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LOCAL SMOOTH SOLUTIONS OF THE RELATIVISTIC EULER EQUATION, II

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1. Introduction

The motion of a relativistic perfect fluid in the Minkowski space-time is governed by

$$(1.1)\qquad \begin{cases} \dfrac{\partial}{\partial t}\Big(\dfrac{\rho c^2+p}{c^2-v^2}-\dfrac{p}{c^2}\Big)+\sum_{k=1}^{3}\dfrac{\partial}{\partial x_k}\Big(\dfrac{\rho c^2+p}{c^2-v^2}v_k\Big)=0, \\ \dfrac{\partial}{\partial t}\Big(\dfrac{\rho c^2+p}{c^2-v^2}v_i\Big)+\sum_{k=1}^{3}\dfrac{\partial}{\partial x_k}\Big(\dfrac{\rho c^2+p}{c^2-v^2}v_i v_k+p\delta_{ik}\Big)=0, \quad i=1,2,3. \end{cases}$$

Here c denotes the speed of light, p the pressure, (v_1, v_2, v_3) the velocity of the fluid particle, ρ the mass-energy density of the fluid (as measured in units of mass in a reference frame moving with the fluid particle) and $v^2=v_1^2+v_2^2+v_3^2$. The fluid is assumed to be bartropic, which means that the equation (1.1) is to be supplemented with the equation of state

$$(1.2)\qquad p=p(\rho),$$

where $p(\rho)$ is a given function of ρ only.

For the case of one space dimension, Smoller and Temple [7] constructed global weak solutions to (1.1) for the isentropic case $p(\rho)=a^2\rho$ with $0<a<c$, and Chen [1] for the case $p(\rho)=a^2\rho^\gamma$ with $a>0$ and $\gamma>1$.

In our previous paper [6], the existence of local smooth solutions was proved for three space dimensions, with $p(\rho)=a^2\rho$, $0<a<c$. Our objective here is to extend this results to the general equation of state (1.2), under the sole assumption that

$$(1.3)\qquad \begin{aligned} &p(\rho)\in C^\infty(\rho_*, \rho^*), \\ &p(\rho)>0, \qquad 0<p'(\rho)<c^2 \qquad \text{for} \quad \rho\in(\rho_*, \rho^*), \end{aligned}$$

where ρ_* and ρ^* are some constants such that $0\leqq\rho_*<\rho^*\leqq\infty$. Note that if $p(\rho)=a^2\rho^\gamma$, then $\rho_*=0$ while $\rho^*=\infty$ if $\gamma=1$ and $\rho^*=\{c^2/(\gamma a^2)\}^{1/(\gamma-1)}$ if $\gamma>1$.

We consider the initial value problem to (1.1) with the initial condition

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$$(1.4)\qquad \begin{cases} \rho|_{t=0}=\rho_0(x), \\ v_i|_{t=0}=v_{0i}(x), \qquad i=1,2,3. \end{cases}$$

The main result of this paper is,

THEOREM 1.1. *Assume* (1.3) *for* $p(\rho)$. *Suppose that the initial data* ρ_0 *and* (v_{01}, v_{02}, v_{03}) *belong to the uniformly local Sobolev space* $H^s_{ul}=H^s_{ul}(\boldsymbol{R}^3)$, $s\geqq 3$, [3] *and that there exist a positive constant* δ *sufficiently small so that*

$$\rho_*+\delta\leqq\rho(x)\leqq\rho^*-\delta,$$

$$v_0^2(x)=v_{01}^2(x)+v_{02}^2(x)+v_{03}^2(x)\leqq(1-\delta)c^2,$$

hold for all $x\in\boldsymbol{R}^3$. *Then, the Cauchy problem* (1.1), (1.2) *and* (1.4) *has a unique solution*

$$(1.5)\qquad (\rho, v_1, v_2, v_3)\in L^\infty(0, T; H^s_{ul})\cap C([0, T]; H^s_{\mathrm{loc}})\cap C^1([0, T]; H^{s-1}_{\mathrm{loc}}),$$

with $\rho_*<\rho(x,t)<\rho^*$ *and* $v^2(x,t)<c^2$. *Here* $T>0$ *depends only on* δ *and the* H^s_{ul}-*norm of the initial data.*

As in [6], we shall prove the theorem by symmetrizing (1.1) and applying the Friedlichs-Lax-Kato theory [3], [5] of symmetric hyperbolic systems. According to Godunov [2], a suitable symmetrizer can be constructed if a strictly convex entropy function exists. In § 3, it is shown that such an entropy function exists for (1.1), and in § 2, the symmetrizer it induces is discussed. Finally in § 4, the non-relativistic limit of the solutions to (1.1) as $c\to\infty$ is shown to be a solution of the non-relativistic Euler equation with the same equation of state (1.2).

2. Symmetrization

Theorem 1.1 can be proved if there is a change of variables

$$(2.1)\qquad z=(\rho, v_1, v_2, v_3)^T \longrightarrow u=(u_0, u_1, u_2, u_3)^T,$$

which reduces the system (1.1) to a system of the form

$$(2.2)\qquad A^0(u)\frac{\partial u}{\partial t}+\sum_{l=1}^{3}A^l(u)\frac{\partial u}{\partial x_l}=0,$$

whose coefficent matrices $A^\alpha(u)$, $\alpha=0, 1, 2, 3$, satisfy the condition

(2.3)
(i) *they are all real symmetric and smooth in* u, *and*
(ii) $A^0(u)$ *is positive definite.*

The system (2.2) satisfying (2.3) is called a symmetric hyperbolic system, see

[3], [5]. We claim that for (1.1), one of such changes of variables is given by

$$
(2.4)\qquad \begin{cases} u_0=-\dfrac{c^3Ke^{\phi(\rho)}}{(\rho c^2+p)(c^2-v^2)^{1/2}}+c^2, \\ u_j=\dfrac{cKe^{\phi(\rho)}}{(\rho c^2+p)(c^2-v^2)^{1/2}}v_j, \qquad j=1,2,3, \end{cases}
$$

where

$$
(2.5)\qquad \phi(\rho)=\int_{\bar\rho}^{\rho}\frac{c^2}{\rho c^2+p(\rho)}d\rho, \qquad K=c^2\bar\rho+p(\bar\rho),
$$

$\bar\rho$ being an arbitrarily fixed number in (ρ_*, ρ^*). The derivation of (2.4), based on the idea of Godunov [2], will be presented in § 3. Here we shall check the condition (2.3). To this end, we shall find the matrices A^α, $\alpha=0, 1, 2, 3$. explicitly. First, note from (2.4) that

$$
v^2=\frac{c^4}{(c^2-u_0)^2}u^2, \qquad u^2=u_1^2+u_2^2+u_3^2.
$$

Substituting this into the first equation of (2.4) and putting

$$
(2.6)\qquad \Phi(\rho)=\frac{Ke^{\phi(\rho)}}{\rho c^2+p(\rho)},
$$

we get

$$
(2.7)\qquad \Phi(\rho)=\frac{1}{c^2}((c^2-u_0)^2-c^2u^2)^{1/2}.
$$

Since $\Phi'(\rho)=-Kp'(\rho)e^{\phi(\rho)}/(\rho c^2+p)^2<0$ from (2.5) and (2.6), (2.7) can be solved uniquely for $\rho\in(\rho_*, \rho^*)$ provided

$$
(2.8)\qquad \Phi(\rho^*-0)^2<\left(1-\frac{u_0}{c^2}\right)^2-\frac{u^2}{c^2}<\Phi(\rho_*+0)^2.
$$

Thus, the map (2.1) defined with (2.4) is a diffeomorphism from

$$
(2.9)\qquad \Omega_z=\{\rho_*<\rho<\rho^*,\ v^2<c^2\}
$$

onto

$$
(2.10)\qquad \Omega_u=\{u_0<c^2,\ (2.8)\ \text{holds.}\}.
$$

After a straight but tedious computation, we find the coefficients $A^\alpha(u)=(A^\alpha_{\beta\gamma})$, α, β, $\gamma=0, 1, 2, 3$, as follows:

$$
(2.11)\qquad \begin{aligned} &A^0_{00}=A_1\Psi(\rho), \qquad A^0_{0i}=A^0_{i0}=A_2\Psi(\rho)v_i, \\ &A^0_{ij}=A_3\Psi(\rho)v_iv_j+A_4\Psi(\rho)\delta_{ij}, \\ &A^l_{00}=A_2\Psi(\rho), \qquad A^l_{0i}=A^l_{i0}=A_3\Psi(\rho)v_iv_l+A_5\Psi(\rho)\delta_{il}, \\ &A^l_{ij}=A_3\Psi(\rho)v_iv_jv_l+A_4\Psi(\rho)(v_i\delta_{jl}+v_j\delta_{il}+v_l\delta_{ij}), \end{aligned}
$$

for $i, j, l=1, 2, 3$, where

$$\Psi(\rho)=\frac{1}{K}(\rho c^2+p)^2 e^{-\phi(\rho)},$$

and

$$A_1=\frac{c^4+3p'v^2}{c^3p'(c^2-v^2)^{3/2}}, \qquad A_2=\frac{c^4+2p'c^2+p'v^2}{c^3p'(c^2-v^2)^{3/2}},$$

$$A_3=\frac{c^2+3p'}{cp'(c^2-v^2)^{3/2}}, \qquad A_4=\frac{1}{c(c^2-v^2)^{1/2}}, \tag{2.12}$$

$$A_5=\frac{1}{c(\rho c^2+p)(c^2-v^2)^{1/2}}.$$

These coefficents can be calculated by the chain rule and the formula

$$\frac{\partial\rho}{\partial u_0}=\frac{A_4}{p'}\Psi(\rho), \qquad \frac{\partial\rho}{\partial u_j}=\frac{A_4}{p'}\Psi(\rho)v_j,$$

$$\frac{\partial v_i}{\partial u_0}=A_6\Psi(\rho)v_i, \qquad \frac{\partial v_i}{\partial u_j}=c^2A_6\Psi(\rho)\delta_{ij}, \qquad i, j=1, 2, 3,$$

with

$$A_6=\frac{(c^2-v^2)^{1/2}}{c^3(\rho c^2+p)}.$$

It is clear from (2.11) that the matrices $A^\alpha(u)$ are all real symmetric and smooth in Ω_z, and hence in Ω_u. To see that $A^0(u)$ is positive definite, let $\Xi=(\xi_0, \xi)^T\in\boldsymbol{R}^4$ be a 4-vector with $\xi\in\boldsymbol{R}^3$. We should calculate the inner product

$$(A^0(u)\Xi\,|\,\Xi)=\Psi(\rho)\{A_1\xi_0^2+2A_2\xi_0(v\,|\,\xi)+A_3(v\,|\,\xi)^2+A_4\xi^2\},$$

A_j being those in (2.12). In the same way as in [6], we can get an estimate

$$(A^0\Xi\,|\,\Xi)\geqq\frac{1}{2}(\kappa_0\xi_0^2+\kappa\xi^2), \tag{2.13}$$

with

$$\kappa_0=\frac{(c^2-v^2)^{1/2}(c^4-p'v^2)\Psi(\rho)}{c^3(c^4v^2+2c^2v^2p'+c^4p')},$$

$$\kappa=\frac{(c^2-v^2)^{1/2}(c^4-p'v^2)\Psi(\rho)}{c^3(c^4+3v^2p')},$$

which implies that (2.3) (ii) is also satisfied in Ω_u since (1.3) is fulfilled. Thus, (2.2) with (2.11) for the elements of the matrices $A^\alpha(u)$ is a symmetric hyperbolic system, which entails the existence of smooth local solutions to (2.2), thanks to the Friedrichs-Lax-Kato theory [3], [5]. Since (2.4) is a diffeomorphism, we can go back from (2.2) to the original system (1.1) to conclude Theorem 1.1.

3. Strictly convex entropy function

In this section, we shall follow Godunov [2] and explain how to find out the change of variables (2.4). First of all, we rewrite (1.1) in the form of the conservation laws,

$$w_t+\sum_{k=1}^{3}(f^k(w))_{x_k}=0\,, \tag{3.1}$$

where $w=(w_0, w_1, w_2, w_3)^T$ and $f^k(w)=(w_k, f_1^k, f_2^k, f_3^k)^T$ are defined by

$$\begin{aligned} w_0&=\frac{\rho c^2+p}{c^2-v^2}-\frac{p}{c^2}\,, \qquad w_j=\frac{\rho c^2+p}{c^2-v^2}v_j\,, \\ f_i^k&=\frac{\rho c^2+p}{c^2-v^2}v_iv_k+p\delta_{ik}\,. \end{aligned} \tag{3.2}$$

A scalar function $\eta=\eta(w)$ is called an entropy function to (3.1) if there exist scalar functions $q^k=q^k(w)$, $k=1, 2, 3$, satisfying

$$D_w\eta(w)D_wf^k(w)=D_wq^k\,. \tag{3.3}$$

Then, the symmetrizing variable u can be given by

$$u=(D_w\eta)^T\,. \tag{3.4}$$

For the detail, see Godunov [2] or Kawashima-Shizuta [4].

Now, we shall solve (3.3). To this end, it is convenient to employ $z=(\rho, v_1, v_2, v_3)$, instead of w of (3.2), as the independent variables in (3.3). This is possible since D_zw is regular;

$$\det D_zw=\frac{(\rho c^2+p)^3(c^4-v^2p')}{c^2(c^2-v^2)^4}>0\,,$$

which comes by noting

$$\begin{aligned} \frac{\partial w_0}{\partial\rho}&=B_1, \qquad \frac{\partial w_0}{\partial v_j}=B_2v_j\,, \\ \frac{\partial w_j}{\partial\rho}&=B_3v_i, \qquad \frac{\partial w_i}{\partial v_j}=B_2v_iv_j+B_4\delta_{ij}\,, \end{aligned} \tag{3.5}$$

where

$$\begin{aligned} B_1&=\frac{c^2+p'}{c_2-v^2}-\frac{p}{c^2}\,, \qquad B_2=\frac{2(\rho c^2+p)}{(c^2-v^2)^2}\,, \\ B_3&=\frac{c^2+p'}{c^2-v^2}\,, \qquad B_4=\frac{\rho c^2+p}{c^2-v^2}\,. \end{aligned}$$

Thus the mapping $z\to w$ is a diffeomorphism in a neighbourhood of each point of Ω_z. Moreover, using (3.5), we get

(3.6) $$(D_z w)^{-1}=(e_{\alpha\beta})_{\alpha,\beta=0,1,2,3}$$

as

$$e_{00}=c^2(c^2+v^2)E_1, \qquad e_{0j}=-2c^2E_1v_j,$$

$$e_{i0}=-c^2(c^2+p')E_1E_2v_i, \qquad e_{ij}=2p'E_1E_2v_iv_j+E_2\delta_{ij},$$

with

$$E_1=\frac{1}{c^4-v^2p'}, \qquad E_2=\frac{c^2-v^2}{\rho c^2+p}.$$

In view of (3.2) and (3.6), (3.3) can now be rewritten as

(3.7) $$D_z\eta C^k=D_zq^k, \qquad k=1, 2, 3,$$

where

$$C^k=(D_zw)^{-1}D_zf^k=(c^k_{\alpha\beta})_{\alpha,\beta=0,1,2,3},$$

are given by

$$c^k_{00}=c^2C_1v_k, \qquad c^k_{i0}=-C_1C_2v_iv_k+C_2\delta_{kj},$$

$$c^k_{0j}=C_3\delta_{ki}, \qquad c^k_{ij}=C_4v_i\delta_{kj}+v_k\delta_{ij},$$

with

(3.8)
$$C_1=\frac{c^2-p'}{c^4-p'v^2}, \qquad C_2=\frac{p'(c^2-v^2)}{\rho c^2+p},$$
$$C_3=\frac{c^2(\rho c^2+p)}{c^4-v^2p'}, \qquad C_4=\frac{p'(c^2-v^2)}{c^4-v^2p'}.$$

Let us solve (3.7) for (η, q^1, q^2, q^3). A quick count shows that (3.7) costitutes 12 equations for 4 unknowns, that is, it forms an over-determined system. We shall look for the solution of the form

(3.9) $$\eta=H(\rho, y), \qquad q^k=Q(\rho, y)v_k,$$

where

$$y=v^2=v_1^2+v_2^2+v_3^2.$$

This ansatz reduces (3.7) to the following system of first order linear partial differential equations;

(3.10) $$H_y=Q_y,$$

(3.11) $$c^2C_1H_\rho+2C_2(-C_1y+1)H_y=Q_\rho,$$

(3.12) $$C_3H_\rho-2C_4yH_y=Q,$$

C_j being as in (3.8). Seemingly, we have still an over-determined system. However, making (3.11)$\times(\rho c^2+p)-$(3.12)$\times(c^2-p')$ and using (3.10), we get a single equation for Q:

(3.13) $$2(c^2-y)p'Q_y=(\rho c^2+p)Q_\rho-(c^2-p')Q\,.$$

On the other hand, it follows from (3.10) that there should exist a function $G=G(\rho)$ of ρ only such that

$$H=Q(\rho,\, y)+G(\rho)\,.$$

Substitution of this into (3.11), together with (3.13), then yields

$$\rho G_\rho=\frac{c^2-y}{\rho c^2+p}Q-\frac{c^2-y}{c^2}Q_\rho\,,$$

or putting $q=(c^2-y)Q$,

(3.14) $$G_\rho=\frac{1}{\rho c^2+p}q-\frac{1}{c^2}q_\rho\,.$$

Since the left hand side of (3.14) is a function of ρ only, q must be of the form

(3.15) $$q=e^{\phi(\rho)}[g(\rho)+h(y)]\,,$$

where $\phi(\rho)$ is as in (2.5) while g and h are arbitrary functions. Substituting (3.15) into (3.13) and separating the variables, we have

$$\frac{\rho c^2+p}{p'}\frac{dg}{d\rho}-g=2(c^2-y)\frac{dh}{dy}+h=\text{constant},$$

which can be easily solved as

(3.16) $$\begin{aligned} q&=e^{\phi(\rho)}[K_1(c^2-y)^{1/2}+K_2' e^{\psi(\rho)}] \\ &=K_1(c^2-y)^{1/2}e^{\phi(\rho)}+K_2(\rho c^2+p)\,, \end{aligned}$$

where K_j's are integration constants and

(3.17) $$\begin{aligned} \psi(\rho)&=\int_{\bar\rho}^{\rho}\frac{p'(\rho)}{\rho c^2+p(\rho)}d\rho \\ &=-\phi(\rho)+\log\frac{\rho c^2+p(\rho)}{\bar\rho c^2+p(\bar\rho)}\,, \end{aligned}$$

$\bar\rho$ being as in (2.5). Now, (3.14) combined with (3.16) gives $G'=-K_2p'/c^2$, so that

(3.18) $$G=-\frac{K_2}{c^2}p+K_3\,,$$

K_3 being also an integration constant. In view of (3.16) and (3.18), we get

(3.19) $$\eta=H=\frac{K_1}{(c^2-v^2)^{1/2}}e^{\phi(\rho)}+K_2\Big(\frac{\rho c^2+p}{c^2-v^2}-\frac{p}{c^2}\Big)+K_3\,,$$

(3.20) $$Q=\frac{K_1}{(c^2-v^2)^{1/2}}e^{\phi(\rho)}+\frac{K_2}{c^2-v^2}(\rho c^2+p)\,.$$

For the later purpose, we wish to choose the constants K_j, $j=1, 2, 3$, so that (3.19) converges, as $c\to\infty$, to the entropy function for the non-relativistic case,

$$\eta^{(\infty)}=\frac{1}{2}\rho v^2+\rho\int_{\bar{\rho}}^{\rho}\frac{dp}{\rho}-p\,, \tag{3.21}$$

which can be obtained exactly in the same way as (3.19). In view of (3.17), $\Phi(\rho)$ of (2.6) equals $e^{-\phi(\rho)}$ so that it can be expanded for large c as

$$\Phi(\rho)=1-\frac{1}{c^2}\int_{\bar{p}}^{p}\frac{dp}{\rho}+O(c^{-4}),$$

for each fixed $\rho\in(\rho_*, \rho^*)$. Insert this into (3.19) to deduce

$$\begin{aligned}\eta=\frac{\rho+p/c^2}{(1-v^2/c^2)^{1/2}}\Big\{\frac{cK_1}{K}&+\frac{K_2}{(1-v^2/c^2)^{1/2}}\\ &-\frac{K_1}{cK}\int_{\bar{p}}^{p}\frac{dp}{\rho}-\frac{K_1}{K}O(c^{-3})\Big\}-\frac{K_2 p}{c^2}+K_3\,,\end{aligned}$$

where K is as in (2.5). Therefore, the right choice is found to be

$$K_1=-cK, \qquad K_2=c^2, \qquad K_3=0,$$

with which (3.19) becomes

$$\eta=-\frac{cK}{(c^2-v^2)^{1/2}}e^{\phi(\rho)}+c^2\Big(\frac{\rho c^2+p}{c^2-v^2}-\frac{p}{c^2}\Big). \tag{3.22}$$

The change of variables (2.4) was derived from (3.22) via the formula (3.4) or

$$u=((D_z w)^T)^{-1}(D_z\eta)^T\,,$$

combined with (3.6). Since the matrix $A^0(u)$ is positive definite in Ω_u as was shown in the preceding section, the entropy function (3.22) is strictly convex there.

4. Non-relativistic limit

In order to study the limit $c\to\infty$, we consider $c\geqq c_0$ with a fixed c_0 sufficiently large and assume, without loss of generality, that (1.3) is satisfied for all $c\geqq c_0$ with the same constants ρ_* and ρ^*. For the sake of simplicity, we discuss only the case $\rho^*<\infty$. The case $\rho^*=\infty$ can be treated similarly. Given $\delta>0$ sufficiently small, define

$$\Omega_z(\delta, c_0)=\{\rho_*+\delta<\rho<\rho^*-\delta,\ v^2<(1-\delta)c_0^2\}. \tag{4.1}$$

Firstly, note that (2.4) is a diffeomorphism from the domain (4.1) onto

$$\Omega_u(\delta, c_0, c)=\left\{u_0<c^2, \left(1-\frac{u_0}{c^2}\right)^2-\frac{u^2}{c_0^2(1-\delta)}>0, \right. \tag{4.2}$$
$$\left. \Phi(\rho^*-\delta)^2<\left(1-\frac{u_0}{c^2}\right)^2-\frac{u^2}{c^2}<\Phi(\rho_*+\delta)^2\right\},$$

cf. (2.9) and (2.10). Secondly, the matrices $A^\alpha(u)$ and all of their derivatives are uniformly bounded in the domain (4.2). Moreover, κ_0 and κ in (2.13) are bounded away from zero uniformly there, as seen from

$$\kappa_0=\frac{\rho}{v^2+p'}+O(c^{-2}),$$
$$\kappa=\rho+O(c^{-2}).$$

This means that the Friedlichs-Kato-Lax theory applies for (2.2) uniformly for all $c\geqq c_0$. Go back to (1.1), which is possible due to the diffeomorphism (2.4), to conclude

THEOREM 4.1. *Let $s\geqq 3$. For any fixed M_0, $c_0>0$ sufficiently large and $\delta_0>0$ sufficiently small, there exist positive constants M and T such that for any initial $z_0=(\rho_0, v_{01}, v_{02}, v_{03})\in H_{ul}^s$ satisfying*

$$\|z_0\|_{H^s}\leqq M_0, \quad z_0(x)\in\Omega_z(\delta_0, c_0) \qquad \text{for any } x\in\boldsymbol{R}^3,$$

and for any $c\geqq c_0$, the Cauchy problem (1.1), (1.2) *and* (1.4) *possesses a unique solution $z=(\rho, v_1, v_2, v_3)$ belonging to the class* (1.7) *and satisfying*

$$\operatorname*{ess\,sup}_{t\in(0,T)}\|z(t)\|_{H_{ul}^s}\leqq M,$$
$$z(t, x)\in\Omega_z(\delta_0/2, c_0) \text{ for all } t\in[0, T] \text{ and } x\in\boldsymbol{R}^3.$$

Let us show that the solutions z thus obtained converge as $c\to\infty$ to the solution of the non-relativistic Euler equation,

$$\begin{cases} \dfrac{\partial\rho}{\partial t}+\displaystyle\sum_{k=1}^3\frac{\partial}{\partial x_k}(\rho v_k)=0 \\ \dfrac{\partial}{\partial t}(\rho v_i)+\displaystyle\sum_{k=1}^3\frac{\partial}{\partial x_k}(\rho v_i v_k+p\delta_{ik})=0, \qquad i=1, 2, 3, \end{cases} \tag{4.3}$$

with the same intial z_0.

The symmetrizing variables for (4.3) associated with the entropy function (3.21) are given by

$$u_0^{(\infty)}=-\frac{1}{2}v^2+\int_{\bar{p}}^{p}\frac{dp}{\rho}, \tag{4.4}$$
$$u_j^{(\infty)}=-v_j, \qquad j=1, 2, 3,$$

and the resulting system is

(4.5) $$A^{(\infty)0}(u^{(\infty)})u_t^{(\infty)}+\sum_{l=1}^{3}A^{(\infty)l}(u^{(\infty)})u_{xl}^{(\infty)}=0\,,$$

with

$$a_{00}^{(\infty)0}=\frac{\rho}{p'},\qquad a_{i0}^{(\infty)0}=a_{0i}^{(\infty)0}=\frac{\rho}{p'}v_i\,,$$

$$a_{ij}^{(\infty)0}=\frac{\rho}{p'}v_iv_j+\rho\delta_{ij}\,,$$

for the matrix elements of $A^{(\infty)0}$ and so on. Between the transformations (2.4) and (4.4), it holds that

$$u(z)=u^{(\infty)}(z)+O(c^{-2})\,,$$

$$A^{\alpha}(u(z))=A^{(\infty)\alpha}(u^{(\infty)}(z))+O(c^{-2}),\qquad \alpha=0,1,2,3\,,$$

uniformly for $c\geqq c_0$ and $z\in\Omega_z(\delta_0/2, c_0)$, which implies, together with the uniform properties stated before Theorem 4.1 and by the arguments in [6], the uniform convergence of the solutions u of (2.2) to the solution of (4.5). Again we can go back to (1.1) and conclude

THEOREM 4.2. *Let $s\geqq 3$. Then, as $c\to\infty$, the solution z of* (1.1), (1.2) *and* (1.4) *given in Theorem* 4.1 *converges to the solution $z^{(\infty)}$ to* (4.3) *with the same initial data, uniformly on the time interval* $[0, T]$ *with T specified in Theorem* 4.1, *strongly in $H_{loc}^{s-\varepsilon}$ for any $\varepsilon>0$.*

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