S. FUKUDA
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ON BASE POINT FREE THEOREM

SHIGETAKA FUKUDA

§1. Introduction

Let X be a non-singular projective variety with $\dim X=n$ over $\boldsymbol{C}$. And let $\Delta=\sum_{i=1}^{s}\Delta_i$ be a reduced divisor on X with only simple normal crossings.

Let **Strata** $(\Delta):=\{\Gamma \mid 1\leqq k\leqq n,\ 1\leqq i_1<i_2<\cdots<i_k\leqq s,\ \Gamma$ is an irreducible component of $\Delta_{i_1}\cap\Delta_{i_2}\cap\cdots\cap\Delta_{i_k}\neq\emptyset\}$. A divisor R on (X, Δ) is, by definition, *nef and log big* if R is nef and big and $R|_\Gamma$ is nef and big for any member Γ of **Strata**(Δ) (due to Reid [8, 10.4]).

Note that if R is ample then R is nef and log big on (X, Δ).

The purpose of this paper is to prove the following:

MAIN THEOREM ("log effective freeness"). *Let L be a nef divisor on X such that $aL-(K_X+\Delta)$ is nef and log big on (X, Δ) for some $a\geqq 0$.*

Then there exists a natural number l_1, depending only on n and a, such that the complete linear system $|l_1L|$ is base point free.

The paper is organized as follows. In Section 2 we recall known base point free theorems. In Section 3 we give a proof to Reid's "log eventual freedom" theorem in the "smooth" case. In Section 4 we prove the main theorem. In Section 5 we show some results concerning the log canonical divisor.

We follow the notation and terminology of [Utah].

The results of the present paper have been announced in [Résumé].

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§2. Known results concerning base point freeness

Concerning base point freeness of linear systems on higher dimensional algebraic varieties, the following results are known.

THEOREM 1 ("eventual freedom") (Kawamata-Shokurov [4, Theorem 3-1-1]). *If L is a nef divisor on X and $aL-(K_X+\Delta)$ is ample for some $a\geqq 0$, then $|lL|$*

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is base point free for every $l \gg 0$.

THEOREM 2 ("effective freeness") (Kollár [5]). *Notation as in Theorem* 1. *There exists a natural number* l_0 *depending only on* n *and* a *such that* $|l_0 L|$ *is base point free.*

Proof. For $0<d\ll 1$, $aL-(K_X+(1-d)\Delta)$ is ample and $(X, (1-d)\Delta)$ is Kawamata-log-terminal. Thus the assertion immediately follows from Kollár [5, 1.1]. Q.E.D.

THEOREM 3 ("log eventual freedom") (Reid [8, 10.4]). *If* L *is a nef divisor on* X *and* $aL-(K_X+\Delta)$ *is nef and log big on* (X, Δ) *for some* $a \geqq 0$, *then* $|lL|$ *is base point free for every* $l \gg 0$.

Remark. This theorem is stated in Reid [8] with the idea for its proof. We give the theorem a proof based on this idea in the next section.

This theorem is not valid under the condition that $aL-(K_X+\Delta)$ is not "nef and log big" on (X, Δ) but "nef and big" (Zariski [4, Remark 3-1-2], Kawamata [3, Introduction]).

The Main Theorem of this paper is the generalization of Theorem 2 to "log effective freeness".

§3. Norimatsu Type Vanishing and proof of Theorem 3

THEOREM 4 (cf. Ein-Lazarsfeld [1, 2.4] and Norimatsu [7]) ("Norimatsu Type Vanishing"). *Let* R *be a nef and log big divisor on* (X, Δ). *Then*

$$H^i(X, \mathcal{O}_X(K_X+\Delta+R))=0 \quad \textit{for } i>0.$$

Proof (similar to the proof of [1, 2.4]). The assertion follows from the following exact sequence by induction on (n, s), by virtue of Kawamata-Viehweg vanishing theorem:

$$\begin{aligned} 0 \longrightarrow \mathcal{O}_X(K_X+\sum_{j<s}\Delta_j+R) &\longrightarrow \mathcal{O}_X(K_X+\Delta+R) \\ &\longrightarrow \mathcal{O}_{\Delta_s}(K_{\Delta_s}+\sum_{j<s}\Delta_j|_{\Delta_s}+R|_{\Delta_s}) \longrightarrow 0, \end{aligned}$$

where $\Delta=\sum_{j=1}^{s}\Delta_j$. Q.E.D.

Now we give Theorem 3 a proof relying on "Norimatsu Type Vanishing" and the method of Kawamata [3].

Proof of Theorem 3 (based on Reid's idea [8, 10.4]). We shall prove the theorem by induction on $n=\dim X$. In the case where $\Delta=0$, this is a Kawamata-Shokurov result [4, Remark 3-1-2]. So we may assume that $\Delta \neq 0$.

For any $i\in\{1, 2, \cdots, s\}$, we consider the following exact sequence:

$$0 \longrightarrow \mathcal{O}_X(lL-\Delta_i) \longrightarrow \mathcal{O}_X(lL) \longrightarrow \mathcal{O}_{\Delta_i}(lL) \longrightarrow 0.$$

We assume that $l\geqq a$. Thus $lL-\Delta_i-(K_X+\sum_{j\neq i}\Delta_j)=lL-(K_X+\Delta)$ is nef and log big on $(X, \sum_{j\neq i}\Delta_j)$.

Hence $H^1(X, \mathcal{O}_X(lL-\Delta_i))=0$ from Theorem 4.

And $aL|_{\Delta_i}-(K_{\Delta_i}+\sum_{j\neq i}\Delta_j|_{\Delta_i})=(aL-(K_X+\Delta))|_{\Delta_i}$ is nef and log big on $(\Delta_i, \sum_{j\neq i}\Delta_j|_{\Delta_i})$. Thus by induction hypothesis, $\mathrm{Bs}|lL|_{\Delta_i}|=\emptyset$ for every $l\gg 0$.

Therefore $\mathrm{Bs}|lL|\cap\mathrm{Supp}\,\Delta=\emptyset$ for every $l\gg 0$.

From now, we use the technique of Kawamata [3, Proof of Lemma 2].

Now fix a prime number p. We claim that $\mathrm{Bs}|p^mL|=\emptyset$ for $m\gg 0$.

Take a sufficient large natural number m_0 such that $\mathrm{Bs}|p^{m_0}L|\cap\mathrm{Supp}\,\Delta=\emptyset$. We assume that $\mathrm{Bs}|p^{m_0}L|\neq\emptyset$.

Take a log resolution $\mu: Y\to X$ (all relevant divisors F_j are smooth and cross normally) such that

(a) $K_Y=\mu^*(K_X+\Delta)+\sum_j a_jF_j$,

(b) $|\mu^*(p^{m_0}L)|=|M|+\sum_j r_jF_j$ ($|M|$ is the movable part and is free), and

(c) $\mu^*(aL-(K_X+\Delta))-\sum_j\delta_jF_j$ is ample (where $\delta_j\in\boldsymbol{Q}$ and $0\leqq\delta_j\ll 1$).

Let $c:=\min_{r_j\neq 0}((a_j+1-\delta_j)/r_j)$.

Here $c>0$, because if $a_j=-1$ then $\mu(F_j)\subset\mathrm{Supp}\,\Delta$ and therefore $r_j=0$.

We may assume that the minimum is attained at exactly one value $j=j_0$. Note that $F_{j_0}\cap\mu^{-1}(\Delta)=\emptyset$.

Put $A:=\sum_j(-cr_j+a_j-\delta_j)F_j$ and $A':=\ulcorner(A+F_{j_0}+\mu^{-1}(\Delta))\urcorner=\ulcorner A\urcorner+F_{j_0}+\mu^{-1}(\Delta)$ (where $\mu^{-1}(\Delta)$ denotes the set theoretical inverse image with a reduced structure).

Then $A'\geqq 0$ and $\mu_*A'=0$.

We consider a $\boldsymbol{Q}$-divisor $N:=\mu^*(p^mL)+A-K_Y\equiv cM+\mu^*((p^m-cp^{m_0})L-(K_X+\Delta))-\sum\delta_jF_j$.

If $p^m-cp^{m_0}\geqq a$, then N is ample and hence $H^1(Y, \mathcal{O}_Y(\mu^*(p^mL)+\ulcorner A\urcorner))=0$.

We consider the following exact sequence:

$$\begin{aligned}0 &\longrightarrow \mathcal{O}_Y(\mu^*(p^mL)+\ulcorner A\urcorner) \longrightarrow \mathcal{O}_Y(\mu^*(p^mL)+A')\\ &\longrightarrow \mathcal{O}_{F_{j_0}}(\mu^*(p^mL)+A')\oplus\mathcal{O}_{\mu^{-1}(\Delta)}(\mu^*(p^mL)+A') \longrightarrow 0.\end{aligned}$$

By the Nonvanishing theorem of Shokurov,

$$H^0(F_{j_0}, \mathcal{O}_{F_{j_0}}(\mu^*(p^mL)+A'))\neq 0 \quad \text{for } m\gg 0,$$

because $p^m\mu^*L|_{F_{j_0}}+(A+F_{j_0}+\mu^{-1}(\Delta))|_{F_{j_0}}-K_{F_{j_0}}=(\mu^*(p^mL)+A-K_Y)|_{F_{j_0}}=N|_{F_{j_0}}$ is ample.

Thus, for $m\gg 0$, $\mathrm{Bs}|p^mL|$ does not include $\mu(F_{j_0})$ but $\mathrm{Bs}|p^{m_0}L|$ includes $\mu(F_{j_0})$. Hence the method of KMM [4, Theorem 3-1-1] implies the theorem.

Q.E.D.

§4. Main theorem (log effective freeness)

MAIN THEOREM ("log effective freeness"). *Assume that* L *is a nef divisor on* X *and that* $aL-(K_X+\Delta)$ *is nef and log big an* (X, Δ) *for some* $a\geqq 0$. *Then there exists a natural number* $f(n, a)$ *(which is* $\geqq a$*), depending only on* n *and* a*, such that the complete linear system* $|f(n, a)L|$ *is base point free.*

LEMMA 1. *Let* L *be a nef divisor on* X *such that* $aL-(K_X+\Delta)$ *is nef and log big on* (X, Δ) *for an integer* $a\geqq 0$. *Then* $\mathrm{Bs}|f(n-1, a)L|\cap\Delta=\emptyset$.

Proof. Put $m_1:=f(n-1, a)$. We consider the exact sequence

$$0 \longrightarrow \mathcal{O}_X(m_1L-\Delta_i) \longrightarrow \mathcal{O}_X(m_1L) \longrightarrow \mathcal{O}_{\Delta_i}(m_1L) \longrightarrow 0$$

for all $1\leqq i\leqq s$. By Theorem 4, $H^1(X, \mathcal{O}_X(m_1L-\Delta_i))=0$ because $m_1L-\Delta_i-(K_X+\sum_{j\neq i}\Delta_j)$ is nef and log big on $(X, \sum_{j\neq i}\Delta_j)$. Here $\mathrm{Bs}|m_1L|_{\Delta_i}|=\emptyset$, because $aL|_{\Delta_i}-(K_{\Delta_i}+\sum_{j\neq i}\Delta_j|_{\Delta_i})$ is nef and log big on $(\Delta_i, \sum_{j\neq i}\Delta_j|_{\Delta_i})$. Thus $\mathrm{Bs}|m_1L|\cap\Delta=\emptyset$. Q.E.D.

Let $g: X\rightarrow S$ be the morphism defined by the linear system $|lL|$ for $l\gg 0$ (Theorem 3). Here there exists a Cartier divisor L_S on S such that L is linearly equivalent to g^*L_S, because $\Phi_{|lL|}=\Phi_{|(l+1)L|}=g$ for $l\gg 0$. We may assume that $L=g^*L_S$.

For any effective divisor H on X which is linearly equivalent to mL for $m\in\boldsymbol{N}$, there exists a divisor H_S on S such that $H=g^*H_S$, because $\Phi_{|lH|}=\Phi_{|(l+1)H|}=g$ for $l\gg 0$.

Because $g_*\mathcal{O}_X=\mathcal{O}_S$, for any Cartier divisor D on S such that g^*D is linearly equivalent to 0, $\mathcal{O}_S\cong g_*g^*\mathcal{O}_S(D)=g_*\mathcal{O}_X\otimes_{\mathcal{O}_S}\mathcal{O}_S(D)=\mathcal{O}_S(D)$.

Thus $|mL|=g^*|mL_S|$ for any $m\in\boldsymbol{N}$.

LEMMA 2. *Let* m *be a natural number such that* $|mL|\neq\emptyset$ *and* $\mathrm{Bs}|mL|\cap\Delta=\emptyset$.

Let Z_S *be an irreducible component of* $\mathrm{Bs}|mL_S|$ *and set* $k=\mathrm{codim}(Z_S, S)$. *Then* $\mathrm{Bs}|(km+j+a+1)L_S|$ *does not include* Z_S *for* $j\geqq 0$ *except at most* $\dim Z_S$ *different values of* j.

Proof (using Kawamata-Shokurov-Kollár's method [5]). Taking general elements $B_i\in|mL|$, put $B=(1/2m)B_0+B_1+\cdots+B_k$. Then $(X, \Delta+B)$ is log canonical outside $\mathrm{Bs}|mL|$ and $(X, \Delta+B)$ is not log canonical at the points belonging to the inverse image of the generic point of Z_S by g (by the argument of Kollár [5, 2.2.1]).

Let $M_0:=aL-(K_X+\Delta)+(1/2)L$.

Take a log resolution $f: Y\rightarrow X$ (*i.e.* Y is smooth and all relevent divisors are smooth and cross normally). Let

$$K_Y = f^*(K_X + \Delta) + \sum_i e_i E_i \quad (e_i \geqq -1);$$

$$f^*B = \sum_i b_i E_i;$$

$$f^*M_0 = A + \sum_i p_i E_i \ (A \text{ is an ample } \boldsymbol{Q}\text{-divisor and } 0 \leqq p_i \ll 1).$$

Put $c := \min\{(e_i + 1 - p_i)/b_i \mid Z_S \subset gf(E_i);\ b_i > 0\}$. By changing the p_i slightly, we may assume that the minimum is achieved for exactly one index. Let us denote the corresponding divisor by E_0.

Put $W := \bigcup_{e_i - b_i < -1} gf(E_i)$.

CLAIM 1. $0 < c < 1$.

Proof of Claim 1. We prove $c > 0$. If $Z_S \subset gf(E_i)$, $b_i > 0$ and $e_i = -1$, then $f(E_i) \subset \Delta$. But this can not occur, because $g^{-1}(Z_S) \cap \Delta = \emptyset$ from $\mathrm{Bs}|mL| \cap \Delta = \emptyset$.

Next we prove $c < 1$. Let z be a scheme-theoretic point on X such that $g(z)$ is the generic point of Z_S. Because $(X, \Delta + B)$ is not log canonical at z, $z \in f(E_j)$ and $e_j - b_j < -1$ for some j. Q.E.D.

CLAIM 2. *Put* $c' := \max\{(e_i + 1)/b_i \mid e_i + 1 < b_i\}$. *Then* $c \leqq c' < 1$ *and* c' *is not affected by* p_i*'s.*

Proof of Claim 2. From the proof of Claim 1, $(e_j + 1)/b_j < 1$ and $gf(E_j) \supset Z_S$ for some j. Q.E.D.

CLAIM 3. *If* W *does not include* $gf(E_i)$, *then* $cb_i - e_i + p_i < 1$ *or* $f(E_i) \subset \Delta$.

Proof of Claim 3. Here $e_i - b_i \geqq -1$. If $b_i \neq 0$, then $e_i - cb_i \geqq e_i - c'b_i > -1$ by Claim 2. If $b_i = 0$ and $e_i > -1$, then $e_i - cb_i = e_i > -1$. If $b_i = 0$ and $e_i = -1$, then $f(E_i) \subset \Delta$. Q.E.D.

CLAIM 4. $gf(E_0) = Z_S$. *If* $cb_i - e_i + p_i \geqq 1$ *and* $i \neq 0$, *then* $gf(E_i)$ *does not include* Z_S.

Proof of Claim 4. If $cb_i - e_i + p_i \geqq 1$, then $gf(E_i) \subset W$ or $f(E_i) \subset \Delta$, by Claim 3. Because $cb_0 - e_0 + p_0 = 1$, $Z_S \subset gf(E_0)$ and $g^{-1}(Z_S) \cap \Delta = \emptyset$, we get $gf(E_0) \subset W$. Here Z_S is an irreducible component of W. Thus $gf(E_0) = Z_S$. If $cb_i - e_i + p_i \geqq 1$ and $Z_S \subset gf(E_i)$, then $p_i < 1 + e_i$, because Δ does not include $f(E_i)$. So $b_i > 0$. Thus $c = (e_i + 1 - p_i)/b_i$ by the definition of c. Hence $i = 0$.
Q.E.D.

CLAIM 5. *If* $cb_i - e_i + p_i < 0$, *then* E_i *is* f*-exceptional.*

Proof of Claim 5. Because $c > 0$ (Claim 1), $e_i > 0$. Thus E_i is f-exceptional.
Q.E.D.

Proof of Lemma 2 *Continued.* Let $N_j := (km+j+a+1)L$ and $N'_j := f^*N_j - \sum_{i\neq 0} \lfloor cb_i - e_i + p_i \rfloor E_i$.

We consider the exact sequence

$$0 \longrightarrow \mathcal{O}_Y(N'_j - E_0) \longrightarrow \mathcal{O}_Y(N'_j) \longrightarrow \mathcal{O}_{E_0}(N'_j) \longrightarrow 0.$$

Note that

$$N'_j - E_0 \equiv K_Y + A + (1-c)f^*B + jf^*L + \sum_{i\neq 0} \{cb_i - e_i + p_i\} E_i$$

and

$$N'_j|_{E_0} \equiv K_{E_0} + (A + (1-c)f^*B + jf^*L)|_{E_0} + \sum_{i\neq 0} \{cb_i - e_i + p_i\} E_i|_{E_0}.$$

By Claim 1, Kawamata-Viehweg vanishing implies $H^1(Y, \mathcal{O}_Y(N'_j - E_0)) = 0$ and $h^0(E_0, \mathcal{O}_{E_0}(N'_j)) = \chi(\mathcal{O}_{E_0}(N'_j))$.

$h^0(E_0, \mathcal{O}_{E_0}(N'_j))$ is a nonzero polynomial of degree $\dim Z_S$ in j for $j \geqq 0$ (by the argument of Kollár [5, p. 600]).

By Claim 4 and Claim 5, $\mathrm{Bs}|N_j|$ does not include $f(E_0)$ for all $j \geqq 0$ except at most $\dim Z_S$ different values of j.

Noting that $\mathrm{Bs}|N_j| = g^{-1}\mathrm{Bs}|(km+j+a+1)L_S|$, we end the proof of this lemma. Q.E.D.

Proof of Main Theorem. By using Lemma 1 and Lemma 2, the same argument as in Kollár [5, 2.3] implies the theorem. Q.E.D.

Remark. The function $f(n, a)$ is given as follows.

When $n=1$, we put $f(1, a) := 2^2(1+1)!(a+1)$.

When $n \geqq 2$, we put $f(n, a) := 2^{n+1}(n+1)!(a+n)f(n-1, a)$.

Thus $f(n, a) = \prod_{i=1}^{n} (2^{i+1}(i+1)!(a+i))$.

Proof of the Remark Above. When $n=1$, if $\Delta \neq 0$, then $\mathrm{Bs}|2(a+1)L| \cap \Delta = \emptyset$ from the proof of Lemma 1, because $2(a+1) \geqq a$. If $\Delta = 0$, then $|2(a+1)L| \neq \emptyset$ from Kollár [5, 2.4]. Therefore $\mathrm{Bs}|2^1(1+1)!\,2(a+1)L| = \emptyset$ from the argument of Kollár [5, 2.3], by using Lemma 2.

When $n \geqq 2$, if $\Delta \neq 0$, then $\mathrm{Bs}|2(a+n)f(n-1, a)L| \cap \Delta = \emptyset$ from Lemma 1. If $\Delta = 0$, then $|2(a+n)L| \neq \emptyset$ from Kollár [5, 2.4], thus $|2(a+n)f(n-1, a)L| \neq \emptyset$. Therefore $\mathrm{Bs}|2^n(n+1)!\,2(a+n)f(n-1, a)L| = \emptyset$ from the argument of Kollár [5, 2.3], by using Lemma 2. Q.E.D.

§ 5. Appendix

In this section we show some results concerning the log canonical divisor $K_X + \Delta$.

THEOREM 5. *Assume that* L *is a nef divisor on* X *and that* $aL-(K_X+\Delta)$ *is nef and log big on* (X, Δ) *for some* $a\geqq 0$. *Let* Γ *be a member of* **Strata**(Δ) *and* $d:=\dim\Gamma$. *Then* $\mathrm{Bs}|mL|$ *does not include* Γ *for every* $m\geqq 2(d+a)$.

Remark. For the proof, we use the following two propositions.

PROPOSITION 1. *Let* R *be a nef and log big divisor on* (X, Δ) *and* Γ *a member of* **Strata**(Δ). *Let* $\pi:\tilde{X}\to X$ *be the blow up with center* Γ *and* E *the exceptional divisor. Then*

$$0\longrightarrow H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}(\pi^*(K_X+\Delta+R)-E)\longrightarrow H^0(\tilde{X}, \mathcal{O}_{\tilde{X}}(\pi^*(K_X+\Delta+R))$$
$$\longrightarrow H^0(E, \mathcal{O}_E(\pi^*(K_X+\Delta+R)))\longrightarrow 0$$

is exact and $\mathcal{O}_E(\pi^*(K_X+\Delta+R))\cong\pi|_E{}^*\mathcal{O}_\Gamma(K_X+\Delta+R)$.

Proof. Let $e:=\mathrm{codim}(\Gamma, X)$. We may assume that $\Gamma\subset\Delta_1\cap\Delta_2\cap\cdots\cap\Delta_e$. Let Δ_i' be the strict transform of Δ_i. Thus $\pi^*(K_X+\Delta+R)-E=K_{\tilde{X}}-(e-1)E+\pi^*(\Delta_1+\Delta_2+\cdots+\Delta_e)+\sum_{i>e}\Delta_i'+\pi^*R-E=K_{\tilde{X}}-(e-1)E+\sum_i\Delta_i'+eE+\pi^*R-E=K_{\tilde{X}}+\sum_i\Delta_i'+\pi^*R$ and π^*R is nef and log big on $(\tilde{X}, \sum_i\Delta_i')$ (where $\sum_i\Delta_i'$ is with only simple normal crossings). Therefore, by Theorem 4, $H^1(\tilde{X}, \mathcal{O}_{\tilde{X}}(\pi^*(K_X+\Delta+R)-E))=0$. Q. E. D.

Remark. Let Γ be a member of **Strata**(Δ). We may assume that

(*) $\Gamma\subset\Delta_1\cap\Delta_2\cap\cdots\cap\Delta_e$ (where $e=\mathrm{codim}(\Gamma, X)$).

Then $(K_X+\Delta)|_\Gamma=(K_X+\Delta_1+\Delta_2+\cdots+\Delta_e+\sum_{i>e}\Delta_i)|_\Gamma=K_\Gamma+\sum_{i>e}\Delta_i|_\Gamma$.

PROPOSITION 2. *Let* L *be a nef divisor on* X *such that* $aL-(K_X+\Delta)$ *is nef and log big on* (X, Δ) *for some* $a\geqq 0$. *Let* Γ *be a member of* **Strata**(Δ) *and* $d:=\dim\Gamma$. *Then* $\mathrm{H}^0(\Gamma, \mathcal{O}_\Gamma(mL))\neq 0$ *for every* $m\geqq 2(d+a)$.

Proof. We may assume (*) in the remark above. If $m\geqq a$, then

$$(mL)|_\Gamma=K_\Gamma+\sum_{i>e}\Delta_i|_\Gamma+(mL-(K_X+\Delta))|_\Gamma,$$

where $(mL-(K_X+\Delta))|_\Gamma$ is nef and log big on $(\Gamma, \sum_{i>e}\Delta_i|_\Gamma)$. Thus $h^0(\Gamma, \mathcal{O}_\Gamma(mL))$ is a polynomial in m for $m\geqq a$, of degree at most d, by Theorem 4. Note that $h^0(\Gamma, \mathcal{O}_\Gamma(mL))$ is a nonzero polynomial by Theorem 3. Thus the same argument as in Kollár [5, 2.4] implies the assertion. Q. E. D.

Proof of Theorem 5. Let $R:=mL-(K_X+\Delta)$. Then the assertion follows from Proposition 1 and Proposition 2. Q. E. D.

THEOREM 6 (Kollár-Matsuki [6, 4.12.1.2], cf. Iitaka [2, Example 11.6]). *Let* $f:Y\to X$ *be a birational morphism between non-singular projective varieties.*

Suppose that $K_Y = f^*(K_X + \varDelta) + \sum_{i=1}^{t} e_i E_i$ *and that the union of* Supp $\sum_{i=1}^{t} E_i$ *and* Exc(f) *is a divisor with only simple normal crossings. Then* $f(E_i) \in \mathbf{Strata}(\varDelta)$ *for all* i *such that* $e_i = -1$.

Remark. We give Theorem 6 an alternative proof, using Iitaka's Logarithmic Ramification formula [2].

Proof. We assume that $e_j = -1$ and that $f(E_j)$ is not a member of $\mathbf{Strata}(\varDelta)$, and we shall derive a contradiction.

Note that $f(E_j) \subset \mathrm{Supp}(\varDelta)$. Let Γ be the minimal member of $\mathbf{Strata}(\varDelta)$ which includes $f(E_j)$. We may assume that Γ is an irreducible component of $\varDelta_1 \cap \varDelta_2 \cap \cdots \cap \varDelta_e$ (where $e = \mathrm{codim}(\Gamma, X)$). Note that $f(E_j) \subsetneqq \Gamma$.

Take a general smooth point p of $f(E_j)$. From now in the proof of this theorem, we consider the problem analytically. There exists a neighborhood U of p in X and a smooth prime divisor $\varDelta_0$ on U, such that $\varDelta_0, \varDelta_1, \varDelta_2, \cdots, \varDelta_e$ are crossing normally and that $f(E_j) \cap U \subset \varDelta_0$. We may assume that $\varDelta_l \cap U = \emptyset$ for any $l > e$.

If we write

$$K_{f^{-1}(U)} = (f|_{f^{-1}(U)})^*(K_U + \varDelta|_U + \varDelta_0) + \left(\sum_{i=1}^{t} d_i E_i|_{f^{-1}(U)} - E_0\right)$$

(where E_0 is the strict transform of $\varDelta_0$ by f), then $d_j \geqq -1$ from Iitaka's Logarithmic Ramification Formula ([2, Theorem 11.5]). Therefore $e_j > -1$ because $(f|_{f^{-1}(U)})^* \varDelta_0 \geqq E_j|_{f^{-1}(U)}$. This is a contradiction. Q.E.D.

Remark. At first the author thought that these two theorems in this section are usefull to get an estimate concerning Main Theorem.

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SUZUKA COLLEGE OF TECHNOLOGY (SUZUKA KŌSEN)
SHIROKO-CHO, SUZUKA CITY, MIE PREFECTURE
510-02, JAPAN