Gödel and the Theory of Everything *

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Among the hopes Gödel's famous *Incompleteness Theorem* is said to dash, one frequently encounters the Theory of Everything (T.O.E), an ideal quite popular among particle physicists. Indeed, Weinberg's *Dreams of a Final Theory* appear quite close to a complete formal system containing all physical laws: "the final theory [is] one that is so rigid that it cannot be warped into some slightly different theory without introducing logical absurdities like infinite energies." ([17], p. 12) According to him, the fact to be *logically isolated* provides an internal check for a theory to be final. "In a logically isolated theory every constant of nature could be calculated from first principles; a small change in the value of any constant would destroy the consistency of the theory." ([17], p. 189). Weinberg is convinced that "string theory has provided our first plausible candidate for a final theory." ([17], p.169.) But his general claims are essentially cosmological ones that are embedded into a big bang type framework. Only one fundamental arbitrariness might remain unexplained by the T.O.E.: the actual value of the cosmological constant Λ. Its existence is consistent with the general symmetry principles of Einstein's equations:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}. \tag{0.1}$$

Does this single undeterminable parameter already constitute a safe haven for Weinberg against Gödel's theorem? Λ appears to be the parameter distinguishing between several consistent T.O.E.'s. How is its measurement represented within the complete final theory? If it is not, there remains much more outside the system than just the numerical value of Λ.

At this point a mathematician or logician perhaps feels rather uneasy being faced with such a specific question without knowing whether the language Weinberg talks about can be gödelized at all. As Weinberg admits: "it is foolhardy to assume that one knows even the terms in which a future final theory will be formulated." ([17], p.137) But then why should there be a T.O.E. at all? As a matter of fact, Weinberg's justification consists in his philosophical convictions of an *objective reductionism* that "is simply true" ([17], p.42) because we can see by subdividing a piece of chalk (and have been taught by the history of physics) that the tinier parts contain the more fundamental physics. Moreover, Weinberg appears as a Platonist believing "in the reality of abstract ideas", in "the reality of the laws of nature." ([17], p.35)

Opponents to the idea of a T.O.E. argue by means of Gödel's theorem also on this general level of beliefs:

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* This paper is in its final form and no similar paper has been or is being submitted elsewhere.

> No matter how far mathematics progresses and no matter how many problems are solved, there will always be, thanks to Gödel, fresh questions to ask and fresh ideas to discover. It is my hope that we may be able to prove the world of physics as inexhaustible as the world of mathematics...If it should turn out that the whole of physical reality can be described by a finite set of equations, I would feel disappointed.([5], p.53)

Quite similar to Freeman Dyson, Tullio Regge concludes:"The universe is infinite not only in duration and extension, but also in its logical structure. Our existence itself and our reason are made possible by the infinity present in reality and from there disconnected fragments are mirrored back."([12], p. 296)

Certainly this never was Gödel's conclusion. Unlike those making use of his theorem, he believed in a capacity of human mind transcending the limits of finite machines and even in the possibility of an absolute mind. Thus, Gödel appears on the optimistic side about achieving objective knowledge, probably even believing in a T.O.E. But his views how such a theory looks like are quite different from those shared by physicists like Weinberg or Hawking. After sketching an 'intermediary' approach towards the T.O.E. question which attributes to the different levels we encounter in present physics more right of their own, I will discuss related thoughts of Gödel, both in the context of logic and cosmology, that support the quest for a more differentiated picture of present physics. I have chosen this order of presentation because it emphasizes how promising Gödel's ideas appear even today. They were written down at a time before high energy physics uncovered deeper and deeper levels of symmetry and successfully predicted particles of higher energies.

1. The T.O.E. and the Levels of Physical Theories

Advocates of a T.O.E. usually parallel the energy or length scale of 'local physics' tested by earthbound accelerators with the stage of evolution in the early universe. Thus, their approach presupposes big bang cosmology, which is, however, constantly questioned by various 'heretic' minority views (See [6] for an overview.). Furthermore, the more foundational theories are also more general in a mathematical sense, e.g. the symmetry groups of 'unified theories' of particle physics contain the standard model as a subgroup. Although T.O.E. proponents admit that the deduction of some particular phenomena from the T.O.E. might fail in practice, they leave no doubt what they consider as the primary duty. Mirroring this value-judgement, I have reserved for the more foundational levels, the 'higher' floors in a pyramid.

Studying concrete examples from physics [1] [14] [15] tells that in most cases 'reductionism' does not grant a pure deduction of the physics of a particular level from 'higher' levels exclusively. Instead one finds multifarious

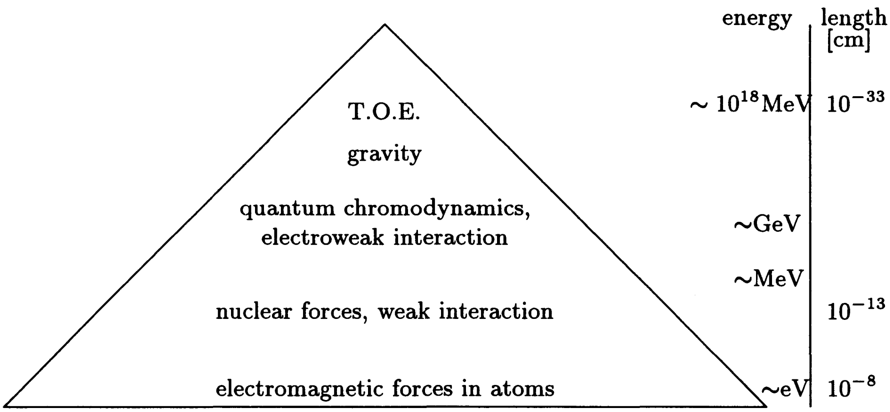

Fig. 1.1. The pyramid of physical laws implicit in T.O.E.

interrelations between the levels that can be summarized in the following theses initially proposed by Walter Thirring [14] [15]:

(i) The phenomena of a lower level are not completely determined by the laws of the higher level, even though they do not contradict the latter. What seems to be a fundamental fact on one level may seem entirely accidental if seen from a higher level.
(ii) The laws of lower levels depend more on the particular circumstances they refer to than on the laws of higher levels. Laws of the higher level, however, may be necessary to resolve internal ambiguities on a lower level.
(iii) The hierarchy of laws has evolved in the course of the evolution of the universe. Newly created laws initially did not exist as laws, but merely as possibilities.

George Ellis [6] distinguishes three types of relations between 'local physics' and cosmology: **a)** given the particular boundary or initial conditions the usual laws of physics are applicable to cosmology and yield properties independent the former, e.g. equilibrium distributions *(physics approach)*; **b)** the Cosmos is very special and has properties, such as the cosmological constant, that cannot be verified 'locally' *(Cosmos approach)*; **c)** the universe alters the laws of physics itself *(interaction approach)*. Thesis (iii) belong to the class c), but it is not intended to block further questions, saying: 'well it is so because it just happened in this way'. Some (not all) proclamations of the Anthropic Principle are in theis style. On the contrary, thesis (iii) should stimulate research in the physics of large systems, where unpredictable phase transitions entailing 'new laws' can be reasonably approached. Moreover, theses (i)–(iii) emphasize the particular role of undeducible fundamental constants that characterize a particular level. Similarly, the 'initial conditions' of a cosmological epoch might be tuned by symmetry-breaking.

Relating different levels is inhibited even more seriously if the concepts of one level neither can be reduced, nor translated into concepts of the other one.

Connecting different levels invokes a metalanguage. Of course, the T.O.E. would be a metatheory to all levels of physics because it defines all their concepts, tunes the fundamental constants, and describes all possible cosmological scenarios. But thesis (i)–(iii) imply that between two levels metalanguage relations exist in *both* directions. If a problem undecidable on the 'lower' level is represented and solved on the 'higher' level, the latter selects the *actual* out of the *possible* scenarios. On the other hand, if an unpredictable phase transition, or an undeducible fundamental constant, chiefly contribute to the explanation of a particular phenomenon, the 'lower' level represents the metatheory telling us that we have to choose a particular representation of the 'higher' level structure or disregard some solutions of its equations as empirically inadequate. This rather multifarious metalanguage problems already suggest that the pyramid is very far from being naively gödelized.

In view of the difficulties of deduction, physicists who are mainly interested in the solution of a problem resort to effective theories or coarse graining. In the latter case, a tinier level is represented by its mean value over a certain scale; in the former there are free parameters, apart from fundamental constants, that can be fitted to experimental data. Explicitly decoupling the levels in this way, the physics community is then kept together by common experimental and mathematical tools, only [13].

Although I concede that physics always sets out to explain particular phenomena, such a strategy, in effect, sacrifices possible deductions and does not exploit the unifying force of mathematics. In my view, mathematics provides the only separation between levels that is suffiently precise. Under certain conditions, quantum mechanical systems can be of macroscopic size. Theories of different levels coexist in one and the same cosmological epoch. On the contrary, the relation between the Lie groups $SU(10)$ and $SU(3) \times SU(2) \times U(1)$, or the relation between a non-commutative C*-algebra describing quantum mechanics and the commutative one of classical mechanics are sharply defined. But, what pyramid order can mathematics bring about for physics? Certainly we cannot substitute a mathematical T.O.E. for the physical one because this definitely contradicts Gödel's theorems. But, mathematics may well provide a 'local' order that comprises some levels and allows us to derive naturally (i.e. without further restrictive assumptions) their physical counterparts from simple first principles and 'simple' fundamental constants. The second type of simplicity holds, if we can relate one level of the mathematical formulation of the theory to a physical one by specifying just one typical parameter, for instance $\hbar$. Of course, the mathematical levels do not consist of entire mathematical disciplines, but of selected objects, concepts, and rules that are chosen to represent physical theory.

Having proposed this modest understanding of the level structure of present physics, I will move on to Gödel's ideas on the subject matter. Apart from the two papers on rotating universes, Gödel's published on matters of physics only a contribution to Einstein's Schilpp volume entitled "A remark about the relationship between relativity theory and idealistic philosophy". It was mainly devoted to the problem of defining cosmological time which turned out to be impossible for his solution of Einstein's equations. After other solutions allowing closed timelike curves even for $\Lambda = 0$ had been found, time became indeed a major problem of cosmology [6]. Recently also the T.O.E. candidate string theory contributed an (unrealizable) proposal for a time machine, apart from the popular (classically relativistic) wormholes. The problem how to protect grandfathers from being killed by their grandsons is quite popular, but unsolved at present.

If one reads through the unpublished papers and lectures of Gödel [8] from 1946 on, one is very surprised by the omnipresence of physics right within the central discussions on Platonism in mathematics. In effect, Gödel develops un understanding of physics that is quite close to that of modern mathematical physics that emerged in those days, not least at Princeton.

2. Objectivity in Physics and Mathematics

Arguments from physics appear quite often in Gödel's attempts to disprove the conventionalist thesis that mathematical concepts and rules are purely syntactical and void of content. Gödel's own incompleteness theorems had rendered untenable Hilbert's program to remain formalist, but simultaneously be convinced that every well-formulated mathematical yes-no alternative is in principle decidable. In the Gibbs lecture Gödel applies the undecidability argument to a finite Turing machine and poses the alternative: "Either mathematics is incomplete in this sense, that its evident axioms can never be comprised in a finite rule, that is to say, the human mind (even within the realm of mathematics) infinitely surpasses the powers of any finite [Turing] machine, or else there exist absolutely unsolvable diophantine problems." ([8], p.310) Gödel definitely objected to the second alternative, whereas his attitude towards the first is less clear (See the introduction of Boolos in [8]). Both imply Platonism, but in different aspects [10]. But Gödel continues that Platonism holds irrespective of the alternative because

> if mathematics were our free creation, ignorance as to the objects we created, it is true, might still occur, but only through lack of a clear realization as to what we have created (or, perhaps, due to the practical difficulty of too complicated computations). Therefore it would have to disappear... as soon as we attain perfect clearness,([8], p.314)

which, empirically, seems to be a daring hypothesis in mathematics. In physics, this argument has far-reaching consequences for non-Platonistic T.O.E. proponents: They have to accept strong reductionism, and everyone becomes a Laplacian demon (at least in principle). In this respect Weinberg's 'objective reductionism' is on the safe Platonistic side.

But can Gödel's logical considerations actually be applied to physics? At first Gödel credits Carnap's syntactical approach for "having pointed out the fundamental difference between mathematical and empirical truth"([8], p. 356f.) He even strengthens the distinction to ontological difference: Mathematical propositions "are true in virtue of *the concepts* occuring in them"([8], p. 356), while "space-time reality...is completely determined by the totality of particularities without any reference to the formal concepts"([8], p. 354). But Gödel makes a surprising second move. Carnap's syntax program added purely linguistic mathematics to empirical science, to 'factual' sentences already present. By the principle *ex falso quodlibet* any inconsistency, implies the truth also of all possible empirical sentences, even those contradicting observation. As of course, no empiricist can consider laws of nature as merely conventional, only those sentences are 'admissible' which do not imply unwanted empirical consequences. Goldfarb ([8], p. 328) reasonably (at least for a verificationist) detects a petitio in Gödel's argument. In order to know that the addition of mathematics does not change the empirical sentences, we need even stronger mathematics. Be that as it may, for Gödel, trusting in the consistency of mathematics has empirical consequences because we need mathematical concepts for expressing and structuring the laws of nature in the first place.

> If it is argued that mathematical propositions have no content because, by themselves they imply nothing about experiences, the answer is that the same is true of laws of nature. For laws of nature without mathematics or logic imply as little about experiences as mathematics without laws of nature. That mathematics, at least in most applications, does add something to the content of the laws of nature is at best seen from examples where one has very simple laws about certain elements, e.g., those about the the reactions of electronic tubes. Here mathematics clearly adds the general laws as to how systems of tubes connected in a certain manner will react. That the latter laws are not contained in the former is seen from the following facts: (a) The latter laws may contain concepts not definable in terms of those occuring in the former (e.g., the concept of a combination of any finite number of elements). (b) In order to understand the laws of nature it is sufficient, as far as the mathematical concepts occuring are concerned, to know rules which decide on their applying or not applying in each particular case. But such rules by no means imply the general laws governing them. (c) These general laws may even require new empirical inductions, namely, in case the mathematical problem in question should be unsolvable. E.g., this may occur in a case like Goldbach's Conjecture, which evidently implies a certain law about the reactions of a computing machine. Note that

> the *general* mathematical laws may even be required for predicting the result of a *single* observation, e.g., in case the latter depends on an infinity (e.g., a continuum) of physical elements... Mathematical propositions, it is true, do not express physical properties of the structures concerned, but rather properties of the *concepts* in which we describe those structures. But this only shows that the properties of those concepts are something quite as objective and independent of our choice as physical properties of matter. This is not surprising, since concepts are composed of primitive ones, which, as well as their properties, we can create as little as the primitive constituents of matter and their properties. ([8], p. 360)

Today one could cite the KAM-theorem as an example for the far-reaching consequences of 'mathematics proper' in physics. For analytically unsolvable many-body problems one can show that resonance properties depend more on number theory than on the particular form of the law of force [14] [15]. If the revolution period of an asteroid around the sun is in a 'small ratio' ($r = p/q \neq 1$, where p and q are small numbers) to that of Jupiter, its orbit is rather unstable. The most irrational number, the golden ratio, provides instead maximal stability.

What Gödel mentions under (a) has nowadays become well-known as 'emergence of new concepts by complexity'. For instance, a single atom is invariant under rotations. But being part of a crystal or a molecule, this symmetry is broken and the concept of an electric dipole moment becomes meaningful [1]. Gödel's example shows that 'lower' levels (on a larger length scale) can require new and very general mathematics. Case (b) occurs if one selects out of many possibilities a certain representation or a value of a fundamental constant without having a 'higher' level law fixing them necessarily. (c) is affirmed by the use of non-standard analysis in solving equations of statistical physics. As a matter of fact, Gödel highly appreciated non-standard analysis as it substantially simplified proofs of deep results and thus facilitated further discoveries. Its long-lasting omission "is largely responsible for the fact that, compared to the enormous development of abstract mathematics, the solution of concrete numerical problems was left far behind. ([7], p. 311)

'Mathematics proper' has proven more fruitful for T.O.E. candidates like string theory than in any other branch. Gödel requires an absolute concept of truth for this core of mathematics. For hypothetico-deductive systems, such as geometry, however, by a proof of undecidability the issue of Euclid's fifth postulate only remains meaningful because "the primitive terms are taken in a definite sense, i.e., as referring to the behavior of rigid bodies, rays of light, etc."([7], p. 267) As the meaning of a mathematical axiom may depend on physics, it may also "be disproved by wrong empirical consequences derived from it in conjunction with well-defined laws of nature."([8], p. 361) At this point the connection between mathematics and physics is already quite tight because falsification concerns a *single* observable fact which does not run into trouble within the syntax program ([8], p. 339, fn. 16); neither does a finite iteration. Nevertheless, without a notion of truth in 'mathematics proper' there would be no axiomatization of geometry ([8], p. 306).

Gödel draws parallels even further: "If mathematics describes an objective world just like physics, there is no reason why inductive methods should not be applied in mathematics just the same as in physics." ([8], p. 313) Thus, by rejecting the logical empiricist's syntax program mathematics "favor[s] the empiricist viewpoint" (ibid.) Moreover, if the attitude "to derive everything by cogent proofs from the definitions (that is, in ontological terminology, from the essences of things)..., if it claims monopoly, is as wrong in mathematics as it was in physics." (ibid.)

Even if we concede to Gödel that "if the objectivity of mathematics is assumed, it follows at once that its objects must be totally different from sensual objects" ([8], p. 312) because they are known precisely without invoking our five senses and say nothing about the space-time world, it remains to determine what objects space-time is about. Here Gödel follows Kant and states that we do not "imagine things to have these properties and relations [in space and time], but on the contrary to each of them there must correspond some *objective relation of the things to us* which subsists (independently of our representaions)" ([8], p. 232; my emphasis). On the other hand, he proposes a modification of Kant, "if one wants to establish agreement between his doctrines and modern physics; i.e., it should be assumed that it is possible for scientific knowledge, at least partially and step by step, to go beyond the appearances and approach the things in themselves" ([8], p. 257). In effect, Gödel lifts the strict border line that Kant's critical philosophy has drawn. "How can this point be deemed 'minor'?" (Howard Stein, [8], p. 224) Furthermore, does the argument really allow Gödel to maintain the strict distinction between mathematical and physical objects in themselves *and* to claim simultaneously that our perceptions and intuitions of those objects are quite similar in every step of objectification ? In particular, Gödel stresses that Kant's rejection of knowledge of things in themselves restricted natural science to "the forms of our sense perception." ([8], p. 244) Köhler [10] here rightly points out that the proper distinction of perceptions and senses is a source of trouble for Gödel's Platonism.

But why did Gödel attempt to give Kant this idealistic 'twist'? In effect, Gödel wanted to maintain the possibility of an 'absolute' cosmology in spite of his own discovery verifying the critical Kant by the relativity of time.

3. Time: How Gödel's Universe Reveals an Inadequacy of Hilbert's Program in Physics

Special relativity (and some vacuum solutions of Einstein's equations, too) treats all uniformly moving observers as equivalent and implies complete relativity of time. "The existence of matter, however,. . . largely distinguishes some of them conspicuously from the rest, namely, those which follow in their motion the mean motion of matter." ([7], p. 203f.) In the solutions known at Gödel's time, the local times of these observers could be fit together into a global cosmological time. Gödel's rotating universe, however, does not admit such an absolute definition of time. Moreover, it contains closed time-like curves that allow for, at least in principle, a travel far out to the horizon and back into one's own past. But, as Gödel emphasized ([8], p. 285), it retains a unique direction of time for any observer — a property not shared by all cosmological scenarios known today [6].

Gödel's solution thus extends the Kantian doctrine of time's being a subjective concept to general relativity. There cannot be any temporal order relation situated in the things in themselves that corresponds to our intuitive concept of time. As time does not 'lapse' or 'pass by' globally, there cannot be objective change either. For change presupposes existence; "The concept of existence, however, cannot be relativized [to observers] without destroying its meaning completely." ([7], p. 203, fn.5) As Gödel's solution is stationary, *spatial* order relations remain absolute, although one cannot separate out a a three-dimensional spacelike hypersurface. This conformity with our a priori concept of space is, however, is not given for other rotating solutions discovered after Gödel.

Gödel's incompleteness theorem rendered Hilbert's program for the axiomatic foundation of mathematics impossible. Gödel's rotating universe tells that also Hilbert's program for the axiomatization of physics failed. After Einstein had specified the symmetry properties of general relativity, Hilbert independently obtained Einstein's equations by way a variational principle ('Hilbert's action') for the special case of matter's being that of Mie's electrodynamics. It is therefore not surprising that Hilbert also proposed an axiomatization of all of field physics (Feldphysik) which he published under the exalted title "The Foundations of Physics" [9]. In order to fulfil the "newly emerged 'ideal of unity of field theory'" ([9], p. 47), Hilbert proposed four axioms: a unified action, general covariance, additivity of gravity and electromagnetism, and a metric axiom. Hilbert, erroneously, was convinced that space and time can be separated globally; thus he claimed that no contradictions arise if causality is defined by means of light cones. "From knowing the state variables at present, all future statements about them follow necessarily and uniquely, *provided they are physically meaningful*" ([9], p. 64), which required that every statement is essentially covariant. Hilbert furthermore held that "only regular solutions of the basic equations of physics represent reality directly" ([9], p. 73). Irregular solutions containing singularities were deemed as of mathematical interest, only. Restricting all matter to Mie's rather peculiar model, Hilbert's axiomatization appears even more narrow-minded. Thus, Einstein found it "too great an audacity to draw already now a picture of the world, since there are still so many things which we cannot remotely anticipate" (from [11], p. 261).

Hilbert's T.O.E. demonstrates an attitude toward the relation between basic axioms or equations and their solutions that is very different from Einstein's and Gödel's. The rotating universe teaches that particular solutions may exhibit drastically diverging properties that can also affect the physical meaning of the axioms. Of course, not in the sense as the incompleteness theorem affected truth, but by limiting the applicability of a physical concept used in the axioms to particular cases. The intuitive concept of time that appears in all dynamical considerations thus cannot be realized within the axiom system due to the existence of certain solutions, such as Gödel's. Should we a priori exclude them? Recent debates in relativity about naked singularities (microscopic black holes) also put the problem of regularity again on the agenda. Should we reject such solutions? Gödel rightly argues against such an apriorism. Basing the impossibility that we actually live in a Gödel universe on the closed time-like curves "presuppose[s] the actual feasibility of the journey into one's own past" ([7], p. 203). As the required loop is very very long, assuming at least its practical impossibility seems to be reasonable. Moreover, as Gödel explicitly acknowledges, his universe contradicts red-shift observations from distant objects. But the problem is a general one:

> The mere compatibility with the laws of nature of worlds in which there is no distinguished absolute time... throws some light on the meaning of time also in those worlds in which an absolute time *can* be defined. For... whether or not an objective lapse of time exists... depends on the particular way in which matter and its motion are arranged in the world. ([7], p. 206)

Gödel considers this situation philosophically unsatisfactory because "a lapse of time... would have to be founded, one should think, in the laws of nature" ([8], p. 238). "If, however, such a world time were to be introduced in these [rotating] worlds as a new entity, independent of all observable magnitudes, it would violate the principle of sufficient reason" ([8], p. 327).

Gödel seems to take an intermediate position. He simultaneously rejects an a priori selection of solutions *and* does not content himself merely in having a solution that agrees with observations. The reason for this lies, to my mind, in his joint effort to combine subjectivity (he even describes the neighborhood of a world line of matter as the region of sensorial contact) and objectivity in the strong sense, i.e. that we can get to the things in themselves. Gödel seems to believe that human beings can surmount the Kantian a priori forms of intuition: "Kant says that for beings with other forms of cognition 'those modifications which we represent to ourselves as changes would give rise to a perception in which the idea of time, and therefore also of change, would not occur at all'" ([8], p. 235). Kant, in fact, had often mentioned that there could be a 'discursive understanding' (though not ours) that perceives ideas. As Gödel held the human mind to be stronger than any computing machine, it is reasonable to conjecture a parallel approach to things in themselves also by a hierarchy of minds that are capable of intuting (mathematically as well as physically) stronger theories.

Hence one can imagine a companion of Laplace's demon for Gödel's concept of cosmology. Laplace's argument tacitly assumed that the motions of the particles of the world were governed by second order differential equations and that the law of gravitational attraction was of finite complexity. Otherwise the hypothesis of the demon only states that there is just *one* function describing all successive states of the *one* world. This form of overall determinism is, however, rather close to tautology. For this reason, I do not consider Howard Stein's argument (that is endorsed by Wang [16]) conclusive that auto-infanticide can never occur in a Gödel universe because "that act would simply not be possible... [because] such a cosmos would have to be regarded as fully deterministic" ([7], p. 199). Neither, perhaps, did Gödel, for he remarked: "A lapse of time, however, which is not a lapse in some definite way seems to me as absurd as a colored object which has no definite colors" ([7], p. 203). Since one cannot impose time from outside, the demon's companion does not calculate a cosmological time-evolution, but he has to determine the universe in a rather different manner. This has consequences for the relation between causality and determinism. In his discussions with Wang, Gödel considered the causal succession as more basic than temporal order (change): "The real idea behind time is causation: the time structure of the world is just its causal structure. Causation in mathematics, in the sense of, say, a fundamental theorem causing its consequences, is not in time, but we take it as a scheme in time... Causation is unchanging in time and does not imply change. It is an empirical— but not a priori—fact that causation is always accompanied by change." ([16], p. 229) As a matter of fact [6], the question about causality and time arises if one considers a primordial quantum state as the beginning of the universe. Accordingly, a universal theory for the demon's companion is neither the axiomatic T.O.E. of Weinberg or Hilbert, nor is it the the classically deterministic world of Stein that a priori prohibits causal loops. As Göedel remarks: "Quantum physics in particular seems to indicate that physical reality is something still more different from the appearances than even the four-dimensional Einstein-Minkowski world. T. Kaluza's fifth dimension points in the same direction" ([8], p. 240). The latter has become the origin of higher-dimensional cosmologies which, as 'higher' level theories, have to explain why our present space-time has been singled out, while the others collapsed to internal degrees of freedom [14].

4. Levels of Objectivation

The quote immediately above is preceded by the sentence: "Perhaps it [the space-time scheme of relativity] is to be considered as only one step beyond the appearances and towards the things (i.e. as one 'level of objectivation', to be followed by others[24]). [Footnote 24 starts:] Cf. in this connection *Bollert 1921*, where one may find a description in more detail of these steps or 'levels of objectivation', each of which is obtained from the preceding one by elimination of certain subjective elements... Kant's world of appearances itself, also must of course be considered as one such level, in which a great many subjective elements of the 'world of sensations' are already eliminated." ([8], p. 240) Karl Bollert subdivides

> the whole of our experiences into a succession of well-defined *levels of objectivation.* The endeavor to combine a constantly wider range of experiences, and finally all accessible ones, into a unity, forces us into more and more abstract systems of order, for the unity lost in widening the range of experiences can only be regained by abstracting from a part of the content of experience and restricting oneself to this which now still is at the basis of all single experiences in the same manner. What does not comply with this condition, remains situated on the lower level of objectivation. Thus, this extension of range is compensated for by an impoverishment of content... Every *level of objectivation* corresponds to a subject concept (Subjektsbegriff) as its complement, which becomes more and more concrete as we descend down the sequence of objectivations.([3], p. 49f.)
>
> Thus, the whole of experience appears as a system of connections conceptually nested into each other, as a sequence of levels of objectivation, among which there remains merely a relation of logical superordination and subordination.([3], p. 55)

According to Bollert, the hierarchy, however, is not a 'causal' one because not all concrete information is inherited upward. The lower levels cannot be deduced from the higher ones. Instead, there are two methods for obtaining knowledge. The *differential method* moves upward by abstraction until it obtains the differential equations as the sentences of the "book of nature" ([3], p. 60). But "the last level of objectivation [does not mark] the origin of all being [Urquell alles Seins], from which the all of reality could be deduced, but merely a complete calculatory connection." ([3], p. 65) While Bollert, strangely, considers this direction 'necessary and unique', as "the law of construction is taken from the richer content of the lower level itself", the *integral* method, "the inverse problem of *reintegration* [of the concrete] process is *ambiguous*" (both [3], p. 66). For instance, the fact that a metrically covariant four-dimensional manifold splits into space and time is accidental and due to the circumstance that the metric components can be regarded as constant in weak gravitational fields prevailing in most places. Thus, Bollert puts his finger on the above-mentioned problem of higher-dimensional cosmologies.

In his argument against conventionalism Gödel applies the subjective-objective distinction to mathematics. Mathematics in the *subjective* sense is "the system of all demonstrable propositions", mathematics in the *objective* sense is "the system of all true mathematical propositions" (both [8], p. 309).

While a well-defined system of mathematical axioms can contain the former, Gödel's theorems prove that this is false for the latter. And they also show that no analogue of Bollert's allegedly unique 'differential' method can exist in mathematics (neither is this the case in physics). In a conversation with Hao Wang on 9 December, 1975, Gödel talks about the formation of concepts and sets: "In some sense the subjective view leads to the objective view. Subjectively [a set is] something you can overview in one thought. If we overview a multitude of objects in one thought in our mind, then this whole contains also as a part the objective unity of the multitude of objects , as well as its relation to our thought. Hence it is natural to assume a common nucleus which is the objective unity... From the idealized subjective view, we can get the power set [i.e., overview the power set of any set which we can overview]. But the definability of V [the set of all sets] can't be got by the subjective view at all. The difference in strength [is exemplified] only when you introduce new principles which make no sense at all in the subjective view." (from [10], p. 18; [] are Wang's) An analogous difference in strength separates levels in physics. 'Higher' levels require stronger mathematics, but thereby reduce the complexity of 'lower' ones. Fundamental constants cannot be generalized without loss of information, i.e. their particular value, for concepts pertaining to them, e.g. an atom for the finestructure constant, lose their meaning on the subatomic level. Thus only part of a 'lower' level can be deduced with mathematical rigor. This difficulty adds to the purely mathematical one of consistency being provable only at the 'higher' level.

This restriction is even in some sense wanted because, in fact, mathematics has to be able to express situations not actualized. "For the main function of mathematics... is exactly to bring the vast manifold of possible situations and particularities of the existent under control." ([8], p. 352). The objective level is thus the common core of the subjective lower levels, the possible worlds. They are not of entirely different nature, since in Göedel's ontology "Possibility is a weakened form of being." ([16], p. 229) By gödelization a subset of the true sentences of the higher level can already be represented on the lower one. Thus, "*subjective intellectual* things (intentions, e.g. concepts and proofs) represented on a lower level may appear as *objective natural* things (bodies or processes) represented on a higher level." ([10], p. 20f.) This shows that mathematical levels never can be separated 'hermetically'; there is always a possibility to reflect parts of every meta-theoretical level on a 'lower' level.

5. Intuition and Induction

> That mathematics *does* have a content... appears from the fact that, in whatever way it, or any part of it, is built up, one always needs certain undefined terms and certain axioms (i.e., deductively unprovable assertions) about them. *For these axioms there exists no*

> *other rational (and not merely practical) foundation except either that they (or propositions implying them) can directly be perceived to be true (owing to the meaning of the terms or by an intuition of the objects falling under them), or that they are assumed (like physical hypotheses) on the grounds of inductive arguments, e.g., their success in the applications... To eliminate mathematical intuition or empirical induction by positing the mathematical axioms to be true by convention is not possible*([8], p. 346f.),

in virtue of the incompleteness theorem. For Gödel, intuition is not some aesthetic epiphenomenon that could easily be disregarded. Instead it shares the level structure of mathematics because for the consistency assumption for any axiom system one needs an intuition of appropriate strength. In a talk never held before the American Philosophical Society, Gödel credits Husserl for having formulated more precisely Kant's idea that, for the derivation of mathematical theorems, we always need new mathematical intuitions. Husserl's phenomenology represents, for Gödel, a promising "systematic method [a technique, not a science proper] for a such a clarification of meaning... [that] consists in... directing our attention... [toward] our own acts in the use of these concepts"([8], p. 383). In analogy to a child that "passes through states of consciousness of various heights... it seems quite possible that a systematic and conscious advance... will also far exceed the expectations one may have a priori."([8], p. 385) Intuition is not directed to 'things' (perhaps, in themselves) but to relations, both in physics and mathematics. Gödel emphasizes this already in a quote from Kant: "the mode of intuition of the subject *in the relation* of the given object to it"([8], p. 231). To stress both that intuition is relational and able to surmount our a priori 'lifeworld' is quite important, for in present foundational debates about quantum theory a different 'intuition' has become popular [2] that has, to my mind, great disadvantages. Gödel's Platonism justifies the objectivity of intuition as a "special kind of experience"([8], p. 351) by admitting fallibilism, which turns out to be a 'surprisingly strong principle' [10].

"However, mathematical intuition in addition produces the conviction that, if these sentences express observable facts and were obtained by applying mathematics to verified physical laws (or if they express ascertainable mathematical facts), then these facts will be brought out by observation (or computation)."([8], p. 340) In this way the certainty guaranteed by mathematical intuition is the basis for the reliability of all physical predictions. But the relation can be inverted: Empirical evidence also justifies our belief in the truth of mathematical axioms.

Already in mathematics inductive methods are used, for instance, when an equality $F(n) = G(n)$ is verified up to very large n. But induction plays a much more general role. It may well be that properties of concepts of mathematical physics "referring to combinations of things... may *not* follow from the definitions or the meanings of the terms (as far as we are able to understand them) but still may be knowable in the same sense as laws of nature". Accordingly, mathematical properties are "even verifiable by sense experience under the hypothesis that certain laws of nature which can be confirmed independently of mathematics hold good"(both [8], p. 349).

Induction as a truth criterion rests not only upon probability estimates (as in Carnap's case), but also invokes the classical principles of theory choice: "fruitfulness in mathematics and...in physics"([7], p. 269): a fruitful new axiom allows one to make proofs "considerably simpler and easier to discover"([7], p. 261), and to contract into one many proofs of such consequences that were already demonstrable without the new axiom—others cannot be compared. In a parenthesis stricken out in the Gibbs lecture (which belongs to [8], p. 313) Gödel mentions other conceivable inductive criteria: "[Others might be based on the (requirement of simplicity, aesthetic value combined with symmetry, plausibility, fruitfulness of a general prop[erty]) of the fundamental law of i.e. the axioms]" (from [10], p. 29). These are exactly those criteria of which physicists claim to have an intuitive understanding.

It is a common demand that the T.O.E. should be expressed in a few simple principles, as it is the case in relativity theory or in C*-algebraic quantum theory. But not all physics can rest upon them. Gödel's basing the belief in mathematical axioms on their empirical implications teaches that one should, perhaps, even be glad that in order to make the principles operative in explaining physical phenomena, one has to supply further information. Insisting on the fallibilism of intuition and induction allows Gödel, however, to maintain a T.O.E. program in the weak (objective) sense. As all precisely formulated questions should ultimately be decidable (though, perhaps, not provable) his concept appears similar to Duhem's 'natural order' [4]. Gödel's 'natural order' comprises many levels of objectivation that are found by induction, simple principles as well as particular solutions. Thus, in contrast to Hilbert, Gödel does not appear as a proponent of a world formula or of a 'world action principle'.

Acknowledgement. A substantial part of the ideas presented here was motivated by discussions with Walter Thirring and Eckehart Köhler.

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