SYMMETRIC SPACES AND PRODUCTS OF SPHERES

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1. INTRODUCTION

In this paper we extend to certain symmetric spaces a result of J.-P. Serre [8] on the comparison of Lie groups and products of spheres. Throughout the paper, G will denote a compact, connected, simply connected Lie group, $\sigma\colon G \to G$ an automorpism of period 2, and K the identity component of the fixed point set of σ. We shall assume that K is totally nonhomologous to zero in G with real coefficients, that is, that the inclusion $K \subset G$ induces an epimorphism in real cohomology. It is known [4] that under these hypotheses G/K has the same real cohomology as a product

$$X = S^{n_1} \times \cdots \times S^{n_\ell} \qquad (n_1 \leq n_2 \leq \cdots \leq n_\ell;\ n_i \text{ odd}).$$

A prime p is *regular* for G/K if there exists $f\colon X \to G/K$ such that

$$f^*\colon H^*(G/K; Z_p) \to H^*(X; Z_p)$$

is an isomorphism.

THEOREM 1. *If* p *is an odd prime,* $p \geq (n_\ell + 1)/2$, *and* G/K *has no* p-*torsion, then* p *is regular for* G/K.

This theorem, the proof of which is given in Section 2, extends Proposition 6 in Chapter V of Serre's paper [8], which under similar hypotheses gives the above conclusion for a Lie group. The converse of Serre's result on the regularity of primes for a Lie group was proved by Serre [8] for classical groups, and by the author [6] for the exceptional groups. Here we give a proof, using the classification of irreducible symmetric spaces, of a partial converse of Theorem 1 in the case where G is classical:

THEOREM 2. *If* G *is a classical group and* G/K *is an irreducible symmetric space different from a sphere, then each prime* $p < (n_\ell + 1)/2$ *is irregular for* G/K.

The irreducible symmetric spaces to which Theorem 1 applies are

(i) $(K \times K)/K$ (K a simple Lie group),

(ii) $SU(2n + 1)/SO(2n + 1)$,

(iii) $SU(2n)/Sp(n)$,

(iv) $Spin(2n)/Spin(2n - 1)$,

(v) E_6/F_4.

In Section 4 we show that the only obstacle to the elimination from Theorem 2 of the hypothesis that G is classical is a proof that the prime 7 is irregular for E_6/F_4.

Received October 20, 1966.

This research was supported in part by a State University of New York Faculty Research Fellowship.

From Theorem 1 and the J. H. C. Whitehead Theorem as stated in [8, p. 276], we obtain the following homotopy information (the subscript p denotes the p-primary component of these groups). For all i,

(1.1) $$\pi_i(SU(2n+1)/SO(2n+1))_p \approx \pi_i(S^5 \times S^9 \times \cdots \times S^{4n+1})_p \textit{ for } p \geq 2n+1,$$

(1.2) $$\pi_i(SU(2n)/Sp(n))_p \approx \pi_i(S^5 \times S^9 \times \cdots \times S^{4n-3})_p \textit{ for } p \geq 2n-1,$$

(1.3) $$\pi_i(E_6/F_4)_p \approx \pi_i(S^9 \times S^{17})_p \textit{ for } p \geq 9.$$

Theorems 1 and 2 may be combined to yield the following.

THEOREM 3. *If* G *is a classical group and* G/K *is an irreducible symmetric space different from a sphere, then* p *is regular for* G/K *if and only if* $p \geq (n_\ell + 1)/2$.

We need merely observe that the only torsion in G/K under the hypotheses is 2-torsion.

The author wishes to acknowledge the assistance of Professors Lawrence Conlon, Bruno Harris, and Henry Tramer in the preparation of this paper.

2. PROOF OF THEOREM 1

Let G, σ, K, p, be as described in the introduction and in Theorem 1. Let ℓ: G → G/K be the natural projection, q: G/K → G the map defined by $q(gK) = g\sigma(g)^{-1}$, and w: G/K × G/K → G/K the product defined by

$$w(g_1K, g_2K) = g_1\sigma(g_1)^{-1}g_2K.$$

Let V be the subalgebra of $H^*(G; Z_p)$ (which is exterior on odd-dimensional generators) generated by those primitive generators of $H^*(G; Z_p)$ that are mapped to their negatives by σ^*. Harris [4] has shown that then

$$q^*: H^*(G; Z_p) \to H^*(G/K; Z_p)$$

maps V isomorphically onto $H^*(G/K; Z_p)$. Moreover, if x is a primitive element of $H^*(G; Z_p)$ with $\sigma^*(x) = -x$, then the generator $q^*(x) \in H^*(G/K; Z_p)$ satisfies the equation $\ell^* q^*(x) = 2x$. Let $x_{n_1}, \cdots, x_{n_\ell}$ $(n_1 \leq \cdots \leq n_\ell)$ be the images under q^* of primitive generators of V. Harris also showed [5] that then the product w induces a map

$$w^*: H^*(G/K; Z_p) \to H^*(G/K; Z_p) \otimes H^*(G/K; Z_p)$$

whose value on x_{n_i} is given by the formula

$$w^*(x_{n_i}) = \ell^* q^*(x_{n_i}) \otimes 1 + 1 \otimes x_{n_i} + d_i,$$

where d_i involves only the generators $x_{n_1}, \cdots, x_{n_{i-1}}$.

Let $X = S^{n_1} \times \cdots \times S^{n_\ell}$, and let Y be the $(n_\ell + 1)$-skeleton of G/K (assumed to be triangulated as a finite simplicial complex). Then $H^*(X; Z_p) \approx H^*(G/K; Z_p)$, and $H^i(Y; Z_p) \approx H^i(G/K; Z_p)$ for $i \leq n_\ell$ (under inclusion). We shall make use of the following lemma due to Serre [8, p. 288].

LEMMA 1 (Serre). *Let* Y *be a finite polyhedron, and* p *an odd prime such that* Y *has no* p*-torsion. Let* n *be an odd integer satisfying the inequalities*

$$\dim Y \leq n + 2p - 3 \quad \textit{and} \quad n \leq \dim Y - 1\,.$$

For each $x \in H^n(Y;\, Z_p)$, *there exist* $f\colon Y \to S^n$ *and* $u \in H^n(S^n;\, Z_p)$ *with* $f^*(u) = x$.

This lemma applies to the case where Y is the $(n_\ell + 1)$-skeleton of G/K, with $n = n_i$, and $p \geq (n_\ell + 1)/2$. To see this, note that $n_i \geq 3$, since G/K is simply connected, and that therefore $\dim Y = n_\ell + 1 \leq 2p \leq 2p + (n_i - 3)$. Thus, if we denote (ambiguously) by $x_{n_1}, \cdots, x_{n_\ell}$ the generators of $H^*(Y;\, Z_p)$ corresponding to $x_{n_1}, \cdots, x_{n_\ell} \in H^*(G/K;\, Z_p)$, we obtain maps $f_i\colon Y \to S^{n_i}$ such that

$$f_i^*\colon H^{n_i}(S^{n_i};\, Z_p) \to H^{n_i}(Y;\, Z_p)$$

maps a generator u_{n_i} of $H^{n_i}(S^{n_i};\, Z_p) \approx Z_p$ onto x_{n_i}; in fact, f_i^* maps $H^{n_i}(S^{n_i};\, Z_p)$ isomorphically onto the subspace of $H^*(Y;\, Z_p)$ spanned by x_{n_i}. Define $h\colon Y \to S^{n_1} \times \cdots \times S^{n_\ell}$ by

$$h(y) = (f_1(y), \cdots, f_\ell(y))\,.$$

Then $h^*\colon H^j(S^{n_1} \times \cdots \times S^{n_\ell};\, Z_p) \to H^j(Y;\, Z_p)$ is an isomorphism for $j \leq n_\ell$. For

$$H^*(S^{n_1} \times \cdots \times S^{n_\ell};\, Z_p) \approx H^*(S^{n_1};\, Z_p) \otimes \cdots \otimes H^*(S^{n_\ell};\, Z_p)$$

is an exterior algebra on u_{n_i} $(i = 1, \cdots, \ell)$, and up to degree n_ℓ, $H^*(Y;\, Z_p)$ is the same on the images of these generators, that is, on the x_{n_i} $(i = 1, \cdots, \ell)$.

The diagram

$$\begin{array}{ccccc}
H^{n_i}(G/K;\, Z_p) & \approx & H_{n_i}(G/K;\, Z_p) & \leftarrow & \pi_{n_i}(G/K) \otimes Z_p \\
\approx\downarrow & & \uparrow & & \approx\uparrow \\
H^{n_i}(Y;\, Z_p) & \approx & H_{n_i}(Y;\, Z_p) & \leftarrow & \pi_{n_i}(Y) \otimes Z_p \\
h^*\uparrow\approx & & h^*\downarrow\approx & & h^*\downarrow\approx \\
H^{n_i}(X;\, Z_p) & \approx & H_{n_i}(X;\, Z_p) & \leftarrow & \pi_{n_i}(X) \otimes Z_p \\
\downarrow & & \uparrow & & \uparrow \\
H^{n_i}(S^{n_i};\, Z_p) & \approx & H_{n_i}(S^{n_i};\, Z_p) & \overset{\approx}{\leftarrow} & \pi_{n_i}(S^{n_i}) \otimes Z_p
\end{array}$$

commutes (here the first column of isomorphisms is obtained by dualization, and the second column of maps consists of Hurewicz homomorphisms). The first and third rows of arrows represent maps induced by the respective inclusions $Y \subset G/K$ and $S^{n_i} \subset X$.

The map $H^{n_i}(G/K;\, Z_p) \to \pi_{n_i}(G/K) \otimes Z_p$ (the composition of the homomorphisms forming the left vertical column, the bottom row, and then the right vertical column of the diagram) sends the generator $x_{n_i} \in H^{n_i}(G/K; Z_p)$ to an element $[g_i] \otimes 1 \in \pi_{n_i}(G/K) \otimes Z_p$, where $g_i\colon S^{n_i} \to G/K$. The map

$$\pi_{n_i}(G/K) \otimes Z_p \to H_{n_i}(G/K; Z_p) \approx H^{n_i}(G/K; Z_p)$$

(the top row of the diagram) sends $[g_i] \otimes 1$ to x_{n_i}. Then

$$g_{i_*}\colon H_*(S^{n_i}; Z_p) \to H_*(G/K; Z_p)$$

maps $H_*(S^{n_i}; Z_p)$ isomorphically onto the subspace spanned by 1 and x_{n_i}. Let $f\colon S^{n_1} \times \cdots \times S^{n_\ell} \to G/K$ be defined as the composition

$$S^{n_1} \times \cdots \times S^{n_\ell} \xrightarrow{g_1 \times \cdots \times g_\ell} G/K \times \cdots \times G/K \xrightarrow{\phi} G/K,$$

where ϕ is the product

$$G/K \times \cdots \times G/K \xrightarrow{w \times 1} G/K \times \cdots \times G/K \to \cdots \to G/K \times G/K \xrightarrow{w} G/K.$$

Our problem is reduced to showing that f induces an isomorphism in homology with coefficients Z_p.

We state and prove this in a more general setting, primarily for the sake of notational convenience.

Let A be an exterior algebra over a field of characteristic different from 2, on generators $x_{n_1}, \cdots, x_{n_\ell}$ $(n_1 \leq \cdots \leq n_\ell)$, and let A_i be the subalgebra generated by 1 and x_{n_i} $(i = 1, \cdots, \ell)$. In referring to *the* basis of A, we shall mean the standard basis consisting of 1, $x_{n_1}, \cdots, x_{n_\ell}$, and all products $x_{n_{i_1}} \cdots x_{n_{i_k}}$ $(i_1 < \cdots < i_k$, $i_j \in \{1, \cdots, \ell\})$. A similar interpretation applies to *the* basis of $A \otimes A \otimes \cdots \otimes A$.

Define $\mu\colon A \to A \otimes A$ to be the algebra homomorphism sending 1 to $1 \otimes 1$, and x_{n_i} to $2x_{n_i} \otimes 1 + 1 \otimes x_{n_i} + d_i$, where d_i is a linear combination of basis elements of $A \otimes A$ that involve only $x_{n_1}, \cdots, x_{n_{i-1}}$. Let $\psi\colon A \to A \otimes A \otimes \cdots \otimes A$ (ℓ factors A) be the composition

$$A \xrightarrow{\mu} A \otimes A \xrightarrow{\mu \otimes 1} A \otimes A \otimes A \to \cdots \to A \otimes \cdots \otimes A \qquad (\ell \text{ factors } A),$$

and P: $A \otimes \cdots \otimes A$ (ℓ factors A) $\to A_1 \otimes \cdots \otimes A_\ell$ the projection onto the subspace $A_1 \otimes \cdots \otimes A_\ell$. Then P is a homomorphism of algebras. This follows from the fact that the subspace of $A \otimes \cdots \otimes A$ spanned by basis vectors other than those of $A_1 \otimes \cdots \otimes A_\ell$ is an ideal in $A \otimes \cdots \otimes A$. Observe that $\psi(1) = 1 \otimes \cdots \otimes 1$, and that

$$\psi(x_{n_i}) = 2^{\ell-1} x_{n_i} \otimes 1 \otimes \cdots \otimes 1 + 2^{\ell-2} 1 \otimes x_{n_i} \otimes 1 \otimes \cdots \otimes 1$$

$$+ \cdots + 1 \otimes \cdots \otimes 1 \otimes x_{n_i} + D_i,$$

where D_i involves only $x_{n_1}, \cdots, x_{n_{i-1}}$. This follows from the definition of μ, by induction.

Thus $P \circ \psi: A \to A_1 \otimes \cdots \otimes A_\ell$ is the algebra homomorphism sending 1 to $1 \otimes \cdots \otimes 1$, and x_{n_i} to $2^{\ell-i}\, 1 \otimes \cdots \otimes 1 \otimes x_{n_i} \otimes 1 \otimes \cdots \otimes 1 + D_i'$, where D_i' is a linear combination of basis elements of $A_1 \otimes \cdots \otimes A_\ell$ involving only $x_{n_1}, \cdots, x_{n_{i-1}}$.

An easy inductive argument establishes the following.

LEMMA 2. *If* $\mathscr{A}$ *is an algebra over the field* F *generated by* $a_1, \cdots, a_n$, d_i *is an element of the subalgebra generated by* $a_1, \cdots, a_{i-1}$, *and* $\bar{a}_i = \alpha_i a_i + d_i$ $(\alpha_i \neq 0$, $\alpha_i \in F)$, *then the elements* $\bar{a}_1, \cdots, \bar{a}_n$ *also generate* $\mathscr{A}$.

Applying Lemma 2 to $A_1 \otimes \cdots \otimes A_\ell$, we see that

$$\text{image } P \circ \psi = A_1 \otimes \cdots \otimes A_\ell .$$

Comparing the dimensions of the vector spaces A and $A_1 \otimes \cdots \otimes A_\ell$, we find that $P \circ \psi: A \xrightarrow{\approx} A_1 \otimes \cdots \otimes A_\ell$.

Dualizing, we see that $\psi^* P^*: A_1^* \otimes \cdots \otimes A_\ell^* \to A^*$ is an isomorphism (and that $P^*: A_1^* \otimes \cdots \otimes A_\ell^* \to A^* \otimes \cdots \otimes A^*$ is the inclusion map).

Taking $A = H^*(G/K; Z_p)$ and $\psi: A \to A \otimes \cdots \otimes A$ to be the map induced by $\phi: G/K \times \cdots \times G/K \to G/K$, we see that the composition

$$A_1^* \otimes \cdots \otimes A_\ell^* \xrightarrow{P^*} A^* \otimes \cdots \otimes A^* \xrightarrow{\phi^*} A^*$$

is an isomorphism, where A_i is the subalgebra of $H^*(G/K; Z_p)$ spanned by 1 and x_{n_i}. We have seen that g_{i_*} maps $H_*(S^{n_i}; Z_p)$ isomorphically onto A_i^*. Hence

$$g_1 \times \cdots \times g_\ell: S^{n_1} \times \cdots \times S^{n_\ell} \to G/K \times \cdots \times G/K$$

maps $H_*(S^{n_1} \times \cdots \times S^{n_\ell}; Z_p)$ isomorphically onto $A_1^* \otimes \cdots \otimes A_\ell^*$, and this in turn is mapped isomorphically onto $H_*(G/K; Z_p)$ by $\phi^* P^*$. Since P^* is the inclusion, it follows that

$$f_*: H_*(S^{n_1} \times \cdots \times S^{n_\ell}; Z_p) \xrightarrow{(g_1 \times \cdots \times g_\ell)_*} H_*(G/K \times \cdots \times G/K; Z_p) \xrightarrow{\phi^*} H_*(G/K; Z_p)$$

is an isomorphism; this establishes Theorem 1.

It should be pointed out that the proof given above follows the suggestion of Serre [8, p. 292], with modifications appropriate to our case.

3. PROOF OF THEOREM 2

The proof of this theorem rests on the classification of irreducible symmetric spaces, and it can be shown [7, p. 497] that the only pairs (G, K) satisfying the hypotheses are the cases (i), (ii), and (iii) following the statement of Theorem 2. Case (i), $(K \times K)/K$ (K a simple Lie group), reduces to the theorem for groups that is established in [6] and [8]. Hence we are left to deal with the cases (ii) $SU(2n+1)/SO(2n+1)$, and (iii) $SU(2n)/Sp(n)$. We shall show that

(3.1) *for* $n \geq 2$, *each prime* $p < 2n+1$ *is irregular for* $SU(2n+1)/SO(2n+1)$ (*in this case,* $X = S^5 \times S^9 \times \cdots \times S^{4n+1}$),

(3.2) *for* $n \geq 3$, *each prime* $p < 2n-1$ *is irregular for* $SU(2n)/Sp(n)$ (*in this case,* $X = S^5 \times S^9 \times \cdots \times S^{4n-3}$).

Note that $SU(3)/SO(3) \approx S^5$ and $SU(4)/Sp(2) \approx S^5$.

Observe that a prime p is irregular for G/K if either

(3.3) *for some* j, $\pi_j(G/K)$ *and* $\pi_j(X)$ *have nonisomorphic* p-*primary components, or*

(3.4) *there exists a nonzero reduced power* $P_p^i: H^j(G/K; Z_p) \to H^{j+2i(p-1)}(G/K; Z_p)$.

(3.3) follows from the J. H. C. Whitehead Theorem [8], and (3.4) from the fact that reduced powers commute with induced maps and that they are zero for products of spheres. We use (3.3) to deal with the case $p = 2$ for both (3.1) and (3.2), and (3.4) to deal with odd primes.

The prime 2 is irregular for $SU(2n+1)/SO(2n+1)$ $(n \geq 2)$, for it is known that

$$\pi_2(SU(2n+1)/SO(2n+1)) \approx \pi_2(U(2n+1)/O(2n+1)) \approx Z_2,$$

for $n \geq 2$ [1]. Since $\pi_2(S^5 \times S^9 \times \cdots \times S^{4n+1}) = 0$, an application of (3.3) completes the proof. Similarly, recall that

$$\pi_5(SU(2n)/Sp(n)) \approx \pi_5(U(2n)/Sp(n)) \approx Z_2 \qquad (n \geq 3);$$

hence 2 is irregular for $SU(2n)/Sp(n)$ $(n \geq 3)$, for $\pi_5(S^5 \times S^9 \times \cdots \times S^{4n-3}) \approx Z$.

We now consider the case of odd primes. It is known that the cohomology algebras (with Z_p-coefficients) of $SU(2n+1)$, $SO(2n+1)$, $SU(2n)$, and $Sp(n)$ are exterior on generators in the following dimensions:

$$SU(2n+1)\colon 3, 5, 7, \cdots, 4n+1,$$

$$SO(2n+1)\colon 3, 7, 11, \cdots, 4n-1,$$

$$SU(2n)\colon 3, 5, 7, \cdots, 4n-1,$$

$$Sp(n)\colon 3, 7, 11, \cdots, 4n-1.$$

The map q^* carries $H^*(SU(2n+1)/SO(2n+1); Z_p)$ isomorphically onto the subalgebra of $H^*(SU(2n+1); Z_p)$ generated by the generators of dimensions $5, 9, \cdots, 4n+1$. We shall show that

$$P_p^1: H^*(SU(2n+1); Z_p) \to H^*(SU(2n+1); Z_p)$$

sends one of these generators to another of these generators. Then the map

$$P_p^1: H^*(SU(2n+1)/SO(2n+1); Z_p) \to H^*(SU(2n+1)/SO(2n+1); Z_p)$$

is nonzero, because P_p^1 is natural. Applying (3.4), we see that p is irregular for SU(2n + 1)/SO(2n + 1). A similar argument shows that p is irregular for SU(2n)/Sp(n). Hence our proof of Theorem 2 will be completed once the following is established.

LEMMA 3. (a) *Let*

$$m = \begin{cases} 2n+1 & \text{if } p \nmid 2n+1, \\ 2n-1 & \text{if } p \mid 2n+1. \end{cases}$$

Then P_p^1 *maps the generator of degree* $2m - 1 - 2(p - 1)$ *in* $H^*(SU(2n+1); Z_p)$ $(n \geq 2)$ *to the generator of degree* $2m - 1$.

(b) *Let*

$$m = \begin{cases} 2n-1 & \text{if } p \nmid 2n-1, \\ 2n-3 & \text{if } p \mid 2n-1. \end{cases}$$

Then P_p^1 *maps the generator of degree* $2m - 1 - 2(p - 1)$ *in* $H^*(SU(2n); Z_p)$ $(n \geq 3)$ *to the generator of degree* $2m - 1$.

Note that in Lemma 3(a) m is odd and p - 1 is even, so that both $2m - 1 - 2(p - 1)$ and $2m - 1$ are congruent to 1 modulo 4. Similarly in (b). One can verify that the restrictions on n, p, and m imply that

$$5 \leq 2m - 1 - 2(p - 1) < 2m - 1 \leq 4n + 1,$$

and

$$5 \leq 2m - 1 - 2(p - 1) < 2m - 1 \leq 4n - 3$$

in (a) and (b), respectively. Hence the generators in (a) and (b) correspond to generators of $H^*(SU(2n+1)/SO(2n+1); Z_p)$ and $H^*(SU(2n)/Sp(n); Z_p)$, respectively. We also note that SU(n) has no p-torsion.

The proof of Lemma 3 depends on a result of Clark, which we state here in a form convenient for our purposes. For a proof, see [2] or [6].

LEMMA 4 (Clark). *Let* G *satisfy the conditions in the introduction. Let* $H^*(G; R)$ *be an exterior algebra on generators* x_{n_i} $(i = 1, \cdots, \ell;\ \deg x_{n_i} = 2m_i - 1)$. *If* p *is prime and* G *has no* p-*torsion, and if there exists a* k $(1 \leq k \leq \ell)$ *such that*

(i) $m_k \not\equiv 0 \pmod p$,

(ii) $m_k > p$,

(iii) *the set* $\{m_1, \cdots, m_\ell\}$ *contains exactly one element* m_j *such that*

$$m_j \equiv 1 - p \pmod{m_k} \quad \textit{and} \quad m_j < m_k,$$

then $P_p^1 x_{2m_j - 1} = x_{2m_k - 1}$.

Proof of Lemma 3. Consider case (a). Let $m_k = m$ as defined in Lemma 3(a). Then (i) of Lemma 4 holds by definition. To prove (ii), observe that $m_k = m \geq 2n - 1$; since $p < 2n - 1$ and p is odd, $p \leq 2n - 1$, hence $m_k \geq p$. But m_k is not a multiple of p, hence $m_k > p$. To prove (iii), note that the only element of the set $\{m_1, \cdots, m_\ell\} = \{2, 3, \cdots, 2n+1\}$ of the form $1 - p + \alpha m_k$ is $1 - p + m_k$. For clearly $2 \leq 1 - p + m_k \leq 2n + 1$, and if $\alpha \geq 2$, then

$$1 - p + \alpha m_k \geq 1 - p + 2m_k = (m_k - p) + 1 + m_k.$$

But $m_k - p > 1$ (since $m_k > p$ and both are odd), hence

$$1 - p + \alpha m_k > 2 + m_k \geq 2 + 2n - 1 = 2n + 1.$$

Clearly, $m_j = 1 - p + m_k < m_k$; hence Lemma 4 applies, and the proof of Lemma 3(a) is complete. The proof of (b) is similar.

4. IRREGULAR PRIMES FOR E_6/F_4

Theorem 1 applies to the symmetric space E_6/F_4, for which the corresponding product of spheres is $S^9 \times S^{17}$. Therefore each prime $p \geq 9$ is regular. Conlon's result [3] $\pi_{16}(E_6/F_4) = 0$, together with the fact that $\pi_{16}(S^9 \times S^{17}) \approx Z_{240}$ [9], shows that the primes 2, 3, and 5 are irregular for E_6/F_4. It is clear that for reasons of degree the reduced powers P_7^i are all zero. All homotopy groups of E_6/F_4 known to the author have 7-primary components isomorphic to those of $S^9 \times S^{17}$. It would be of interest to settle the question of the irregularity of the prime 7, since, by the above argument, this is the only obstacle to the removal from Theorem 2 of the hypothesis that G is classical.

REFERENCES

1. R. Bott, *The stable homotopy of the classical groups*, Ann. of Math. (2) 70 (1959), 313-337.
2. A. Clark, *On* π_3 *of finite dimensional* H-*spaces*, Ann. of Math. (2) 78 (1963), 193-196.
3. L. Conlon, *An application of the Bott suspension map to the topology of* EIV, Pacific J. Math. (to appear).
4. B. Harris, *On the homotopy groups of the classical groups*, Ann. of Math. (2) 74 (1961), 407-413.
5. ———, *Suspensions and characteristic maps for symmetric spaces*, Ann. of Math. (2) 76 (1962), 295-305.
6. P. G. Kumpel, Jr., *Lie groups and products of spheres*, Proc. Amer. Math. Soc. 16 (1965), 1350-1356.
7. ———, *On the homotopy groups of the exceptional Lie groups*, Trans. Amer. Math. Soc. 120 (1965), 481-498.
8. J.-P. Serre, *Groupes d'homotopie et classes de groupes abéliens*, Ann. of Math. (2) 58 (1953), 258-294.
9. H. Toda, *Composition methods in homotopy groups of spheres*, Annals of Mathematics Studies, No. 49, Princeton Univ. Press, Princeton, N.J., 1963.

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