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KODAI MATH. J.
38 (2015), 687–689

CORRECTION TO "SURFACES WITH PARALLEL MEAN CURVATURE VECTOR IN $P^2(C)$"

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Abstract

We describe a condition under which the claims in the paper cited above hold.

1. Correction

It has been pointed out by Hirakawa [3] that a previous paper by Ogata [5] contained a mistake. In fact, the claim made in line 3, page 401 in [5], which states that "λ is a real-valued function defined on U," is not generally correct. We now give a geometric condition for the claim to hold. We follow the notation used in [5].

Lemma. *Suppose that the immersion in* [5] *satisfies a condition $a = \bar{a}$ on M. Then, there exists a complex coordinate w on a neighborhood of a point of M such that $\phi = \mu\, dw$, where μ is real valued.*

Using this lemma, we can state the following:

Correction. *For the claims given in* [5] *to hold, we add the condition $a = \bar{a}$ to the immersion.*

Since Kenmotsu and Zhou [4] and Hirakawa [2] used the results given by Ogata [5], those papers also need the additional assumption $a = \bar{a}$ for the immersion.

2. Proof of Lemma

Set $\phi = \lambda\, dz$, where λ is a non-zero complex valued function on a simply connected domain U with complex coordinate z. Although the lemma can be

2010 *Mathematics Subject Classification*. 53C42, 53B25.
Key words and phrases. Parallel mean curvature vector, complex space form.
The first author is partly supported by JSPS Grant-in-Aid for Scientific Research (C-25400062).
Received December 24, 2014; revised April 3, 2015.

proved using (2.4) in [5], we employ a slightly modified formula here. By (2.4) of [5], we have

$$\lambda_{\bar{z}} = -\lambda\bar{\lambda}(\bar{a} - b) \cot \alpha, \tag{2.1}$$

$$\alpha_{\bar{z}} = \bar{\lambda}(\bar{a} + b), \tag{2.2}$$

$$a_{\bar{z}} = \bar{\lambda}\left(2a(\bar{a} - b) + \frac{3\rho}{2} \sin^2 \alpha\right) \cot \alpha, \tag{2.3}$$

$$c_z = 2\lambda c(a - b) \cot \alpha. \tag{2.4}$$

We note that (2.8) in [5] is not generally correct.

First, we prove the lemma for the case in which α is constant on M. By (2.2), we have $a = -b = \bar{a} =$ constant. By (2.6) of [5], $|c|^2$ is constant. Set $c = |c| \exp(i\theta)$, where θ is a real-valued function on U. Then, using (2.4), we have $i\theta_z = -4b\lambda \cot \alpha$. If we take the partial derivative with respect to $\bar{z}$, then (2.1) can be used to obtain $8b^2\lambda\bar{\lambda} \cot^2 \alpha + i\theta_{z\bar{z}} = 0$. Since $\theta_{z\bar{z}}$ is real valued, this implies $\cot \alpha = 0$. Therefore, we have $\lambda_{\bar{z}} = 0$ by (2.1). Hence, λ is holomorphic. Define the complex coordinate w as $w = \int \lambda\, dz$. Then, we have $\phi = \lambda\, dz = dw$, which proves the lemma for the case $\alpha =$ constant.

When α is not constant, we need the following claim to prove the lemma:

Claim. Suppose that $a = \bar{a}$ on M. If α is not constant, then a is a function of α.

Proof. By the assumption, we see $a_z = (\bar{a})_z = \overline{a_{\bar{z}}}$. By (2.2) and (2.3), we have

$$d\alpha = (a + b)(\phi + \bar{\phi}),$$
$$da = \left(2a(a - b) + \frac{3}{2}\rho \sin^2 \alpha\right) \cot \alpha \cdot (\phi + \bar{\phi}).$$

Canceling out $(\phi + \bar{\phi})$ in the above formulas, we have a differential equation in a for α, which proves the claim.

Proof of Lemma. Using the above claim, we write $a = a(\alpha)$, and define a real-valued function $F(\alpha)$ as

$$F(\alpha) = \frac{(a(\alpha) - b)^2 + 3\rho/2 \sin^2 \alpha}{(a(\alpha) + b)^2} \cot \alpha.$$

Taking the partial derivative of (2.2) with respect to z and using (2.1) and (2.3), we have a second-order partial differential equation $\alpha_{z\bar{z}} - F(\alpha)\alpha_z\alpha_{\bar{z}} = 0$. It follows that $(\alpha_z \exp(-\int F(\alpha)\, d\alpha))_{\bar{z}} = 0$. Hence, there exists a holomorphic function $G(z)$ on U such that $\alpha_z = G(z) \exp(\int F(\alpha)\, d\alpha)$. Setting

$$w = \int G(z)\, dz, \quad \mu = \frac{\exp(\int F(\alpha)\, d\alpha)}{a(\alpha) + b},$$

the lemma is proved by the conjugate of (2.2).

Remark. Briefly, we explain the geometric meanings for these quantities used in (2.1)–(2.4). The real valued function α is the Kaehler angle of the immersion, the positive number b is two times of the length of the mean curvature vector, and the complex valued functions a and c determine the second fundamental tensors of the immersion. The ambient space is a complex 2-dimensional Kaehler manifold of constant holomorphic sectional curvature 4ρ. These were first introduced in Chern and Wolfson [1].

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