PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, Central Missouri State University, Warrensburg, MO 64093-5247 or via email to ccooper@cmsuvmb.cmsu.edu.

Problems which are new or interesting old problems which are not well-known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk $(*)$ after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than April 1, 1997, although solutions received after that date will also be considered until the time when a solution is published.

92. *Proposed by Joseph B. Dence, University of Missouri-St. Louis, St. Louis, Missouri.*

It is easy to show that the homogeneous quadratic expression $A_n^2 - 2B_n^2$ is invariant for all members of the sequence $\{\gamma_n\}_{n=1}^{\infty}$, defined by $\gamma_n = (3+2\sqrt{2})^n = A_n + B_n\sqrt{2}$. Find a homogeneous cubic expression that is invariant for all members of the sequence $\{I_n\}_{n=1}^{\infty}$, defined by $I_n = (1+\sqrt[3]{2}+\sqrt[3]{4})^n = A_n + B_n\sqrt[3]{2} + C_n\sqrt[3]{4}$.

93. *Proposed by Alan H. Rapoport, Ashford Medical Center, Santurce, Puerto Rico.*

(a) What is the minimum number of people needed in order to assure a greater than 50% chance of at least three having the same birthday?

(b) What is the minimum number of people you must query in order to have a 50% chance of finding at least two having your birthday?

94. *Proposed by Curtis Cooper and Robert E. Kennedy, Central Missouri State University, Warrensburg, Missouri.*

Let n be a nonnegative integer. Prove

$$\sum_{k=0}^{n} \binom{n}{k} (-1)^k F_k = -F_n,$$

where F_n denotes the nth Fibonacci number.

95. *Proposed by Stanley Rabinowitz, Westford, Massachusetts and Robert E. Kennedy and Curtis Cooper, Central Missouri State University, Warrensburg, Missouri.*

Let n be a positive integer. It is known that

$$\sum_{\substack{k=1 \\ \gcd(k,n)=1}}^{n} 1 = \phi(n),$$

where ϕ is Euler's phi function.

(a) Prove

$$\sum_{\substack{k=1 \\ \gcd(k,n)=1}}^{n} k = \frac{1}{2} n \cdot \phi(n).$$

(b) Prove

$$\sum_{\substack{k=1 \\ \gcd(k,n)=1}}^{n} k^2 = \frac{1}{6} n\bigl(2n \cdot \phi(n) + \phi^{-1}(n)\bigr),$$

where ϕ^{-1} is the Dirichlet inverse of Euler's phi function.

(c)* Let m be a positive integer, $m \geq 3$. Find a formula for

$$\sum_{\substack{k=1 \\ \gcd(k,n)=1}}^{n} k^m.$$

96. *Proposed by Herta T. Freitag, Roanoke, Virginia.*

Let an (m by 2) determinant

$$\det\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & \cdots & a_{1,m-1} & a_{1,m} \\ a_{2,1} & a_{2,2} & a_{2,3} & \cdots & a_{2,m-1} & a_{2,m} \end{pmatrix}, \quad m \geq 3$$

be defined by $S(m-1) + \overline{D}$, where

$$S(m-1) = \sum_{r=1}^{m-1} \det\begin{pmatrix} a_{1,r} & a_{1,r+1} \\ a_{2,r} & a_{2,r+1} \end{pmatrix}$$

and

$$\overline{D} = \det\begin{pmatrix} a_{1,m} & a_{1,1} \\ a_{2,m} & a_{2,1} \end{pmatrix}.$$

Consider an (m by 2) determinant D whose elements are polygonal numbers $P_{n,k}$ ($P_{5,3}$ is the 5th triangular number) and such that

$$a_{i,j} = P_{j,i+2},$$

where $i = 1, 2$ and $j = 1, 2, 3, \ldots, m$. Evaluate D.