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ON $(f, g, u_{(k)}, \alpha_{(k)})$-STRUCTURES

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§ 0. Introduction.

Yano and Okumura [6] have studied hypersurfaces of a manifold with (f, g, u, v, λ)-structure. These submanifolds admit under certain conditions what we call an $(f, g, u_{(k)}, \alpha_{(k)})$-structure. In particular, a hypersurface of an even-dimensional sphere carries an $(f, g, u_{(k)}, \alpha_{(k)})$-structure (see also Blair, Ludden and Okumura [2]). Submanifolds of codimension 2 in an almost contact metric manifold also admit the same kind of structure (see Yano and Ishihara [5]).

The main purpose of the present paper is to study the $(f, g, u_{(k)}, \alpha_{(k)})$-structure and hypersurfaces of an even-dimensional sphere. In § 1, we define and discuss $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure and $(f, g, u_{(k)}, \alpha_{(k)})$-structure. In § 2, we recall the definition of (f, g, u, v, λ)-structure and give examples of the manifold with $(f, g, u_{(k)}, \alpha_{(k)})$-structure. In § 3, we study non-invariant hypersurfaces of a manifold with normal (f, g, u, v, λ)-structure. In the last section, we study hypersurfaces of an even-dimensional sphere S^{2n} under certain conditions by using of the following theorem proved by Ishihara and one of the present authors [3]:

THEOREM A. *Let (M, g) be a complete and connected hypersurface immersed in a sphere $S^{m+1}(1)$ with induced metric g_{ji} and assume that there is in (M, g) an almost product structure P_i^h of rank p such that $\nabla_j P_i^h=0$. If the second fundamental tensor H_{ji} of the hypersurface (M, g) has the form $H_{ij}=aP_{ji}+bQ_{ji}$, a and b being non-zero constants, where $P_{ji}=P_j^t g_{it}$ and $Q_{ji}=g_{ji}-P_{ji}$, and, if $m-1\geqq p\geqq 1$, then the hypersurface (M, g) is congruent to the hypersurface $S^p(r_1)\times S^{m-p}(r_2)$ naturally embedded in $S^{m+1}(1)$, where $1/r_1^2=1+a^2$ and $1/r_2^2=1+b^2$.*

§ 1. $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure.

Let M be an m-dimensional differentiable manifold of class C^∞. We assume there exist on M a tensor field f type (1,1), vector fields U, V and W, 1-forms u, v and w, functions α, β and γ satisfying the conditions (1.1)~(1.7):

$$f^2X=-X+u(X)U+v(X)V+w(X)W \tag{1.1}$$

for any vector field X,

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(1.2) $$fU=-\gamma V+\beta W\,, \qquad u\circ f=\gamma v-\beta w\,,$$

(1.3) $$fV=\gamma U+\alpha W\,, \qquad v\circ f=-\gamma u-\alpha w\,,$$

(1.4) $$fW=-\beta U-\alpha V\,, \qquad w\circ f=\beta u+\alpha v\,,$$

where 1-forms $u\circ f$, $v\circ f$ and $w\circ f$ are respectively defined by

$$(u\circ f)(X)=u(fX)\,, \qquad (v\circ f)(X)=v(fX)\,, \qquad (w\circ f)(X)=w(fX)$$

for any vector field X, and

(1.5) $$u(U)=1-\beta^2-\gamma^2\,, \qquad u(V)=-\alpha\beta\,, \qquad u(W)=-\alpha\gamma\,,$$

(1.6) $$v(U)=-\alpha\beta\,, \qquad v(V)=1-\alpha^2-\gamma^2\,, \qquad v(W)=\beta\gamma\,,$$

(1.7) $$w(U)=-\alpha\gamma\,, \qquad w(V)=\beta\gamma\,, \qquad w(W)=1-\alpha^2-\beta^2\,.$$

In this case, we say that the manifold M has *an $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure.* We first prove

LEMMA 1.1. *In a manifold with $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure, the vectors U, V and W (or the covectors u, v and w) are linearly dependent if and only if*

$$\alpha^2+\beta^2+\gamma^2=1\,.$$

Proof. If there are three numbers a, b and c such that

$$aU+bV+cW=0\,,$$

then, using (1.5), (1.6) and (1.7), we obtain

$$(1-\beta^2-\gamma^2)a-\alpha\beta b-\alpha\gamma c=0\,,$$

$$-\alpha\beta a+(1-\alpha^2-\gamma^2)b+\beta\gamma c=0\,,$$

$$-\alpha\gamma a+\beta\gamma b+(1-\alpha^2-\beta^2)c=0\,.$$

Since we obtain

(1.8) $$\det\begin{vmatrix} 1-\beta^2-\gamma^2 & -\alpha\beta & -\alpha\gamma \\ -\alpha\beta & 1-\alpha^2-\gamma^2 & \beta\gamma \\ -\alpha\gamma & \beta\gamma & 1-\alpha^2-\beta^2 \end{vmatrix}=(1-\alpha^2-\beta^2-\gamma^2)^2\,,$$

we can immediately derive our result.

In the next place, we prove that a manifold with $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure is odd-dimensional. Let P be a point of M at which $\alpha^2+\beta^2+\gamma^2\neq 1$. Then the vectors U, V and W are linearly independent at this point P by virture of Lemma 1.1. Thus we can choose m linearly independent vectors $X_1=U$, $X_2=V$, $X_3=W$, $X_4, \cdots, X_m$ which span the tangent space $T_p(M)$ of M at P and such that $u(X_i)=0$, $v(X_i)=0$ and $w(X_i)=0$, for $i=4, \cdots, m$. Consequently, we have from (1.1)

$$f^2X_i=-X_i\,,\qquad i=4,\cdots,m\,,$$

which shows that f is an almost complex structure in the subspace V_p of $T_p(M)$ at P spanned by $X_4,\cdots,X_m$ and that V_p is even-dimensional. Thus $T_p(M)$ is odd-dimensional.

Next, let P be a point of M at which $\alpha^2+\beta^2+\gamma^2=1$. Then u, v and w are linearly dependent at this point by virtue of Lemma 1.1. Let say,

$$u=av+bw\,, \tag{1.9}$$

where a and b are numbers. Then, from (1.2), (1.3), (1.4) and $u\circ f=av\circ f+bw\circ f$, we have

$$\gamma v-\beta w=-a\gamma(av+bw)-a\alpha w+b\beta(av+bw)+b\alpha v\,,$$

or,

$$0=(-\gamma-a^2\gamma+ab\beta+b\alpha)v+(\beta-ab\gamma-a\alpha+b^2\beta)w\,. \tag{1.10}$$

Moreover, from (1.9), we get $u(W)=av(W)+bw(W)$, or, using (1.7),

$$-\alpha\gamma=a\beta\gamma+b\gamma^2 \tag{1.11}$$

by virtue of $\gamma^2=1-\alpha^2-\beta^2$.

If $\gamma(P)\neq 0$, then, from (1.10) and (1.11), we find

$$0=-(1+a^2+b^2)\gamma v+(1+a^2+b^2)\beta w\,.$$

This means that any two of covectors u, v and w are also linearly dependent at this point. Since $w\neq 0$ at P, we can choose m linearly independent covectors $w_1=w$, w_2, $w_3,\cdots,w_m$ which span the cotangent space ${}^cT_p(M)$ of M at P. We denote the dual basis by $(X_1,\cdots,X_m)$. Then we have

$$f^2X_i=-X_i\,,\qquad i=2,3,\cdots,m\,,$$

which shows that f is an almost complex structure in the subspace V_p of $T_p(M)$ which is spanned by $X_2,\cdots,X_m$ and that dim V_p=even, and consequently $T_p(M)$ is of odd-dimensional.

If $\gamma(P)=0$, then $\beta(P)\neq 0$ because of $\alpha^2=-a\alpha\beta-b\alpha\gamma$ and $\alpha^2+\beta^2+\gamma^2=1$. Moreover, from $-\alpha\beta=a\beta^2+b\beta\gamma$ and (1.10), we have $0=(ab\beta+b\alpha)v+(1+a^2+b^2)\beta w$.

On the other hand, two covectors u and v are not zero at the same time. Thus we can get the same result as above in this case.

The cases left to examine are in which

$$v=a_1u+b_1w\,,\qquad w=a_2u+b_2v\,,$$

where a_i's and b_i's $(i=1,2)$ are numbers. But, in these cases, we can also prove the same results as above by the similar method. Thus we have

THEOREM 1.2. *A differentiable manifold with $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure is odd-dimensional.*

Suppose that $(2n-1)$-dimensional manifold M has an $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure. Now, we consider the product manifold $M\times R^3$, R^3 being a 3-dimensional Euclidean space. We define in $M\times R^3$ a tensor field F of type (1, 1) with local components $F_B{}^A$ given by

$$(F_B{}^A)=\begin{bmatrix} f_c{}^b & U^a & V^a & W^a \\ -u_c & 0 & -\gamma & \beta \\ -v_c & \gamma & 0 & \alpha \\ -w_c & -\beta & -\alpha & 0 \end{bmatrix} \tag{1.12}$$

in $\{N\times R^3, x^A\}$, $\{N, x^a\}$ being a coordiante neighborhood of M and $x^{\bar{1}}$, $x^{\bar{2}}$, $x^{\bar{3}}$ Cartesian coordiantes in R^3, where $f_c{}^a$, U^a, V^a, W^a, u_c, v_c and w_c are respectively local components of f, U, V, W, u, v and w in $\{N, x^a\}$. (The indices $A, B, C, \cdots$ run over the range $\{1, 2, \cdots, 2n+2\}$ and a, b, c, d, e run over the range $\{1, 2, \cdots, 2n-1\}$. We denote $2n$, $2n+1$, $2n+2$ by $\bar{1}$, $\bar{2}$ and $\bar{3}$ respectively.) Then, taking account of (1.1)$\sim$(1.7), we can easily check that $F^2=-I$ holds in $M\times R^3$. Thus we have

PROPOSITION 1.3. *If there is given an $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure in M, then the tensor field F defined by* (1.12) *is an almost complex structure in $M\times R^3$.*

Denoting $\partial/\partial x^A$ by ∂_A, then Nijenhuis tensor $[F, F]$ of F has local components

$$S_{CB}{}^A=F_C{}^E\partial_E F_B{}^A-F_B{}^E\partial_E F_C{}^A-(\partial_C F_B{}^E-\partial_B F_C{}^E)F_E{}^A \tag{1.13}$$

in $M\times R^3$. Thus, using (1.12), we can write down $S_{CB}{}^A$ as follows;

$$\begin{aligned} S_{cb}{}^a=&f_c{}^e\partial_e f_b{}^a-f_b{}^e\partial_e f_c{}^a-(\partial_c f_b{}^e-\partial_b f_c{}^e)f_e{}^a\\ &+(\partial_c u_b-\partial_b u_c)U^a+(\partial_c v_b-\partial_b v_c)V^a\\ &+(\partial_c w_b-\partial_b w_c)W^a, \end{aligned} \tag{1.14}$$

$$\begin{aligned} S_{cb}{}^{\bar{1}}=&-f_c{}^e\partial_e u_b+f_b{}^e\partial_e u_c+u_e(\partial_c f_b{}^e-\partial_b f_c{}^e)\\ &-\gamma(\partial_c v_b-\partial_b v_c)+\beta(\partial_c w_b-\partial_b w_c), \end{aligned} \tag{1.15}$$

$$\begin{aligned} S_{cb}{}^{\bar{2}}=&-f_c{}^e\partial_e v_b+f_b{}^e\partial_e v_c+v_e(\partial_c f_b{}^e-\partial_b f_c{}^e)\\ &+\gamma(\partial_c u_b-\partial_b u_c)+\alpha(\partial_c w_b-\partial_b w_c), \end{aligned} \tag{1.16}$$

$$\begin{aligned} S_{cb}{}^{\bar{3}}=&-f_c{}^e\partial_e w_b+f_b{}^e\partial_e w_c+w_e(\partial_c f_b{}^e-\partial_b f_c{}^e)\\ &-\beta(\partial_c u_b-\partial_b u_c)-\alpha(\partial_c v_b-\partial_b v_c), \end{aligned} \tag{1.17}$$

$$\begin{aligned} S_{c\bar{1}}{}^a=&f_c{}^e\partial_e U^a-U^e\partial_e f_c{}^a-(\partial_c U^e)f_e{}^a-(\partial_c\gamma)V^a\\ &+(\partial_c\beta)W^a, \end{aligned} \tag{1.18}$$

(1.19) $$S_{c\bar{2}}{}^{a}=f_c{}^e\partial_e V^a-V^e\partial_e f_c{}^a-(\partial_c V^e)f_e{}^a+(\partial_c\gamma)U^a +(\partial_c\alpha)W^a\,,$$

(1.20) $$S_{c\bar{3}}{}^{a}=f_c{}^e\partial_e W^a-W^e\partial_e f_c{}^a-(\partial_c W^e)f_e{}^a-(\partial_c\beta)U^a -(\partial_c\alpha)V^a\,,$$

$$\cdots.$$

Specially, if $S_{cb}{}^a=0$, then we say that the $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure is *normal.* We assume that, in M with $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$-structure, there exists a positive definite Riemannian metric g such that

(1.21) $$g(U, X)=u(X)\,,\qquad g(V, X)=v(X)\,,\qquad g(W, X)=w(X)$$

and

(1.22) $$g(fX, fY)=g(X, Y)-u(X)u(Y)-v(X)v(Y)-w(X)w(Y)$$

for any vector fields X and Y of M. We call such a structure a *metric* $(f, U_{(k)}, u_{(k)}, \alpha_{(k)})$*-structure* and denote it by $(f, g, u_{(k)}, \alpha_{(k)})$.

Finally, we define a tensor field of type (0, 2) of M by

(1.23) $$\theta(X, Y)=g(fX, Y)$$

for any vector fields X and Y of M. Then we can easily verify that

(1.24) $$\theta(X, Y)=-\theta(Y, X)$$

because of (1.1)$\sim$(1.4) and (1.21)$\sim$(1.23).

§ 2. Examples.

Let $\tilde{M}$ be a $2n$-dimensional differentiable manifold with (f, g, u, v, λ)-structure, that is, a Riemannian manifold admitting a tensor field $f_i{}^h$ of type (1, 1), Riemannian metric g_{ji}, two 1-forms u_i and v_i (or two vector fields $u^h=u_i g^{ih}$ and $v^h=v_i g^{ih}$) and a function λ which satisfy

(2.1) $$\begin{cases} f_t{}^h f_j{}^t=-\delta_j{}^h+u_j u^h+v_j v^h\,, \\ f_j{}^t f_i{}^s g_{ts}=g_{ji}-u_j u_i-v_j v_i\,, \\ f_j{}^t u_t=\lambda v_j\,, \qquad f_j{}^t v_t=-\lambda u_j\,, \\ u^t f_t{}^h=-\lambda v^h\,, \qquad v^t f_t{}^h=\lambda u^h\,, \\ u_t u^t=v_t v^t=1-\lambda^2\,, \qquad u_t v^t=0\,, \end{cases}$$

where $(g^{ji})=(g_{ji})^{-1}$, here and in the sequel the indices $h, j, i, \cdots$ running over the range $\{1, 2, \cdots, 2n\}$.

If we put $f_{ji}=f_j{}^t g_{ti}$, we can easily see that f_{ji} is skew-symmetric.

We put

$$S_{ji}{}^{h}=[f, f]_{ji}{}^{h}+(\nabla_{j}u_{i}-\nabla_{i}u_{j})u^{h}+(\nabla_{j}v_{i}-\nabla_{i}v_{j})v^{h}\,, \tag{2.2}$$

$[f, f]_{ji}{}^{h}$ denoting the Nijenhuis tensor formed with $f_{i}{}^{h}$ and ∇_{i} the operator of covariant differentiation with respect to the Christoffel symbols $\left\{{h \atop ji}\right\}$ formed with g_{ji}. If $S_{ji}{}^{h}$ vanishes, it is said that the (f, g, u, v, λ)-structure is *normal* ([7]).

The following theorem is well known (cf. [4], [8]):

THEOREM 2.1. *Let $\tilde{M}$ be a manifold with normal (f, g, u, v, λ)-structure satisfying $\nabla_{j}v_{i}-\nabla_{i}v_{j}=2f_{ji}$ (or equivalently $\nabla_{j}u_{i}+\nabla_{i}u_{j}=-2\lambda g_{ji}$). If the function $\lambda(1-\lambda^{2})$ does not vanish almost everywhere, then we have*

$$\nabla_{j}f_{i}{}^{h}=g_{ji}(\phi u^{h}-v^{h})-\delta_{j}{}^{h}(\phi u_{i}-v_{i})\,, \tag{2.3}$$

$$\nabla_{j}u_{i}=-\lambda g_{ji}-\phi f_{ji}\,, \qquad \nabla_{j}v_{i}=-\phi\lambda g_{ji}+f_{ji}\,, \tag{2.4}$$

$$\nabla_{j}\lambda=u_{j}+\phi v_{j}\,, \tag{2.5}$$

ϕ being constant. Moreover, if $\tilde{M}$ is complete and $\dim \tilde{M}>2$, *then $\tilde{M}$ is isometric with an even-dimensional sphere.*

An even-dimensional sphere S^{2n} induces a normal (f, g, u, v, λ)-structure and satisfies differential equations (2.3)$\sim$(2.5) with $\phi=0$ (cf. [1]).

We consider a $(2n-1)$-dimensional manifold M covered by a system of coordinate neighborhoods $\{U; x^{a}\}$, where here and throughout the paper the indices $a, b, c, d, e, \cdots$ run over the range $\{1, 2, \cdots, 2n-1\}$. We assume that the manifold M is immersed in $\tilde{M}$ by the immersion ι: $M\rightarrow\tilde{M}$ as a hypersurface $i(M)$ of $\tilde{M}$ and that the equations of $i(M)$ in $\tilde{M}$ are

$$y^{h}=y^{h}(x^{b})\,.$$

If we put $B_{b}{}^{h}=\partial_{b}y^{h}(\partial_{b}=\partial/\partial x^{b})$, then the Riemannian metric induced on $i(M)$ from that of $\tilde{M}$ is given by $g_{cb}=g_{ji}B_{c}{}^{j}B_{b}{}^{i}$. We identify $i(M)$ with M itself.

Moreover, if we choose a unit vector N^{h} of $\tilde{M}$ normal to M in such a way that $2n$ vectors $B_{b}{}^{h}$, N^{h} give the positive orientation of $\tilde{M}$, then the transforms $f_{i}{}^{h}B_{b}{}^{i}$ of $B_{b}{}^{i}$ by $f_{i}{}^{h}$ can be expressed as linear combinations of $B_{e}{}^{h}$ and N^{h}, that is,

$$f_{i}{}^{h}B_{b}{}^{i}=f_{b}{}^{e}B_{e}{}^{h}+w_{b}N^{h}\,, \tag{2.6}$$

where $f_{b}{}^{e}$ is a tensor field of type (1, 1) and w_{b} is a 1-form on M. Similarly, the transform $f_{i}{}^{h}N^{i}$ of N^{i} by $f_{i}{}^{h}$ and vectors u^{h}, v^{h} can be written as

$$f_{i}{}^{h}N^{i}=-w^{e}B_{e}{}^{h}\,, \tag{2.7}$$

$$u^{h}=u^{e}B_{e}{}^{h}+\beta N^{h}\,, \tag{2.8}$$

$$v^{h}=v^{e}B_{e}{}^{h}+\alpha N^{h}\,, \tag{2.9}$$

where $w^e = w_a g^{ae}$, u^e and v^e are vectors, α and β are functions on M.

Transvecting (2.6) with $f_h{}^j$ and taking account of (2.1), (2.6) itself and (2.7), we find

$$(-\delta_i{}^j + u_i u^j + v_i v^j) B_b{}^i = f_b{}^e (f_e{}^a B_a{}^j + w_e N^j) + w_b(-w^a B_a{}^j),$$

or, using (2.8) and (2.9),

$$\begin{aligned} -B_b{}^j + u_b(u^a B_a{}^j + \beta N^j) + v_b(v^a B_a{}^j + \alpha N^j) \\ = f_b{}^e (f_e{}^a B_a{}^j + w_e N^j) + w_b(-w^a B_a{}^j), \end{aligned}$$

from which,

$$f_b{}^e f_e{}^a = -\delta_b{}^a + u_b u^a + v_b v^a + w_b w^a, \tag{2.10}$$

$$f_b{}^e w_e = \beta u_b + \alpha v_b. \tag{2.11}$$

Transvecting (2.7) with $f_h{}^j$ and making use of (2.1), (2.6), (2.8) and (2.9), we have

$$w_e w^e = 1 - \alpha^2 - \beta^2. \tag{2.12}$$

Transvecting (2.8) and (2.9) with $f_h{}^j$ and using (2.1), (2.6) and (2.7), we get

$$f_e{}^a u^e = -\lambda v^a + \beta w^a, \tag{2.13}$$

$$f_e{}^a v^e = \lambda u^a + \alpha w^a, \tag{2.14}$$

$$u_e w^e = -\alpha\lambda, \qquad v_e w^e = \beta\lambda. \tag{2.15}$$

Similarly, transvecting (2.8) and (2.9) with u^h and v^h, we obtain

$$u_e u^e = 1 - \beta^2 - \lambda^2, \qquad v_e v^e = 1 - \alpha^2 - \lambda^2, \qquad u_e v^e = -\alpha\beta. \tag{2.16}$$

On the other hand we find, from the second equation of (2.1) and (2.6),

$$g_{ea} f_c{}^e f_b{}^a = g_{cb} - u_c u_b - v_c v_b - w_c w_b. \tag{2.17}$$

Therefore, equations (2.10)~(2.17) mean that M admits an $(f, g, u_{(k)}, \alpha_{(k)})$-structure. If we put $f_{cb} = f_c{}^e g_{eb}$, then f_{cb} is skew-symmetric because f_{ji} is skew-symmetric.

Next, we assume that $\bar{M}$ be a $(2n+1)$-dimensional almost contact metric manifold covered by a system of coordinate neighborhoods $\{\bar{U}; y^\kappa\}$, i. e.,

$$f_\mu{}^\kappa f_\lambda{}^\mu = -\delta_\lambda{}^\kappa + v_\lambda v^\kappa, \tag{2.18}$$

$$f_\lambda{}^\kappa v^\lambda = 0, \qquad v_\lambda v^\lambda = 1, \tag{2.19}$$

$$g_{\kappa\nu} f_\mu{}^\kappa f_\lambda{}^\nu = g_{\mu\lambda} - v_\mu v_\lambda, \tag{2.20}$$

where $f_\mu{}^\kappa$ is a tensor field of type (1, 1), $g_{\mu\lambda}$ is the Riemannian metric of $\bar{M}$, v_λ is a 1-form and $v^\kappa = v_\lambda g^{\lambda\kappa}$, the indices $\lambda, \mu, \nu, \cdots$ running over the range $\{1, 2, \cdots, 2n+1\}$ in this section.

Let M be a $(2n-1)$-dimensional manifold covered by a system of coordinate neighborhoods $\{U ; x^b\}$, which is differentiably immersed in $\bar{M}$ as a submanifold of codimension 2 by the equations $y^\kappa = y^\kappa(x^b)$. If we put $B_b{}^\kappa = \partial_b y^\kappa$, $\partial_b = \partial/\partial x^b$, then $B_b{}^\kappa$ are $2n-1$ linearly independent local vector fields of $\bar{M}$ tangent to M, and the Riemannian metric induced on M from that of $\bar{M}$ is given by $g_{cb} = g_{\mu\nu} B_c{}^\mu B_b{}^\nu$. If we choose two unit vectors C^κ and D^κ of $\bar{M}$ normal to M in such a way that $2n+1$ vectors $B_b{}^\kappa$, C^κ, D^κ give the positive orientation of $\bar{M}$, then we can write equations of the form

$$f_\lambda{}^\kappa B_b{}^\lambda = f_b{}^e B_e{}^\kappa + w_b C^\kappa + u_b D^\kappa , \tag{2.21}$$

$$f_\lambda{}^\kappa C^\lambda = -w^e B_e{}^\kappa + \beta D^\kappa , \qquad f_\lambda{}^\kappa D^\lambda = -u^e B_e{}^\kappa - \beta C^\kappa , \tag{2.22}$$

where $u^e = u_a g^{ae}$, $w^e = w_a g^{ae}$, $f_b{}^a$ is a global tensor field of type (1, 1), u_a and w_a are 1-forms and β is a function in M. We can easily see that β is independent of the choice of C and D. The vector field v^κ has the form

$$v^\kappa = v^e B_e{}^\kappa + \alpha C^\kappa + \gamma D^\kappa , \tag{2.23}$$

where v^e defines vector field in M and α, γ are functions of M.

In this case, we also verify that a submanifold M of codimension 2 in an almost contact metric manifold admits an $(f, g, u_{(k)}, \alpha_{(k)})$-structure (cf. [4]).

§ 3. Hypersurfaces of a manifold with normal (f, g, u, v, λ)-structure.

Let $\tilde{M}$ be a manifold with normal (f, g, u, v, λ)-structure such that the function $\lambda(1-\lambda^2)$ is non-zero almost everywhere and satisfies $\nabla_j v_i - \nabla_i v_j = 2f_{ji}$ (or equivalently $\nabla_j u_i + \nabla_i u_j = -2\lambda g_{ji}$). In this section we consider a differentiable manifold M which is a hypersurface immersed in such a manifold $\tilde{M}$.

Denoting by ∇_c the operator of covariant differentiation with respect to the Christoffel symbols $\left\{ {a \atop c\ b} \right\}$ formed with g_{cb}, then the equations of Gauss and Weingarten for M are given by

$$\nabla_c B_b{}^h = h_{cb} N^h , \qquad \nabla_c N^h = -h_c{}^e B_e{}^h , \tag{3.1}$$

where $h_c{}^a = h_{ce} g^{ea}$, $\nabla_c B_b{}^h = \partial_c B_b{}^h + \left\{ {h \atop j\ i} \right\} B_c{}^j B_b{}^i - \left\{ {a \atop c\ b} \right\} B_a{}^h$ is the so-called van der Waerden-Borbtolotti covariant derivative of $B_b{}^h$ and h_{cb} the second fundamental tensor.

Differentiating (2.6) covariantly along M and using (2.3) and (3.1), we find

$$\{g_{ji}(\phi u^h - v^h) - \delta_j{}^h(\phi u_i - v_i)\} B_c{}^j B_b{}^i + (f_i{}^h N^i) h_{cb}$$
$$= (\nabla_c f_b{}^e) B_e{}^h + (h_{ce} f_b{}^e) N^h + (\nabla_c w_b) N^h - w_b h_c{}^e B_e{}^h ,$$

form which,

$$g_{cb}(\phi u^e B_e{}^h+\phi\beta N^h-v^e B_e{}^h-\alpha N^h)-\delta_c{}^e(\phi u_b-v_b)B_e{}^h-h_{cb}w^e B_e{}^h$$
$$=(\nabla_c f_b{}^e-w_b h_c{}^e)B_e{}^h+(\nabla_c w_b+h_{ce}f_b{}^e)N^h$$

by virtue of (2.7)~(2.9) and consequently

$$\nabla_c f_b{}^a=g_{cb}(\phi u^a-v^a)-\delta_c{}^a(\phi u_b-v_b)-h_{cb}w^a+h_c{}^a w_b\,, \tag{3.2}$$

$$\nabla_c w_b=(\phi\beta-\alpha)g_{cb}-h_{ce}f_b{}^e\,. \tag{3.3}$$

Differentiating also (2.8) and (2.9) covariantly and taking account of (2.4), (2.6) and (3.1), we obtain

$$\nabla_c u_b=-\lambda g_{cb}+\beta h_{cb}-\phi f_{cb}\,, \tag{3.4}$$

$$\nabla_c v_b=-\phi\lambda g_{cb}+\alpha h_{cb}+f_{cb}\,, \tag{3.5}$$

$$\nabla_c\alpha=-h_{ce}v^e+w_c\,,\qquad \nabla_c\beta=-h_{ce}u^e-\phi w_c\,. \tag{3.6}$$

Transvecting (2.5) with $B_c{}^j$ and using (2.8) and (2.9), we have

$$\nabla_c\lambda=u_c+\phi v_c\,. \tag{3.7}$$

In section 1, we introduced several tensors on M determined by the Nijenhuis tensor $[F, F]$ of the complex structure tensor F on $M\times R^3$. Substituting (3.2)~(3.7) into (1.14)~(1.20), $\cdots$ we have respectively

$$S_{cb}{}^a=(f_c{}^e h_e{}^a-h_c{}^e f_e{}^a)w_b-(f_b{}^e h_e{}^a-h_b{}^e f_e{}^a)w_c\,, \tag{3.8}$$

$$S_{cb}{}^{\bar 1}=(h_{ce}u^e)w_b-(h_{be}u^e)w_c+(u_c v_b-u_b v_c)\,, \tag{3.9}$$

$$S_{cb}{}^{\bar 2}=(h_{ce}v^e)w_b-(h_{be}v^e)w_c+\phi(u_c v_b-u_b v_c)\,, \tag{3.10}$$

$$S_{cb}{}^{\bar 3}=(h_{ce}w^e)w_b-(h_{be}w^e)w_c-(v_c w_b-v_b w_c)+\phi(u_c w_b-u_b w_c)\,, \tag{3.11}$$

$$S_{c\bar 1}{}^a=\beta(f_c{}^e h_e{}^a-h_c{}^e f_e{}^a)-\phi(v_c v^a+w_c w^a)-w_c(h_e{}^a u^e)-v_c u^a\,, \tag{3.12}$$

$$S_{c\bar 2}{}^a=\alpha(f_c{}^e h_e{}^a-h_c{}^e f_e{}^a)+(u_c u^a-w_c w^a)-w_c(h_e{}^a v^e)+\phi u_c v^a\,, \tag{3.13}$$

$$S_{c\bar 3}{}^a=-h_{ed}f_c{}^e f^{ad}+h_c{}^a-w_c(h_e{}^a w^e)+(\phi u_c-v_c)w^a\,, \tag{3.14}$$

$$\cdots\cdots.$$

We now prove

LEMMA 3.1. *Let M be a hypersurface of 2n-dimensional manifold $\tilde M$ with normal (f, g, u, v, λ)-structure such that the function $\lambda(1-\lambda^2)$ is not zero almost everywhere on $\tilde M$ and satisfies $\nabla_j v_i-\nabla_i v_j=2f_{ij}$ (or equivalently $\nabla_j u_i+\nabla_i u_j=-2\lambda g_{ji}$). Then*

$$\alpha^2+\beta^2+\lambda^2=1\,, \tag{3.15}$$

if and only if λ is constant, where α, β are defined on (2.8) *and* (2.9).

Proof. Suppose that $\alpha^2+\beta^2+\lambda^2=1$. Then we know in Lemma 1.1 that u_c, v_c

and w_c are linearly dependent. Thus we can put

$$u_c = a v_c + b w_c\,, \tag{3.16}$$

where a and b are numbers.

Transvecting (3.16) with $f_b{}^c$ and using (2.11), (2.13), (2.14) and (3.16) itself, we find

$$(\lambda + a^2\lambda - ab\beta - b\alpha)v_b - (\beta + b^2\beta - a\alpha - ab\lambda)w_b = 0\,. \tag{3.17}$$

On the other hand, transvecting (3.16) with u^c, v^c and w^c and using (2.12), (2.15), (2.16) and (3.15), we get

$$\alpha(\alpha + a\beta + b\lambda) = \beta(\alpha + a\beta + b\lambda) = \lambda(\alpha + a\beta + b\lambda) = 0\,,$$

or, using (3.15),

$$\alpha + a\beta + b\lambda = 0\,. \tag{3.18}$$

Substituting this into (3.17), we have

$$(1 + a^2 + b^2)(\lambda v_b - \beta w_b) = 0\,,$$

from which,

$$\lambda v_b - \beta w_b = 0\,. \tag{3.19}$$

Comparing (3.19) with (3.16) and taking account of (3.18), we obtain

$$\beta u_b + \alpha v_b = 0\,, \qquad \lambda u_b + \alpha w_b = 0\,. \tag{3.20}$$

Differentiating the first equation of (3.20) covariantly along M and using (3.4), (3.5) and (3.6) we get

$$\begin{aligned} 0 = &-(h_{ce}u^e u_b + h_{ce}v^e v_b) + w_c(v_b - \phi u_b) \\ &- \lambda(\phi\alpha + \beta)g_{cb} + (\alpha^2 + \beta^2)h_{cb} + (\alpha - \phi\beta)f_{cb}\,, \end{aligned} \tag{3.21}$$

from which, multiplying this equation by α^2 and making use of (3.20) in the equation obtained,

$$\begin{aligned} 0 = &-(\alpha^2 + \beta^2)h_{ce}u^e u_b + \lambda(\phi\alpha + \beta)u_c u_b \\ &- \alpha^2\lambda(\phi\alpha + \beta)g_{cb} + \alpha^2(\alpha^2 + \beta^2)h_{cb} + \alpha^2(\alpha - \phi\beta)f_{cb}\,, \end{aligned}$$

or, taking the skew-symmetric part with respect to c and b,

$$-(\alpha^2 + \beta^2)(h_{ce}u^e u_b - h_{be}u^e u_c) + 2\alpha^2(\alpha - \phi\beta)f_{cb} = 0\,. \tag{3.22}$$

Transvecting (3.22) with u^b and using (2.13) and (3.15), we get

$$-(\alpha^2 + \beta^2)\{\alpha^2 h_{ce}u^e - (h_{ea}u^e u^a)u_c\} + 2\alpha^2(\alpha - \phi\beta)(\lambda v_c - \beta w_c) = 0\,.$$

or, using (3.19),

$$(\alpha^2 + \beta^2)\{\alpha^2 h_{ce}u^e - (h_{ea}u^e u^a)u_c\} = 0\,.$$

Substituting last equation into (3.22), we have

$$\alpha^4(\alpha-\phi\beta)f_{cb}=0\,,$$

from which, transvecting f^{cb} and using (3.15), $\alpha^4(\alpha-\phi\beta)=0$, which implies

$$\alpha^2(\alpha-\phi\beta)=0\,. \tag{3.23}$$

Similarly, from (3.21), we can prove that

$$\beta^2(\alpha-\phi\beta)=0\,, \qquad \lambda^2(\alpha-\phi\beta)=0 \tag{3.24}$$

by virtue of (3.15), (3.19) and (3.20).

Adding (3.23) to (3.24) and making use of (3.15), we find

$$\alpha-\phi\beta=0\,. \tag{3.25}$$

Differentiating (3.15) covariantly and taking account of (3.6), we obtain

$$2\alpha(-h_{ce}v^e+w_c)+2\beta(-h_{ce}u^e-\phi w_c)+\nabla_c(\lambda^2)=0\,,$$

or,

$$-h_{ce}(\alpha v^e+\beta u^e)+(\alpha-\phi\beta)w_c+1/2\,\nabla_c(\lambda^2)=0$$

and consequently $\nabla_c(\lambda^2)=0$ by virtue of (3.20) and (3.25). Thus λ=const. on M.

Conversely, if we suppose λ=const., then we have from (3.7)

$$u_c=-\phi v_c\,, \tag{3.26}$$

which means that u^a, v^a and w^a are linearly dependent vectors.

According to Lemma 1.1, we see

$$\alpha^2+\beta^2+\lambda^2=1\,.$$

This completes the proof of Lemma 3.1.

LEMMA 3.2. *Under the same assumptions as those in Lemma* 3.1, *the four conditions* $S_{cb}{}^{\bar{1}}=0$, $S_{c\bar{1}}{}^a=0$, (3.15) *and* λ=*const. are equivalent to each other.*

Proof. Assume that $S_{cb}{}^{\bar{1}}=0$, that is,

$$(h_{ce}u^e)w_b-(h_{be}u^e)w_c+(u_cv_b-u_bv_c)=0\,. \tag{3.27}$$

Transvecting (3.27) with w^b, we find

$$(1-\alpha^2-\beta^2)h_{ce}u^e=-\beta\lambda u_c-\alpha\lambda v_c+(h_{ea}u^ew^a)w_c\,, \tag{3.28}$$

from which, combining (3.28) and (3.27),

$$0=(1-\alpha^2-\beta^2)(u_cv_b-u_bv_c)-\alpha\lambda(v_cw_b-v_bw_c)+\beta\lambda(w_cu_b-w_bu_c)\,, \tag{3.29}$$

or, transvecting (3.29) with f^{cb} and using (2.11)$\sim$(2.15),

$$\lambda(1-\alpha^2-\beta^2-\lambda^2)=0\,.$$

If we put $N_0=\{P:(1-\alpha^2-\beta^2-\lambda^2)(P)\neq 0\}$, then $\lambda=0$, i.e., λ=const. on N_0, which means $1-\alpha^2-\beta^2-\lambda^2=0$ on N_0 by virtue of Lemma 3.1. Therefore we find (3.15) on M.

Conversely, suppose that (3.15) satisfies, then (3.20), (3.25) and (3.26) are implied.

Differentiating (3.26) covariantly and making use of (3.4) and (3.5), we obtain

$$\beta h_{cb}-\lambda g_{cb}=0\,.$$

On $N_1=\{P:\beta(P)=0\}$, $\alpha=0$ and $\lambda=0$ as consequences of (3.25) and the above equation, respectively. This is contradiction to (3.15). It follows that N_1 is void. Thus $\beta\neq 0$ on M. Therefore we have

$$h_{cb}=\frac{\lambda}{\beta}g_{cb}\,. \tag{3.30}$$

Substituting (3.20) and (3.30) into (3.9), we get $S_{cb}{}^{\bar{1}}=0$. Therefore, the two conditions $S_{cb}{}^{\bar{1}}=0$ and (3.15) are equivalent.

Next, hypothesize $S_{c\bar{1}}{}^{a}=0$, that is,

$$\beta(f_c{}^e h_{ea}+f_a{}^e h_{ec})-\phi(v_c v_a+w_c w_a)-w_c(h_{ea}u^e)-v_c u_a=0\,,$$

from which, taking the skew-symmetric part,

$$w_c(h_{ea}u^e)-w_a(h_{ec}u^e)+v_c u_a-u_c v_a=0\,,$$

which is the same equation as (3.27).

Hence, by the same method, we can verify that the two conditions $S_{c\bar{1}}{}^{a}=0$ and (3.15) are equivalent.

Therefore, combining these and Lemma 3.1, we obtain Lemma 3.2.

Now, if (3.15) holds, then, substituting (3.19), (3.20) and (3.30) into (3.8), (3.10), (3.11), (3.13) and (3.14), we find $S_{cb}{}^{a}=S_{cb}{}^{\bar{2}}=S_{cb}{}^{\bar{3}}=S_{c\bar{2}}{}^{a}=S_{c\bar{3}}{}^{a}=\cdots=0$. Thus we obtain

THEOREM 3.3. *Let M be a hypersurface of $2n$-dimensional manifold $\tilde{M}$ with normal (f, g, u, v, λ)-structure such that the function $\lambda(1-\lambda^2)$ is not zero almost everywhere on $\tilde{M}$ and satisfies $\nabla_j v_i-\nabla_i v_j=2f_{ji}$ (or equivalently $\nabla_j u_i+\nabla_i u_j=-2\lambda g_{ji}$). Then*

$$\alpha^2+\beta^2+\lambda^2=1\,, \tag{3.31}$$

$$S_{cb}{}^{\bar{1}}=0\,,\qquad S_{c\bar{1}}{}^{a}=0\qquad or\qquad \lambda=\text{const.}$$

implies $S_{cb}{}^{a}=S_{cb}{}^{\bar{2}}=S_{cb}{}^{\bar{3}}=S_{c\bar{2}}{}^{a}=S_{c\bar{3}}{}^{a}=\cdots=0$. If one equation of (3.31) satisfies, then M is totally umbilical.

PROPOSITION 3.4. *Let M be a hypersurface of $2n$-dimensional manifold $\tilde{M}$ with normal (f, g, u, v, λ)-structure such that the function $\lambda(1-\lambda^2)$ is not zero almost everywhere on $\tilde{M}$ and satisfies $\nabla_j v_i-\nabla_i v_j=2f_{ji}$ (or equivalently $\nabla_j u_i+\nabla_i u_j=-2\lambda g_{ji}$). Then the necessary and sufficient condition that the induced $(f, g, u_{(k)}, \alpha_{(k)})$-structure on M is normal is*

$$f_c{}^e h_e{}^a - h_c{}^e f_e{}^a = 0\,. \tag{3.32}$$

Proof. The proof of the necessity is trivial.

Let $(f, g, u_{(k)}, \alpha_{(k)})$-structure be normal, that is, $S_{cb}{}^a=0$. Putting $T_c{}^a = f_c{}^e h_e{}^a - h_c{}^e f_e{}^a$, (3.12) becomes

$$T_c{}^a w_b - T_b{}^a w_c = 0\,, \tag{3.33}$$

from which, contracting with respect to c and b,

$$T_c{}^e w_e = 0\,. \tag{3.34}$$

Transvecting (3.33) with w^b and using (3.34), we get

$$(1-\alpha^2-\beta^2)T_c{}^a = 0\,.$$

On $N_2 = \{P \in M : T_c{}^a(P) \neq 0\}$, $1-\alpha^2-\beta^2=0$ from which $w_c=0$. Thus it follows that $f_c{}^e w_e = \beta u_c + \alpha v_c = 0$ on N_2. Since the last equation means that u_c and v_c are linearly dependent, we get (3.15). Hence, owing to (3.15) and $1-\alpha^2-\beta^2=0$, $h_{cb}=0$ holds on this set. Thus we find $T_c{}^a=0$ on N_2, which implies $T_c{}^a=0$ on M. Therefore, the sufficiency is also proved.

§ 4. Hypersurfaces of an even-dimensional sphere.

In this section, we consider a manifold M admitting $(f, g, u_{(k)}, \alpha_{(k)})$-structure as a hypersurface of even-dimensional sphere S^{2n}.

According to the structure equations of S^{2n} given in section 2, we can see that M satisfies differential equations (3.2)$\sim$(3.7) with $\phi=0$ (cf. [2]), i.e.,

$$\nabla_c f_b{}^a = -g_{cb} v^a + \delta_c{}^a v_b - h_{cb} w^a + h_c{}^a w_b\,, \tag{4.1}$$

$$\nabla_c w_b = -\alpha g_{cb} - h_{ce} f_b{}^e\,, \tag{4.2}$$

$$\nabla_c u_b = -\lambda g_{cb} + \beta h_{cb}\,, \qquad \nabla_c v_b = \alpha h_{cb} + f_{cb}\,, \tag{4.3}$$

$$\nabla_c \alpha = -h_{ce} v^e + w_c\,, \qquad \nabla_c \beta = -h_{ce} u^e\,, \tag{4.4}$$

$$\nabla_c \lambda = u_c\,. \tag{4.5}$$

Since we consider S^{2n} as a space of constant curvature, M also satisfies

$$\nabla_c h_{ba} - \nabla_b h_{ca} = 0\,. \tag{4.6}$$

Remark. If we assume that $\lambda=0$, then $u_c=0$ from (4.5), from which $\beta^2=1$ by virtue of (1.5). Hence we find $h_{cb}=0$ from (4.3). This means that M is totally geodesic. Afterward we consider the case in which $\lambda \neq 0$ almost everywhere.

Now, we suppose that $S_{cb}{}^a=0$ and $S_{cb}{}^{\bar{3}}=0$, or equivalently,

$$h_{ce} f_b{}^e + h_{be} f_c{}^e = 0\,, \tag{4.7}$$

and

$$(h_{ce}w^e)w_b-(h_{be}w^e)w_c-(v_cw_b-v_bw_c)=0\,. \tag{4.8}$$

Transvecting (4.8) with v^b, we have

$$\beta\lambda h_{ce}w^e=\beta\lambda v_c+\{(h_{ea}v^ew^a)-(1-\alpha^2-\lambda^2)\}w_c\,. \tag{4.9}$$

Transvecting (4.9) with u^c, we get

$$0=\beta(h_{ea}u^ew^a)+\alpha(h_{ea}v^ew^a)-\alpha(1-\alpha^2-\beta^2-\lambda^2) \tag{4.10}$$

because of (1.5).

On the other hand, transvecting (4.7) with w^cw^b and using (1.4), we also find

$$\beta(h_{ea}u^ew^a)+\alpha(h_{ea}v^ew^a)=0\,. \tag{4.11}$$

Comparing (4.10) and (4.11), we find

$$\alpha(1-\alpha^2-\beta^2-\lambda^2)=0\,.$$

If we put $M_1=\{P:\alpha(P)\neq 0\}\subset M$, then $\alpha^2+\beta^2+\lambda^2=1$ on M_1. It is easily shown that $\alpha=0$ on M_1 by the same method as that in the proof of Lemma 3.1. Thus M_1 is void, that is, $\alpha=0$ on M.

Using (1.7), (4.4) and the fact $\alpha=0$, we have

$$h_{ce}v^e=w_c \tag{4.12}$$

and

$$h_{ce}v^cw^e=1-\beta^2\,. \tag{4.13}$$

On $M_2=\{P:\beta(P)=0\}$, transvecting (4.8) with w^b and taking account of $\alpha=0$, we obtain

$$h_{ce}w^e=v_c+(h_{ea}w^ew^a)w_c\,,$$

and consequently

$$h_{ce}v^cw^e=1-\lambda^2$$

by virtue of (1.6).

Substituting this equation into (4.13), we find $\lambda=0$ on M_2. Thus M_2 is null, i. e., $\beta\neq 0$ on M.

On the other hand, substituting (4.13) into (4.9), we have

$$\beta\lambda h_{ce}w^e=\beta\lambda v_c-(\beta^2-\lambda^2)w_c\,. \tag{4.14}$$

Transvecting (4.7) with v^c and using (1.3), (1.4) and (4.12), we find

$$\lambda h_{ce}u^e=-\beta u_c\,. \tag{4.15}$$

Differentiating (4.14) covariantly, we obtain

$$\begin{aligned}&\nabla_b(\beta\lambda)(h_{ce}w^e)+\beta\lambda(\nabla_bh_{ce})w^e+\beta\lambda h_{ce}\nabla_bw^e\\&=(\nabla_b(\beta\lambda))v_c+\beta\lambda\nabla_bv_c-2(\beta\nabla_b\beta-\lambda\nabla_b\lambda)w_c-(\beta^2-\lambda^2)\nabla_bw_c\,,\end{aligned}$$

from which, using (4.2)~(4.5), (4.14) and (4.15), we also have

$$\beta\lambda(\nabla_b h_{ce})w^e - \beta\lambda h_{ce}h_{ba}f^{ea} = \beta\lambda f_{bc} + (\beta^2-\lambda^2)h_{be}f_c{}^e .$$

Taking the skew-symmetric part of this equation and making use of (4.6) and (4.7), we get

$$\beta\lambda(h_{ce}h_a{}^e f_b{}^a + f_{cb}) + (\beta^2-\lambda^2)h_{ce}f_b{}^e = 0 . \tag{4.16}$$

Transvecting (4.16) with $f_d{}^b$, we aftain

$$\beta\lambda h_c{}^e h_{ed} + (\beta^2-\lambda^2)h_{cd} - \beta\lambda g_{cd} = 0 . \tag{4.17}$$

On the other hand, owing to (4.4), (4.5) and (4.15), λ/β is covariantly constant, and consequently, $\lambda=\beta c$ for suitable non-zero constant c.

Thus, we can get, from (4.17),

$$h_c{}^e h_{ed} = \frac{c}{c^2-1} h_{cd} + g_{cd} .$$

From this relation we can easily verify that eigenvalues of $(h_b{}^c)$ are c and $-1/c$.

Now we define a (1, 1)-type tensor $P_b{}^c$ as the form:

$$P_b{}^c = -\frac{c}{c^2+1}(h_b{}^c - c\delta_b{}^c) . \tag{4.18}$$

Then we can easily see that

$$P_c{}^e P_{eb} = P_{cb} , \tag{4.19}$$

that is, $P_b{}^c$ is an almost product structure, and

$$\nabla_d P_b{}^c = 0 \tag{4.20}$$

because of (4.6).

Moreover, from (4.15) we can classify our development in two cases;

1st case: M is totally umbilical:

2nd case: $1 \leqq \text{rank of } (P_c{}^b) \leqq 2n-2$.

In the 1st case, we find that M is a $(2n-1)$-dimensional sphere S^{2n-1}.

In the 2nd case, taking account of $h_{cb} = -P_{cb}/c + cQ_{cb}$, (4.18), (4.19) and (4.20), where $P_{ce} = P_c{}^e g_{eb}$ and $Q_{cb} = g_{cb} - P_{cb}$, we can apply the Theorem A to our discussion.

Summing up, we have

THEOREM 4.1. *Let M be a complete and connected hypersurface of an even-dimensional sphere S^{2n}. If the induced $(f, g, u_{(k)}, \alpha_{(k)})$-structure is normal, $S_{cb}{}^{\bar{s}}=0$ and the function λ is almost everywhere non-zero on M, then M is congruent to S^{2n-1} or the hypersurface $S^p \times S^{2n-1-p}$ naturally embedded in S^{2n}, where p is the rank of $(P_c{}^b)$.*

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