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Classification of the irreducible representations of the affine Hecke algebra of type B_2 with unequal parameters

By

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1. Introduction

The representation theory of the affine Hecke algebras has two different approaches. One is a geometric approach and the other is a combinatorial one.

In the equal parameter case, affine Hecke algebras are constructed using equivariant K-groups, and their irreducible representations are constructed on Borel-Moore homologies. By this method, their irreducible representations are parameterized by the index triples ([CG], [KL]). On the other hand, G. Lusztig classified the irreducible representations in the unequal parameter case. His ideas are to use equivariant cohomologies and graded Hecke algebras ([Lus89], [LusI], [LusII], [LusIII]).

Although the geometric approach will give us a powerful method for the classification, but it does not tell us the detailed structure of irreducible representations. Thus it is important to construct them explicitly in combinatorial approach.

Using semi-normal representations and the generalized Young tableaux, A. Ram constructed calibrated irreducible representations with equal parameters ([Ram1]). Furthermore C. Kriloff and A. Ram constructed irreducible calibrated representations of graded Hecke algebras ([KR]). However, in general, we don't know the combinatorial construction of non-calibrated irreducible representations.

A. Ram classified irreducible representations of affine Hecke algebras of type A_1, A_2, B_2, G_2 in equal parameter case ([Ram2]). But there are some mistakes in his list of irreducible representations and his construction of induced representation of type B_2. For example, he missed the case $\chi_d^{(5)}$ (see Example 3.1).

In this paper, we will correct his list about type B_2 and also classify the irreducible representations in the unequal parameter case. There are three one-parameter families of calibrated irreducible representations and some other

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irreducible representations. We will use the Kato's criterion for irreducibility (see Theorem 2.1).

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2. Preliminaries

2.1. Affine Hecke algebra

We will use following notations.

(R, R^+, Π)	a root system of finite type, its positive roots and simple roots,
Q, P	the root lattice and the weight lattice of R,
$Q^\vee, P^\vee$	the coroot lattice and the coweight lattice of R
W	the Weyl group of R,
$\ell(w)$	the length of $w \in W$

We put $\Pi = \{\alpha_i\}_{i \in I}$, and denote by s_i the simple reflection associated with α_i.
First we define the Iwahori-Hecke algebra of W.

Definition 2.1. Let $\{q_i\}_{i \in I}$ be indeterminates. Then the *Iwahori-Hecke algebra* $\mathcal{H}$ of W is the associative algebra over $\mathbb{C}(q_i)$ defined by following generators and relations;

$$\begin{aligned} \text{generators } & T_i \quad (i \in I) \\ \text{relations } & (T_i - q_i)(T_i + q_i^{-1}) = 0 \quad (i \in I), \\ & \overbrace{T_i T_j T_i \cdots}^{m_{ij}} = \overbrace{T_j T_i T_j \cdots}^{m_{ij}}, \end{aligned}$$

where $m_{ij} = 2, 3, 4, 6$ according to $\langle \alpha_i, \alpha_j^\vee \rangle \langle \alpha_j \alpha_i^\vee \rangle = 0, 1, 2, 3$.

Remark 1. The indeterminates q_i, q_j must be equal if and only if α_i, α_j are in the same W-orbit in R. If all q_i are equal, we call the *equal parameter case*, and otherwise, *the unequal parameter case*.

For a reduced expression $s_{i_1} s_{i_2} \cdots s_{i_r}$ of $w \in W$, we define $T_w = T_{i_1} T_{i_2} \cdots T_{i_r}$. This does not depend on the choice of reduced expressions.

Let us define the affine Hecke algebras.

Definition 2.2. The *affine Hecke algebra* $\widehat{\mathcal{H}}$ is the associative algebra over $\mathbb{C}(q_i; i \in I)$ defined by following generators and relations;

$$\begin{aligned}
&\text{generators } T_w X^\lambda \quad (w \in W, \lambda \in P^\vee),\\
&\text{relations } (T_i - q_i)(T_i + q_i^{-1}) = 0 \quad (i \in I),\\
&\qquad T_w T_{w'} = T_{ww'} \quad \text{if} \quad \ell(w) + \ell(w') = \ell(ww') \ (w, w' \in W),\\
&\qquad X^\lambda X^\mu = X^{\lambda+\mu} \quad (\lambda, \mu \in P^\vee),\\
&\qquad X^\lambda T_i = T_i X^{s_i\lambda} + (q_i - q_i^{-1}) \frac{X^\lambda - X^{s_i\lambda}}{1 - X^{-\alpha_i^\vee}} \quad (i \in I).
\end{aligned}$$

2.2. Principal series representations and their irreducibility

Let us put $X^{P^\vee} = \{X^\lambda | \lambda \in P^\vee\}$ and let $\chi : X^{P^\vee} \to \mathbb{C}^*$ be a character of $X^{P^\vee}$.

Definition 2.3. Let $\mathbb{C}v_\chi$ be the one-dimensional representation of $\mathbb{C}[X]$ defined by

$$X^\lambda \cdot v_\chi = \chi(X^\lambda) v_\chi.$$

We call $M(\chi) = \mathrm{Ind}_{\mathbb{C}[X]}^{\widehat{\mathcal{H}}} \mathbb{C}v_\chi = \widehat{\mathcal{H}} \otimes_{\mathbb{C}[X]} \mathbb{C}v_\chi$ the *principal representation of* $\widehat{\mathcal{H}}$ associated with χ.

Note that $\mathrm{Res}_{\mathcal{H}}^{\widehat{\mathcal{H}}} M(\chi)$ is isomorphic to the regular representation of $\mathcal{H}$, so that $\dim M(\chi) = |W|$.

We put

$$q_\alpha = q_i \text{ for } \alpha^\vee \in W\alpha_i^\vee \ (i \in I).$$

Theorem 2.1 (Kato's Criterion of Irreducibility)**.** *Let us put*

$$P(\chi) = \{\alpha^\vee > 0 | \chi(X^{\alpha^\vee}) = q_\alpha^{\pm 2}\}.$$

Then $M(\chi)$ is irreducible if and only if $P(\chi) = \phi$.

For any finite-dimensional representation of $\widehat{\mathcal{H}}$ we put

$$\begin{aligned}
M_\chi &= \{v \in M | X^\lambda v = \chi(X^\lambda) v \text{ for any} X^\lambda \in X\},\\
M_\chi^{\mathrm{gen}} &= \left\{ v \in M \;\middle|\; \begin{array}{l} \text{there exists } k > 0 \text{ such that} \\ (X^\lambda - \chi(X^\lambda))^k v = 0 \text{ for any } X^\lambda \in X \end{array} \right\}.
\end{aligned}$$

Then $M = \bigoplus_{\chi \in T} M_\chi^{\mathrm{gen}}$ is the generalized weight decomposition of M.

Proposition 2.1. *If M is a simple $\widehat{\mathcal{H}}$-module with $M_\chi \neq 0$, then M is a quotient of $M(\chi)$.*

Definition 2.4. A finite-dimensional representation M of $\widehat{\mathcal{H}}$ is *calibrated*(or *X-semisimple*) if $M_\chi^{\mathrm{gen}} = M_\chi$ (for all χ).

2.3. W-action Lemma

Let us define the action of Weyl group W as the following;

$$(w \cdot \chi)(X^{\lambda}) = \chi(X^{w^{-1}\lambda}) \ (w \in W, \lambda \in P^{\vee}).$$

The following proposition is well known.

Proposition 2.2 (W-action Lemma [Ram1], [Rog]).
(1) *If $M(\chi) \cong M(\chi')$, then there exists $w \in W$ such that $\chi' = w\chi$.*
(2) *The representations $M(\chi)$ and $M(w\chi)$ have the same composition factors.*

2.4. Specialization lemma

Let $\mathbb{K}$ be a field and $\mathbb{S}$ a discrete valuation ring such that $\mathbb{K}$ is the fraction field of $\mathbb{S}$. Let us denote the $\mathfrak{m} = (\pi)$ the maximal ideal of $\mathbb{S}$ and let $\mathbb{F} = \mathbb{S}/\mathfrak{m}$ be the residue field of $\mathbb{S}$. Let $K(\widehat{\mathcal{H}}_{\mathbb{F}}\text{-mod})$ be the Grothendieck group of the category of finite-dimensional representations of $\widehat{\mathcal{H}}_{\mathbb{F}}$.

the following lemma is well-known (e.g. see [Ari, Lemma 13.16].)

Lemma 2.1 (Specialization Lemma). *Let V be an $\widehat{\mathcal{H}}_{\mathbb{K}}$-module and L an $\widehat{\mathcal{H}}_{\mathbb{S}}$-submodule of V which is an $\mathbb{S}$-lattice of full rank. Then $[L \otimes \mathbb{F}] \in K(\widehat{\mathcal{H}}_{\mathbb{F}}\text{-mod})$ is determined by V and does not depend on the choice of L.*

2.5. Key results for type B_2

Let us consider the type B_2;

$$P^{\vee} = \mathbb{Z}\varepsilon_1 \oplus \mathbb{Z}\varepsilon_2, \ R^{\vee} = \{\alpha_1^{\vee} = \varepsilon_1 - \varepsilon_2, \alpha_2^{\vee} = 2\varepsilon_2\}, \ X_i = X^{\varepsilon_i}.$$
$$s_1\varepsilon_1 = \varepsilon_2, \ s_1\varepsilon_2 = \varepsilon_1, \quad s_2\varepsilon_1 = \varepsilon_1, \ s_2\varepsilon_2 = -\varepsilon_2$$

Let us recall the definition of affine Hecke algebra of type B_2 with unequal parameters.

Definition 2.5. The *affine Hecke algebra $\widehat{\mathcal{H}}$ of type B_2* is the associative algebra over $\mathbb{C}(p,q)$ defined by the following generators and relations;

$$\begin{array}{lll} \text{generators} & T_1, T_2, X_1, X_2 & \\ \text{relations} & (T_1 - q)(T_1 + q^{-1}) = 0, & (T_2 - p)(T_2 + p^{-1}) = 0, \\ & T_1T_2T_1T_2 = T_2T_1T_2T_1, & \\ & T_1X_2T_1 = X_1, & T_2X_2^{-1}T_2 = X_2, \\ & T_2X_1 = X_1T_2, & X_1X_2 = X_2X_1. \end{array}$$

We will use the following four subalgebras of $\widehat{\mathcal{H}}(B_2)$;

$$\widehat{\mathcal{H}}_1 = \langle T_1, X_1, X_2 \rangle, \quad \widehat{\mathcal{H}}_2 = \langle T_2, X_1, X_2 \rangle, \quad \mathcal{H} = \langle T_1, T_2 \rangle, \quad \mathbb{C}[X_1, X_2] \subset \widehat{\mathcal{H}}.$$

Lemma 2.2 (Decomposition Lemma). *Suppose $\chi(X^{\alpha_i}) = q_i^2$, and let ρ_1, ρ_2 be the following 1-dimensional representations of $\widehat{\mathcal{H}}_i = \langle T_i, X_j (1 \le j \le 2) \rangle \subset \widehat{\mathcal{H}}$;*

$$\rho_1(X_j) = \chi(X_j), \ \rho_1(T_i) = q_i, \ \rho_2(X_j) = (s_i\chi)(X_j), \ \rho_2(T_i) = -q_i^{-1}.$$

Then there exists the following short exact sequence;

$$0 \to \mathrm{Ind}_{\widehat{\mathcal{H}}_i}^{\widehat{\mathcal{H}}} \rho_2 \to M(\chi) \to \mathrm{Ind}_{\widehat{\mathcal{H}}_i}^{\widehat{\mathcal{H}}} \rho_1 \to 0$$

3. Classification

3.1. Method

Let M be an irreducible representation which is not principal. Then M appears in some $M(\chi)$. By Kato's criterion (Theorem 2.1), $P(\chi) \neq \phi$. Using W-action Lemma (Lemma 2.2), we may assume $P(\chi) \ni \alpha_1$ or α_2. thus we obtain the following Lemma. We will use the notation $-\chi$ defined by $(-\chi)(X_i) = -\chi(X_i)$ $(i = 1, 2)$.

Lemma 3.1. *Except irreducible principal series representations, any finite-dimensional irreducible representation appears in the principal representations associated with the following characters as their composition factors*;

χ	χ_a	χ_b	χ_c	$\chi_d^{(1)}$	$\chi_d^{(2)}$	$\chi_d^{(3)}$	$\chi_d^{(4)}$	$\chi_d^{(5)}$	$\chi_f(v)$	$\chi_g(u)$
$\chi(X_1)$	q^2p	q^2p^{-1}	$-p^{-1}$	q^2	q	p	1	-1	pv	q^2u
$\chi(X_2)$	p	p^{-1}	p	1	q^{-1}	p	p	p	p	u

and $-\chi_a, -\chi_b, -\chi_d^{(1)}, -\chi_d^{(2)}, -\chi_d^{(3)}, -\chi_d^{(4)}, -\chi_d^{(5)}, -\chi_f(v)$, *where*

$$v \neq \pm p^{-2}, \pm p^{-1}, \pm 1, q^{\pm 2}, q^{\pm 2}p^{-2},$$
$$u \neq \pm p^{\pm 1}, \pm 1, \pm q^{-2}, \pm q^{-1}, \pm q^{-2}p^{\pm 1}.$$

Note 1. Two principal series representations $M(-\chi_c)$ and $M(\chi_c)$ have same composition factors, because of W-action lemma (Lemma 2.2). By replacing u with $-u$, we don't need to consider $-\chi_g(u)$.

Finally, we must determine the composition factors of $M(\chi)$ for above characters, and we must prove their irreducibility. But using the decomposition lemma, we consider the representations induced from $\widehat{\mathcal{H}}_i$. We will show the examples and some proofs in the following section.

3.2. Some examples and proofs

Example 3.1. We consider the principal series representation $M(\chi_d^{(5)})$. Let $\rho_1^{d^{(5)}}$ and $\rho_2^{d^{(5)}}$ be the following 1-dimensional representations of $\widehat{\mathcal{H}}_2$;

	X_1	X_2	T_2
$\rho_1^{d^{(5)}}$	-1	p	p
$\rho_2^{d^{(5)}}$	-1	$-p^{-1}$	$-p^{-1}$

Since $\chi_d^{(5)}(\alpha_2^\vee) = p^2$, we can apply the decompose lemma (Lemma 2.2) to $M(\chi_d^{(5)})$.

Lemma 3.2. *Suppose $p \neq -q^{\pm 2}$. Then $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_1^{d^{(5)}}$ and $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_2^{d^{(5)}}$ are 4-dimensional non-calibrated irreducible representations.*

Proof. We consider the case of $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_1^{d^{(5)}}$. These simultaneous eigenvalues of X_1 and X_2 are $(p,-1),(-1,p)$, and the multiplicity of each eigenvalues is two. We can find the following representation matrices;

$$T_1 = \begin{pmatrix} \frac{p(q^2-1)}{(1+p)q} & -\frac{(p-1)(q^2-1)}{(1+p)q} & 1 & -\frac{p(q^2-1)^2}{(1+p)^2q^2} \\ 0 & \frac{p(q^2-1)}{(1+p)q} & 0 & \frac{(p+q^2)(1+pq^2)}{(1+p)^2q^2} \\ \frac{(p+q^2)(1+pq^2)}{(1+p)^2q^2} & \frac{(1-p+p^2)(q^2-1)^2}{(1+p)^2q^2} & \frac{(q^2-1)}{(1+p)q} & \frac{(p-1)(q^2-1)(p+q^2)(1+pq^2)}{(1+p)^3q^3} \\ 0 & 1 & 0 & \frac{(q^2-1)}{(1+p)q} \end{pmatrix},$$

$$T_2 = \begin{pmatrix} & 1 & & \\ 1 & \frac{(p^2-1)}{p} & & \\ & & p & \\ & & & p \end{pmatrix},$$

$$X_1 = \begin{pmatrix} p & & & \\ & p & & \\ & & -1 & -\frac{(-1+p)(p+q^2)(1+pq^2)}{p(1+p)q^2} \\ & & & -1 \end{pmatrix}, \quad X_2 = \begin{pmatrix} -1 & -\frac{p^2-1}{p} & & \\ & -1 & & \\ & & p & \\ & & & p \end{pmatrix}.$$

Since $p \neq -q^{\pm 2}$ and p, q are not a root of unity, the non-diagonal component with respect to $(p,-1),(-1,p)$ in X_1 and X_2 don't vanish. Thus the dimension of each simultaneous eigenspaces is just one. Let v_1, v_2 be the simultaneous eigenvectors with respect to $(p,-1),(-1,p)$. We have

$$T_1 v_1 = \frac{p(q^2-1)}{(1+p)q} v_1 + \frac{(p+q^2)(1+pq^2)}{(1+p)^2q^2} v_2, \quad T_1 v_2 = \frac{q^2-1}{(1+p)q} v_2 + v_1,$$

and $p \neq -q^{\pm 2}$. If there exists a submodule $0 \neq U$ of $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_1^{d^{(5)}}$, then U contains v_1 or v_2. If v_2 is contained in U, then v_1 is contained in U, and vice versa. Therefore $\langle v_1, v_2, T_2 v_1, T_1 T_2 v_1 \rangle \subset U$. This implies that $U = \mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_1^{d^{(5)}}$, and $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_1^{d^{(5)}}$ is irreducible. Similarly, we can show that $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_2^{d^{(5)}}$ is irreducible. □

Example 3.2. We consider $M(\chi_a)$. Let ρ_1^a and ρ_2^a be the following 1-dimensional representations of $\widehat{\mathcal{H}}_2$;

	X_1	X_2	T_2
ρ_1^a	$q^2 p$	p	p
ρ_2^a	$q^2 p$	$-p^{-1}$	$-p^{-1}$

Since $\chi_a(\alpha_2^\vee) = p^2$, we can apply the decompose lemma (Lemma 2.2) to $M(\chi_a)$.

Lemma 3.3. *Suppose $p \neq \pm q^{-1}, \pm q^{-2}, p^2 \neq -q^{-2}$. Then $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_1^a$ and $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_2^a$ have 1- and 3-dimensional calibrated irreducible composition factors. More precisely,*

(1) *$\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_1^a$ have two composition factors which are presented by the following representation matricies;*

- $X_1 = pq^2,\ X_2 = p,\ T_1 = q,\ T_2 = p.$
- U_a^1:

$$X_1 = \begin{pmatrix} p & & \\ & p & \\ & & p^{-1}q^{-2} \end{pmatrix},\ X_2 = \begin{pmatrix} pq^2 & & \\ & p^{-1}q^{-2} & \\ & & p \end{pmatrix},$$

$$T_1 = \begin{pmatrix} -q^{-1} & & \\ & \frac{p^2q(q^2-1)}{(p^2q^2-1)} & \frac{(p^2-1)(p^2q^4-1)}{(p^2q^2-1)^2} \\ & 1 & -\frac{(q^2-1)}{q(p^2q^2-1)} \end{pmatrix},\ T_2 = \begin{pmatrix} \frac{p(p^2-1)q^4}{(p^2q^4-1)} & \frac{(q^4-1)(p^4q^4-1)}{(p^2q^4-1)^2} & \\ 1 & -\frac{p^2-1}{p(p^2q^4-1)} & \\ & & p \end{pmatrix}.$$

(2) $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_2^a$ *have two composition factors which are presented by the following representation matricies*;

- $X_1 = p^{-1}q^{-2},\ X_2 = p^{-1},\ T_1 = -q^{-1},\ T_2 = -p^{-1}.$
- U_a^2:

$$X_1 = \begin{pmatrix} p^{-1} & & \\ & pq^2 & \\ & & p^{-1} \end{pmatrix},\ X_2 = \begin{pmatrix} pq^2 & & \\ & p^{-1} & \\ & & p^{-1}q^{-2} \end{pmatrix},$$

$$T_1 = \begin{pmatrix} -\frac{q^2-1}{q(p^2q^2-1)} & 1 & \\ \frac{(p^2-1)(p^2q^4-1)}{(p^2q^2-1)^2} & \frac{p^2q(q^2-1)}{(p^2q^2-1)} & \\ & & q \end{pmatrix},\ T_2 = \begin{pmatrix} \frac{p(p^2-1)}{(p^2q^4-1)} & & \frac{(q^4-1)(p^4q^4-1)}{(p^2q^4-1)^2} \\ & -p^{-1} & \\ 1 & & -\frac{p^2-1}{p(p^2q^4-1)} \end{pmatrix}.$$

Example 3.3. We consider $M(\chi_b)$. Let ρ_1^b and ρ_2^b be the following 1-dimensional representations of $\widehat{\mathcal{H}}_1$;

	X_1	X_2	T_1
ρ_1^b	q^2p^{-1}	p^{-1}	q
ρ_2^b	p^{-1}	q^2p^{-1}	$-q^{-1}$

Since $\chi_a(\alpha_1^\vee) = q^2$, we can apply the decompose lemma (Lemma 2.2) to $M(\chi_b)$.

Lemma 3.4. (1) *Suppose* $p \neq \pm q, \pm q^2, p^2 \neq -q^2$. *Then* $\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_1^b$ *and* $\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_2^b$ *have* 1- *and* 3-*dimensional calibrated irreducible composition factors which are calibrated and presented by the following representation matrices*;

(i) *case* $\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_1^b$;

- $X_1 = q^2p^{-1},\ X_2 = p^{-1},\ T_1 = q,\ T_2 = -p^{-1}.$
- U_b^1:

$$X_1 = \begin{pmatrix} q^2p^{-1} & & \\ & p & \\ & & p \end{pmatrix},\ X_2 = \begin{pmatrix} p & & \\ & pq^{-2} & \\ & & q^2p^{-1} \end{pmatrix},$$

$$T_1 = \begin{pmatrix} \frac{q(q^2-1)}{q^2-p^2} & & -\frac{(p^2-1)(q^4-p^2)}{(q^2-p^2)^2} \\ & q & \\ 1 & & -\frac{p^2(q^2-1)}{(q^2-p^2)q} \end{pmatrix},\ T_2 = \begin{pmatrix} p & & \\ & \frac{p(p^2-1)}{p^2-q^4} & 1 \\ & -\frac{(p^2-q^2)(q^4-1)(p^2+q^2)}{(p^2-q^4)^2} & -\frac{(p^2-1)q^4}{p(p^2-q^4)} \end{pmatrix}.$$

(ii) *case* $\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_2^b$;

- $X_1 = pq^{-2},\ X_2 = p,\ T_1 = -q^{-1},\ T_2 = -p.$

- U_b^2:

$$X_1 = \begin{pmatrix} pq^{-2} & & \\ & p^{-1} & \\ & & p^{-1} \end{pmatrix},\ X_2 = \begin{pmatrix} p^{-1} & & \\ & pq^{-2} & \\ & & q^2p^{-1} \end{pmatrix},$$

$$T_1 = \begin{pmatrix} -\frac{p^2(q^2-1)}{q^2-p^2} & 1 & \\ -\frac{(p^2-1)(q^4-p^2)}{(q^2-p^2)^2} & \frac{(q^2-1)q}{(q^2-p^2)} & \\ & & -q^{-1} \end{pmatrix},$$

$$T_2 = \begin{pmatrix} -p^{-1} & & \\ & \frac{p(p^2-1)}{p^2-q^4} & 1 \\ & -\frac{(p^2-q^2)(q^4-1)(p^2+q^2)}{(p^2-q^4)^2} & -\frac{(p^2-1)q^4}{p(p^2-q^4)} \end{pmatrix}.$$

(2) *Suppose* $p = q$. *Then they have* 1-*dimensional composition factor and* 3-*dimensional non-calibrated composition factor which are presented by the following representation matrices*;

(i) *case* $\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_1^b$;

- $X_1 = q,\ X_2 = q^{-1},\ T_1 = q,\ T_2 = -q^{-1}.$
- U_b^1:

$$X_1 = \begin{pmatrix} q & & \\ & q & q^2 \\ & & q \end{pmatrix},\ X_2 = \begin{pmatrix} q^{-1} & & \frac{1+2q^2}{q} \\ & q & -q^2 \\ & & q \end{pmatrix},$$

$$T_1 = \begin{pmatrix} q & \frac{1+2q^2}{q^2} & \\ & -q^{-1} & \\ & \frac{q^2-1}{q^2} & q \end{pmatrix},\ T_2 = \begin{pmatrix} -q^{-1} & & \frac{1+q^2}{q(q^2-1)} \\ 1 & q & -\frac{1}{q^2-1} \\ & & q \end{pmatrix}.$$

(ii) *case* $\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_2^b$;

- $X_1 = q^{-1},\ X_2 = q,\ T_1 = -q^{-1},\ T_2 = q.$
- U_b^2:

$$X_1 = \begin{pmatrix} q^{-1} & & -\frac{q^2-1}{q^3} \\ & q^{-1} & \\ & & q^{-1} \end{pmatrix},\ X_2 = \begin{pmatrix} q^{-1} & & \frac{q^2-1}{q^3} \\ & q & \frac{(q^2-1)(q^2+2)}{q} \\ & & q^{-1} \end{pmatrix},$$

$$T_1 = \begin{pmatrix} q & & \\ q(2+q^2) & -q^{-1} & \\ -q & & -q^{-1} \end{pmatrix},\ T_2 = \begin{pmatrix} -q^{-1} & q^{-1} & q \\ & q & q(q^2+1) \\ & & -q^{-1} \end{pmatrix}.$$

(3) *Suppose* $p = q^2$. *Then they have* 1-*dimensional composition factor and* 3-*dimensional non-calibrated composition factor which are presented by the following representation matrices*;

(i) *case* $\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_1^b$;

- $X_1 = 1,\ X_2 = q^{-2},\ T_1 = q,\ T_2 = -q^{-2}.$

- U_b^1:

$$X_1 = \begin{pmatrix} 1 & & \\ & q^2 & \\ & & q^2 \end{pmatrix}, \ X_2 = \begin{pmatrix} q^2 & & \\ & 1 & \frac{q^4-1}{q^2} \\ & & 1 \end{pmatrix},$$

$$T_1 = \begin{pmatrix} -q^{-1} & & -\frac{(q^2+1)^2}{q^2} \\ 1 & q & \frac{q^2+1}{q} \\ & & q \end{pmatrix}, \ T_2 = \begin{pmatrix} q^2 & & \\ & & 1 \\ & 1 & \frac{q^4-1}{q^2} \end{pmatrix}.$$

(ii) *case* $\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_2^b$:

- $X_1 = 1, \ X_2 = q^2, \ T_1 = -q^{-1}, \ T_2 = q^2.$
- U_b^2:

$$X_1 = \begin{pmatrix} q^{-2} & & \\ & q^{-2} & \\ & & 1 \end{pmatrix}, \ X_2 = \begin{pmatrix} 1 & \frac{q^4-1}{q^2} & \\ & 1 & \\ & & q^{-2} \end{pmatrix},$$

$$T_1 = \begin{pmatrix} -q^{-1} & \frac{q^2+1}{q} & \frac{(q^2+1)^2}{q^2} \\ & -q^{-1} & \\ & 1 & q \end{pmatrix}, \ T_2 = \begin{pmatrix} & 1 & \\ 1 & \frac{q^4-1}{q^2} & \\ & & -q^{-2} \end{pmatrix}.$$

Example 3.4. We consider $M(\chi_c)$. Let ρ_1^c and ρ_2^c be the following 1-dimensional representations of $\widehat{\mathcal{H}}_2$;

	X_1	X_2	T_2
ρ_1^c	$-p^{-1}$	p	p
ρ_2^c	$-p^{-1}$	$-p^{-1}$	$-p^{-1}$

Since $\chi_c(\alpha_2^\vee) = p^2$, we can apply the decompose lemma (Lemma 2.2) to $M(\chi_c)$.

Lemma 3.5. (1) *Suppose* $p^2 \neq -q^{\pm 2}$. $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_1^c$ *and* $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_2^c$ *have two* 2*-dimensional irreducible calibrated composition factors which are presented by the following representation matricies*;

composition factors of $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_1^c$;

	X_1	X_2	T_1	T_2
U_c^1	$\begin{pmatrix} p & \\ & -p \end{pmatrix}$	$\begin{pmatrix} -p & \\ & p \end{pmatrix}$	$\begin{pmatrix} \frac{q^2-1}{2q} & \frac{(1+q^2)^2}{4q^2} \\ 1 & \frac{q^2-1}{2q} \end{pmatrix}$	$\begin{pmatrix} p & \\ & p \end{pmatrix}$
U_c^3	$\begin{pmatrix} -p^{-1} & \\ & p \end{pmatrix}$	$\begin{pmatrix} p & \\ & -p^{-1} \end{pmatrix}$	$\begin{pmatrix} \frac{q^2-1}{(p^2+1)q} & \frac{(p^2+q^2)(1+p^2q^2)}{(p^2+1)^2q^2} \\ 1 & \frac{p^2(q^2-1)}{(p^2+1)q} \end{pmatrix}$	$\begin{pmatrix} p & \\ & -p^{-1} \end{pmatrix}$

composition factors of $\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_2^c$;

	X_1	X_2	T_1	T_2
U_c^2	$\begin{pmatrix} -p^{-1} & \\ & p^{-1} \end{pmatrix}$	$\begin{pmatrix} p^{-1} & \\ & -p^{-1} \end{pmatrix}$	$\begin{pmatrix} \frac{q^2-1}{2q} & \frac{(1+q^2)^2}{4q^2} \\ 1 & \frac{q^2-1}{2q} \end{pmatrix}$	$\begin{pmatrix} -p^{-1} & \\ & -p^{-1} \end{pmatrix}$
U_c^4	$\begin{pmatrix} p^{-1} & \\ & -p \end{pmatrix}$	$\begin{pmatrix} -p & \\ & p^{-1} \end{pmatrix}$	$\begin{pmatrix} \frac{q^2-1}{(p^2+1)q} & \frac{(p^2+q^2)(1+p^2q^2)}{(p^2+1)^2q^2} \\ 1 & \frac{p^2(q^2-1)}{(p^2+1)q} \end{pmatrix}$	$\begin{pmatrix} p & \\ & -p^{-1} \end{pmatrix}$

(2) *Suppse* $p^2 = -q^2$. *They have one* 2*-dimensional irreducible calibrated composition factor and two* 1*-dimensional composition factors. And their representation matrices are obtained by putting* $p^2 = -q^2$ *in above matrices, since specialization lemma* (*Lemma* 2.1). *More precisely,* U_c^1, U_c^2 *are irreducible, but* U_c^3, U_c^4 *have two 1-dimensional composition factors.*

3.3. Classification Theorem

By the preceding Examples and Lemmas, we obtain the following classification theorem.

First, let us define the 1-dimensional representations of $\widehat{\mathcal{H}}_i$ in addition to the notation in the preceding Examples and Lemmas;

$\widehat{\mathcal{H}}_1$	$\rho_1^{d^{(1)}}$	$\rho_2^{d^{(1)}}$	$\rho_1^{d^{(2)}}$	$\rho_2^{d^{(2)}}$	$\rho_1^g(u)$	$\rho_2^g(u)$
X_1	q^2	1	q	q^{-1}	q^2u	u
X_2	1	q^2	q^{-1}	q	u	q^2u
T_1	q	$-q^{-1}$	q	$-q^{-1}$	q	$-q^{-1}$

$\widehat{\mathcal{H}}_2$	$\rho_1^{d^{(3)}}$	$\rho_2^{d^{(3)}}$	$\rho_1^{d^{(4)}}$	$\rho_2^{d^{(4)}}$	$\rho_1^f(v)$	$\rho_2^f(v)$
X_1	p	p	1	1	pv	pv
X_2	p	p^{-1}	p	p^{-1}	p	p^{-1}
T_2	p	$-p^{-1}$	p	$-p^{-1}$	p	$-p^{-1}$

Theorem 3.1. *Suppose that* p *and* q *are not a root of unity. The finite-dimensional irreducible representations of type* B_2 *with unequal parameters are given by the following lists depending on the relation of parameters.*
(0) *The principal series representations* $M(\chi)$, *where* $\chi \neq \pm\chi_a, \pm\chi_b, \chi_c, \pm\chi_d^{(j)}$ $(1 \leq j \leq 5), \pm\chi_f(v), \chi_g(u)$ *and their* W*-orbits, are irreducible.*
(1) *For any* p, q, *there are eight 1-dimensional* (*irreducible*) *representations defined by*

X_1	q^2p	$q^{-2}p^{-1}$	q^2p^{-1}	$q^{-2}p$	$-q^2p$	$-q^{-2}p^{-1}$	$-q^2p^{-1}$	$-q^{-2}p$
X_2	p	p^{-1}	p^{-1}	p	$-p$	$-p^{-1}$	$-p^{-1}$	$-p$
T_1	q	$-q^{-1}$	q	$-q^{-1}$	q	$-q^{-1}$	q	$-q^{-1}$
T_2	p	$-p^{-1}$	$-p^{-1}$	p	p	$-p^{-1}$	$-p^{-1}$	p

(2) *For any* p, q,

$$\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_1^f(v),\ \mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_2^f(v),\ \mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} (-\rho_1^f(v)),\ \mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} (-\rho_2^f(v))$$
$$\text{with } v \neq \pm p^{-2}, \pm p^{-1}, \pm 1, q^{\pm 2}, q^{\pm 2}p^{-2}$$
$$\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_1^g(u),\quad \mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_2^g(u) \text{ with } u \neq \pm p^{\pm 1}, \pm 1, \pm q^{-2}, \pm q^{-1}, \pm q^{-2}p^{\pm 1}$$

are 4*-dimensional one parameter families of irreducible representations and calibrated. They are not isomorphic to each other.*
(3) *When* p, q *are generic i.e.* $p \neq \pm q^{\pm 2}, \pm q^{\pm 1}$ *and* $p^2 \neq -q^{\pm 2}$, *the remaining finite-dimensional irreducible representations are the following*;

(I) U_c^i $(1 \le i \le 4)$ *which are* 2*-dimensional and calibrated.*
(II) $U_a^i, U_b^i, U_{-a}^i, U_{-b}^i$ $(i = 1, 2)$ *which are* 3*-dimensional and calibrated.*
(III)

$$\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_j^{d^{(i)}}, \mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} (-\rho_j^{d^{(i)}}) \ (j = 1, 2, i = 1, 2),$$
$$\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_j^{d^{(i)}}, \mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} (-\rho_j^{d^{(i)}}) \ (j = 1, 2, i = 3, 4, 5)$$

which are 4*-dimensional and non-calibrated.*
(4) *When $p = q^2$, the remaining finite-dimensional irreducible representations are the following*;
(I) U_c^i $(1 \le i \le 4)$ *which are* 2*-dimensional and calibrated.*
(II) U_a^i, U_{-a}^i, $(i = 1, 2)$ *which are* 3*-dimensional and calibrated.*
(III) U_b^i, U_{-b}^i, $(i = 1, 2)$ *which are* 3*-dimensional and non-calibrated.*
(IV)

$$\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_j^{d^{(i)}}, \mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} (-\rho_j^{d^{(i)}}) \ (j = 1, 2, i = 2),$$
$$\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_j^{d^{(i)}}, \mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} (-\rho_j^{d^{(i)}}) \ (j = 1, 2, i = 3, 5)$$

which are 4*-dimensional and non-calibrated.*
(5) *When $p = q$, the remaining finite-dimensional irreducible representations are the following*;
(I) U_c^i $(1 \le i \le 4)$ *which are* 2*-dimensional and calibrated.*
(II) U_a^i, U_{-a}^i, $(i = 1, 2)$ *which are* 3*-dimensional and calibrated.*
(III) U_b^i, U_{-b}^i, $(i = 1, 2)$ *which are* 3*-dimensional and non-calibrated.*
(IV)

$$\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_j^{d^{(i)}}, \mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} (-\rho_j^{d^{(i)}}) \ (j = 1, 2, i = 1),$$
$$\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_j^{d^{(i)}}, \mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} (-\rho_j^{d^{(i)}}) \ (j = 1, 2, i = 4, 5)$$

which are 4*-dimensional and non-calibrated.*
(6) *When $p^2 = -q^2$, the remaining finite-dimensional irreducible representations are the following*;
(I) U_c^i $(i = 1, 2)$ *which are* 2*-dimensional and calibrated.*
(II) U_a^i, U_{-a}^i, $(1 \le i \le 2)$ *which are* 3*-dimensional and calibrated.*
(III)

$$\mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} \rho_j^{d^{(i)}}, \mathrm{Ind}_{\widehat{\mathcal{H}}_1}^{\widehat{\mathcal{H}}} (-\rho_j^{d^{(i)}}) \ (j = 1, 2, i = 1, 2),$$
$$\mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} \rho_j^{d^{(i)}}, \mathrm{Ind}_{\widehat{\mathcal{H}}_2}^{\widehat{\mathcal{H}}} (-\rho_j^{d^{(i)}}) \ (j = 1, 2, i = 3, 4, 5)$$

which are 4*-dimensional and non-calibrated.*
(7) *Using the following automorphisms of $\widehat{\mathcal{H}}$*

$$X_1 \mapsto X_1, X_2 \mapsto X_2, T_1 \mapsto T_1, T_2 \mapsto -T_2, q \mapsto q, p \mapsto \mp p^{\pm 1}$$

the cases of $p = \pm q^{-2}, -q^2$ *reduces the case* (4)*. Similarly, the cases of* $p = \pm q^{-1}, -q$ *reduces the case* (5)*. The case of* $p^2 = -q^{-2}$ *also reduces the case* (6)*.*

Note 2. In [Ram2], Ram dealt equal parameter case. However he missed the case $\chi_d^{(5)}$ and did not explicitly list the isomorphism classes of irreducible representations $-\chi_a, -\chi_b, -\chi_d^{(j)}$ and $-\chi_f$.

4. Tables of irreducible representations

We will summarize about the dimension of composition factors and their calibratability. Note that we will omit the principal series representation $M(-\chi)$ and their composition factors in the following tables.

4.1. p, q generic case (i.e. $p \neq \pm q^{\pm 1}, \pm q^{\pm 2}$ and $p^2 \neq -q^{\pm 2}$)

	$\chi(X_1)$	$\chi(X_2)$	$P(\chi)$	dim	calibrated?
χ_a	q^2p	p	$\{\alpha_1, \alpha_2\}$	1	○
				3	○
				3	○
				1	○
χ_b	q^2p^{-1}	p^{-1}	$\{\alpha_1, \alpha_2\}$	1	○
				3	○
				3	○
				1	○
χ_c	$-p^{-1}$	p	$\{\alpha_2, 2\alpha_1+\alpha_2\}$	2	○
				2	○
				2	○
				2	○
$\chi_d^{(1)}$	q^2	1	$\{\alpha_1, \alpha_1+\alpha_2\}$	4	×
				4	×
$\chi_d^{(2)}$	q	q^{-1}	$\{\alpha_1\}$	4	×
				4	×
$\chi_d^{(3)}$	p	p	$\{\alpha_2, 2\alpha_1+\alpha_2\}$	4	×
				4	×
$\chi_d^{(4)}$	1	p	$\{\alpha_2\}$	4	×
				4	×
$\chi_d^{(5)}$	-1	p	$\{\alpha_2\}$	4	×
				4	×
$\chi_f(v)$	pv	p	$\{\alpha_2\}$	4	○
				4	○
$\chi_g(u)$	q^2u	u	$\{\alpha_1\}$	4	○
				4	○

4.2. $p = q$ case; equal parameter case

	$\chi(X_1)$	$\chi(X_2)$	$P(\chi)$	dim	calibrated?
χ_a	q^3	q	$\{\alpha_1, \alpha_2\}$	1	○
				3	○
				3	○
				1	○
χ_b	q	q^{-1}	$\{\alpha_1, \alpha_2, 2\alpha_1 + \alpha_2\}$	1	○
				3	×
				3	×
				1	○
χ_c	$-q^{-1}$	q	$\{\alpha_2, 2\alpha_1 + \alpha_2\}$	2	○
				2	○
				2	○
				2	○
$\chi_d^{(1)}$	q^2	1	$\{\alpha_1, \alpha_1 + \alpha_2\}$	4	×
				4	×
$\chi_d^{(4)}$	1	q	$\{\alpha_2\}$	4	×
				4	×
$\chi_d^{(5)}$	-1	p	$\{\alpha_2\}$	4	×
				4	×
$\chi_f(v)$	qv	q	$\{\alpha_2\}$	4	○
				4	○
$\chi_g(u)$	q^2u	u	$\{\alpha_1\}$	4	○
				4	○

4.3. $p = q^2$ case

	$\chi(X_1)$	$\chi(X_2)$	$P(\chi)$	dim	calibrated?
χ_a	q^4	q^2	$\{\alpha_1, \alpha_2\}$	1	○
				3	○
				3	○
				1	○
χ_b	1	q^{-2}	$\{\alpha_1, \alpha_2, \alpha_1 + \alpha_2\}$	1	○
				3	×
				3	×
				1	○
χ_c	$-q^{-2}$	q^2	$\{\alpha_2, 2\alpha_1 + \alpha_2\}$	2	○
				2	○
				2	○
				2	○
$\chi_d^{(2)}$	q	q^{-1}	$\{\alpha_1\}$	4	×
				4	×
$\chi_d^{(3)}$	q^2	q^2	$\{\alpha_2, 2\alpha_1 + \alpha_2\}$	4	×
				4	×
$\chi_d^{(5)}$	-1	q^2	$\{\alpha_2\}$	4	×
				4	×
$\chi_f(v)$	q^2v	q^2	$\{\alpha_2\}$	4	○
				4	○
$\chi_g(u)$	q^2u	u	$\{\alpha_1\}$	4	○
				4	○

4.4. $p^2 = -q^2$ case

	$\chi(X_1)$	$\chi(X_2)$	$P(\chi)$	dim	calibrated?
χ_a	$-p^3$	p	$\{\alpha_1, \alpha_2\}$	1 3 3 1	○ ○ ○ ○
χ_c	$-p^{-1}$	p	$\{\alpha_1, \alpha_2, 2\alpha_1 + \alpha_2\}$	1 1 1 1 2 2	○ ○ ○ ○ ○ ○
$\chi_d^{(1)}$	$-p^2$	1	$\{\alpha_1, \alpha_1 + \alpha_2\}$	4 4	× ×
$\chi_d^{(2)}$	$\pm p\sqrt{-1}$	$\pm p\sqrt{-1}$	$\{\alpha_1\}$	4 4	× ×
$\chi_d^{(3)}$	p	p	$\{\alpha_2, 2\alpha_1 + \alpha_2\}$	4 4	× ×
$\chi_d^{(4)}$	1	p	$\{\alpha_2\}$	4 4	× ×
$\chi_d^{(5)}$	-1	p	$\{\alpha_2\}$	4 4	× ×
$\chi_f(v)$	pv	p	$\{\alpha_2\}$	4 4	○ ○
$\chi_g(u)$	$-p^2u$	u	$\{\alpha_1\}$	4 4	○ ○

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