PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093 or via email to cnc8851@cmsu2.cmsu.edu.

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than August 15, 1998, although solutions received after that date will also be considered until the time when a solution is published.

109. *Proposed by Kenneth Davenport, P. O. Box 99901, Pittsburgh, Pennsylvania.*

Let n be a positive integer and $a \geq 2$ be a positive integer. Show that

$$\int_0^\infty \frac{dx}{1^a + x^a} + \int_0^\infty \frac{dx}{2^a + x^a} + \cdots + \int_0^\infty \frac{dx}{n^a + x^a}$$

$$= \left[\frac{1}{1^{a-1}} + \frac{1}{2^{a-1}} + \cdots + \frac{1}{n^{a-1}}\right] \frac{\pi/a}{\sin \pi/a}.$$

110. *Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.*

Let α, β, and γ be the three angles of any triangle. Show that

$$\frac{\sin\alpha}{1+\sin\beta\sin\gamma} + \frac{\sin\beta}{1+\sin\alpha\sin\gamma} + \frac{\sin\gamma}{1+\sin\alpha\sin\beta} < 2.$$

111. *Proposed by Herta T. Freitag, Roanoke, Virginia.*

D is a 3 by 3 determinant whose elements are polygonal numbers $P_{n,k}$ such that

$$a_{i,j} = P_{n+3i+j-4,k}, \quad k \geq 3,$$

where $P_{n,k}$ is the nth polygonal number of k "dimensions" ($P_{5,3}$ is the 5th triangular number). Show that D is a cube independent of n.

112. *Proposed by Mathew Timm, Bradley University, Peoria, Illinois.*

Let Y be a connected, first countable Hausdorff space. Then Y is h-connected if and only if where $p: X \to Y$ is a finite-sheeted covering projection from a connected space X onto Y, it follows that X is homeomorphic to Y. Y is trivially h-connected if and only if whenever $p: X \to Y$ is a connected finite-sheeted covering projection of X onto Y, it follows that p is a homeomorphism of X onto Y.

Note that for non-trivially h-connected spaces, the covering projection $p: X \to Y$ is not required to be a homeomorphism, only that some homeomorphism exist between X and Y. Examples of non-trivially h-connected spaces include the circle S^1, the torus $S^1 \times S^1$, and, more generally, the n-tori $S^1 \times \cdots \times S^1$. Examples of trivially h-connected spaces include any simply connected finite simplicial complex or, more generally, any finite simplicial complex whose fundamental group has no proper finite index subgroups.

Recall that a topological space Y has the fixed point property if and only if, for every continuous function $f: Y \to Y$, there is a $y \in Y$ such that $f(y) = y$.

Now assume that Y is a first countable, Hausdorff, connected, locally path connected, semi-locally 1-connected space. Show that if Y is h-connected and has the fixed point property, then Y is trivially h-connected.