tilde{M} I_p$ for some $\tilde{M} \in B^{inv}(H^2, H^p)$. Moreover it is not difficult to see that $\pi_p(M) = \|\tilde{M}\|$. This completes the proof.

Let $\ell^{q,\infty} = \left\{(m_j)_{j\geq 0} : \sup_{k\geq 1}\left(\sum_{2^{k-1}\leq j+1<2^k} |m_j|^q\right)^{1/q} < +\infty\right\}$. It is known (cf. [1], [4]) that $M \in B^{inv}(H^1, H^2)$ if and only if $\hat{M} \in \ell^{2,\infty}$, and if $\hat{M} \in \ell^{2p(2-p),\infty}$ then $M \in B^{inv}(H^p, H^2)$ for $1 < p < 2$; if $p \geq 2$ then $M \in B^{inv}(H^p, H^2)$ if and only if $\hat{M} \in \ell^{\infty}$ (trivial).

Our next result is in fact equivalent to Theorem 1 stated in the introduction.

THEOREM 3. *Every bounded translation invariant operator* $M : H^2 \to A$ *has* 1-*absolutely summing adjoint. Equivalently, there is an absolute constant* K *independent of* M *such that*

$$\pi_1(M^*) \leq K\|M\|. \tag{9}$$

Proof: Recall that (cf. [3], [7]).

(a) an operator $T : X \to Y$ has 1-absolutely summing adjoint if and only if UT is nuclear for every bounded linear operator $U : Y \to \ell^1$; moreover

$$\pi_1(T^*) = \sup\{n(UT) : U : Y \to \ell^1, \|U\| = 1\}.$$

(b) Let $\mathscr{H}$ be a Hilbert space and Y a Banach space. An operator $S : \mathscr{H} \to Y$ is nuclear if and only if for every finite dimensional $V : Y \to \mathscr{H}$, VS is nuclear; moreover $n(S) = \sup\{|\operatorname{tr} VS| : V : Y \to \mathscr{H}, \|V\| = 1, \dim V(Y) < +\infty\}$, where tr T denotes the trace of a nuclear operator $T : \mathscr{H} \to \mathscr{H}$.

By (a) and (b), it is enough to show that there exists an absolute constant $K > 0$ such that

$$\sup|\operatorname{tr}(VUM)| \leq K\|M\|, \tag{10}$$

where the supremum extends over all operators $U : A \to \ell^1$ with $\|U\| = 1$ and $V : \ell^1 \to H^2$ with $\|V\| = 1$ and $\dim V(\ell^1) < \infty$.

Fix U and V as above. The translation invariantness of M and the well known property of the trace yield

$$\operatorname{tr}(VUM) = \operatorname{tr}(T_\alpha VUMT_\alpha^{-1}) = \operatorname{tr}(T_\alpha VUT_\alpha^{-1}M) \tag{11}$$

for every translation $T_\alpha\ (0 \leq \alpha < 2\pi)$. Clearly, for every $f \in A$, the function $\alpha \to (T_\alpha VUT_\alpha^{-1})(f)$ is continuous and therefore the integral

$$\frac{1}{2\pi}\int_0^{2\pi} (T_\alpha VUT_\alpha^{-1})(f)\, d\alpha$$

exists. Define $B : A \to H^2$ by $B(f) = \frac{1}{2\pi}\int_0^{2\pi} (T_\alpha VUT_\alpha^{-1})(f)\, d\alpha$ for $f \in A$. Clearly B is a bounded linear operator with the following property

(12) there is a sequence (B_m) of finite convex combinations of the operators $T_\alpha VUT_\alpha^{-1}$ such that $\lim_m \|B_m(f) - B(f)\| = 0$ for every $f \in A$.

(As the B_m's one may take the Riemann sums of the function $\alpha \to T_\alpha VUT_\alpha^{-1}$.)

Since V is finite dimensional, $n(V) < \infty$. Thus, by (12),

(13) $$n(B_m) \le \sup_{0\le\alpha<2\pi} n(T_\alpha VUT_\alpha^{-1}) \le n(V) \qquad \text{for } n = 1, 2, \ldots .$$

Next recall that the space $N(A, H^2)$ of all the nuclear operators from A into H^2 can be identified with the dual of the space $K(H^2, A)$ of all the compact operators from H^2 into A , and the duality is given by the trace (cf. [3], [7]). Therefore the ball $\{T \in N(A, H^2) : n(T) \le n(V)\}$ is compact in the $\sigma(N(A, H^2), K(H^2, A))$-topology. Thus it follows from (12) and (13) that the sequence (B_m) converges to B in the $\sigma(N(A, H^2), K(H^2, A))$-topology and

$$n(B) \le \overline{\lim_m}\, n(B_m) .$$

Thus $\operatorname{tr}(BM) = \lim_m \operatorname{tr}(B_m M) = \operatorname{tr}(UVM)$, because, by (11),

$$\operatorname{tr} B_m M = \operatorname{tr}(UVM) \qquad \text{for every } m .$$

Hence

(14) $$|\operatorname{tr}(VUM)| \le n(B)\|M\| \le \overline{\lim_m}\, n(B_m).$$

Obviously $\gamma_1(T_\alpha VUT_\alpha^{-1}) \le 1$ for $0 \le \alpha < 2\pi$. Thus $\gamma_1(B_m) \le 1$ for every m because the B_m's are finite convex combinations of the operators $T_\alpha VUT_\alpha^{-1}$. Hence, by (8), $n(B_m) \le K = \inf_{0<p<1} \frac{K_p}{C_p}$ for every m which combined with (14) yields $|\operatorname{tr}(VUM)| \le K\|M\|$. This implies (10) and therefore (9), and completes the proof.

COROLLARY. *Every translation invariant operator* $M : L^1/H_0^1 \to H_-^2$ *is absolutely summing. Here*

$$H_0^1 = \{f \in H^1 : \hat f(0) = 0\} \quad \textit{and} \quad H_-^2 = \{f \in L^2 : \hat f(n) = 0 \text{ for } n > 0\} .$$

Proof: An operator $M : L^1/H_0^1 \to H_-^2$ is translation invariant if and only if $Mq : L^1 \to H_-^2$ is translation invariant ($q : L^1 \to L^1/H_0^1$ is the quotient map); equivalently there is a sequence $(m_k)_{k\ge0}$ with $\sum_{k=0}^{\infty} |m_k|^2 < +\infty$ such that

$$M(\{e^{-ikt} + H_0^1\}) = m_k e^{-ikt} \qquad \text{for } k = 0, 1, \ldots .$$

Define $M_* : H^2 \to A$ by $M_*(e^{ikt}) = m_k e^{ikt}$ for $k = 0, 1, \ldots$. Clearly M_* is bounded, in fact $\|M_*\| = \left(\sum_{k=0}^{\infty} |m_k|^2\right)^{1/2}$. Furthermore M is the restriction of the adjoint of M_* to L^1/H_0^1 (we identify L^1/H_0^1 with a subspace of A^* using the fact that, by the F and M Riesz Theorem, H_0^1 coincides with the annihilator of A in $(C_{2\pi})^*$). The desired conclusion follows now from Theorem 3; in fact

(15) $$\pi_1(M) \le K\|M\| = K\left(\sum_{k=0}^{\infty} |m_k|^2\right)^{1/2}$$

Proof of Theorem 1. Since the dual of L^1/H_0^1 can be identified with H^∞ (cf. [1]), the condition (1) simply means that the cosets $\{f_j + H_0^1\} \in L^1/H_0^1$ form a weakly unconditionally summable sequence, i.e.

$$\sum_{j=1}^{\infty} |x^*(f_j)| < +\infty \qquad \text{for every } x^* \in (L^1/H_0^1)^* .$$

Now the standard Baire category technique yields the existence of a constant $c = c((f_j))$ such that $\sum_{j=1}^{\infty} |x^*(f_j)| \leq c\,\|x^*\|$ for every $x^* \in (L^1/H_0^1)$. Thus for every 1-absolutely summing operator $M : L^1/H_0^1 \to H_-^2$,

$$\sum_{j=1}^{\infty} \|M(\{f_j + H_0^1\})\|_2 \leq c\pi_1(M) . \tag{16}$$

Finally suppose that $M : L^1/H_0^1 \to H_-^2$ is translation invariant and let

$$M(\{e^{-ikt} + H_0^1\}) = m_k e^{-ikt} \qquad \text{for } k = 0, 1, 2, \ldots .$$

Then, by Corollary to Theorem 3 (cf. formula (15)), the inequality (16) gives

$$\sum_{j=1}^{\infty} \|M(\{f_j + H_0^1\})\|_2 \leq cK \left(\sum_{k=0}^{\infty} |m_k|^2\right)^{1/2} . \tag{17}$$

On the other hand $M(\{f_j + H_0^1\}) = \sum_{k=0}^{\infty} m_k \hat{f}_j(-k) e^{-ikt}$. Thus

$$\sum_{j=1}^{\infty} \|M(\{f_j + H_0^1\})\|_2 = \sum_{j=1}^{\infty} \left(\sum_{k=0}^{\infty} |m_k \hat{f}_j(-k)|^2\right)^{1/2} . \tag{18}$$

Obviously (17) and (18) implies (2). This completes the proof.

REFERENCES

1. P. Duren, *Theory of H^p spaces.* Pure and Applied Mathematics, Vol. 38. Academic Press, New York-London, 1970.

2. A. Grothendieck, *Résumé de la théorie métrique des produits tensoriels topologiques.* Bol. Soc. Mat. São Paulo 8 (1953), 1-79 (1956).

3. ———, *Produits tensoriels topologiques et espaces nucleaires,* Mem. Amer. Math. Soc. 16 (1975).

4. J. H. Hedlund, *Multipliers of H^p spaces.* J. Math. Mech. 18 (1968/69), 1067-1074.

5. J. Lindenstrauss and A. Pełczyński, *Absolutely summing operators in* L_p*-spaces and their applications.* Studia Math. 29 (1968), 275-326.

6. B. Maurey, *Théorèmes de factorisation pour les opérateurs linéaires à valeurs dans les espaces* L^p. Astérisque 11 (1974), 1–163.

7. A. Persson und A. Pietsch, *p-nukleare und p-integrale Abbildungen in Banachräumen.* Studia Math. 33 (1969), 19–62.

8. A. Pietsch, *Absolut* p-*summierende Abbildungen in normierten Räumen.* Studia Math. 28 (1966/67), 333–353.

Department of Mathematics
University of Warsaw
Warsaw, POLAND
and
Institute of Mathematics
Polish Academy of Sciences
Warsaw, POLAND