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The Jacobian Problem for singular surfaces

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Abstract

We consider normal affine surfaces X with étale endomorphisms. It is proved that if X has at least one singular point which is not a quotient singular point then such an endomorphism is an isomorphism.

1. Introduction

In this paper we will consider the Jacobian Problem for normal affine surfaces. Let X be an irreducible normal affine surface defined over $\mathbb{C}$. For notational simplicity, let X_u ("upper" X) and X_ℓ ("lower" X) be two copies of X. Let $\varphi : X_u \to X_\ell$ be an étale morphism. The Generalized Jacobian Problem (GJP, for short) asks if φ is a finite morphism. In this generality the answer is negative [7, §4, Example 3]. For some positive results we refer the reader to [7, 6], where the surface X is assumed to be smooth. We do not know if GJP is true for a singular quotient $\mathbb{C}^2/G$, where G is a finite group of automorphisms of $\mathbb{C}^2$. The authors feel that proving GJP for this case will be an important step for proving the classical Jacobian Problem when $X = \mathbb{C}^2$.

In this paper we will prove the following result.

Theorem 1. *With the above notation, let $\widetilde{X}$ be the normalization of X_ℓ in the function field of X_u. Then $\widetilde{X} - X_u$ is a disjoint union of irreducible curves C_i, each of which is isomorphic to $\mathbb{A}^1$. Further, each singularity of $\widetilde{X}$ which is not contained in X_u is a cyclic quotient singularity. Any irreducible component of $\widetilde{X} - X_u$ contains at most one such singular point.*

Our proof of this result uses the classification theory of non-complete algebraic surfaces developed by S. Iitaka, Y. Kawamata, T. Fujita, M. Miyanishi, T. Sugie, S. Tsunoda and others.

The next result is our main motivation for this paper.

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Corollary 2. *Let X be an affine normal surface which has at least one singularity which is not a quotient singularity. Then any étale map $\varphi : X \to X$ is an isomorphism.*

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2. Preliminaries

We will only consider complex algebraic varieties. For any variety Z let $\chi(Z)$ denote the topological Euler-Poincaré characteristic of Z. For a smooth quasi-projective variety Z, let $\overline{\kappa}(Z)$ denote the logarithmic Kodaira dimension as defined by S. Iitaka [4].

Let $\mathbb{A}^1$ denote the affine line as an algebraic variety. Similarly, let $\mathbb{C}^*$ (or $\mathbb{A}^{1*}$) denote the algebraic curve $\mathbb{A}^1 - \{\text{one point}\}$. A morphism $f : Z \to B$ from a normal affine surface to a smooth algebraic curve is called an $\mathbb{A}^1$-fibration if a general fiber of f is isomorphic to $\mathbb{A}^1$. Similarly, a $\mathbb{C}^*$-fibration and a $\mathbb{P}^1$-fibration is defined.

A simple normal crossing divisor on a smooth algebraic surface will be called an SNC divisor. An embedding $Z \subset V$ of a normal quasi-projective surface Z into a normal projective surface V is called an *SNC completion* if V is smooth outside Z and $V - Z$ is an SNC divisor. A smooth irreducible rational curve with self-intersection $-n$ on a smooth algebraic surface will be called a $(-n)$*-curve*. An irreducible component C of an SNC divisor Δ is called an *extraneous* (-1)*-curve* if C meets at most two other irreducible components of Δ.

For an SNC divisor we use the terms like "twig", branch point, etc, from [2]. For a normal variety Z, let Sing Z denote the singular locus of Z and let Z^0 denote $Z - \text{Sing } Z$.

Let (V, p) be a germ of a normal surface singularity. We say that (V, p) is a *quasi-rational* singularity if the exceptional divisor in a suitable resolution of singularities of (V, p) is a tree of smooth rational curves.

3. Proof of Theorem 1

We begin with the following useful result which is an approximation to the main result we want to prove.

Proposition 3. *$\widetilde{X} - X_u$ is a disjoint union of contractible irreducible curves and any singularity of $\widetilde{X}$ not contained in X_u is quasi-rational.*

Proof. Let $\widetilde{X} \subset \widetilde{V}$ be an SNC completion such that $\widetilde{\triangle} := \widetilde{V} - \widetilde{X}$ is an SNC divisor and $\widetilde{V}$ is smooth outside $\widetilde{X}$. Let $\widetilde{N}$ be a nice tubular neighborhood of $\widetilde{\triangle}$ in $\widetilde{X}$. Then $\widetilde{\triangle}$ is a strong deformation retract of $\widetilde{N}$. Hence $H_1(\widetilde{N}, \mathbb{R})$ is isomorphic to $H_1(\widetilde{\triangle}, \mathbb{R})$. We consider a good compactification $X_\ell \subset V$ as above and a tubular neighborhood N of $\triangle = V - X_\ell$. The properness of $\tilde{X} \to X$

implies that there is a continuous proper map with finite fibers $\widetilde{N} \to N$. The following result can be proved using either transfer homomorphism in the theory of finite transformation groups or triangulation of complex analytic varities [3].

Lemma 4. *Let V, W be normal complex analytic varieties and let $f : V \to W$ be a finite complex analytic map. Then the induced homomorphisms $H_i(V, \mathbb{R}) \to H_i(W, \mathbb{R})$ are surjective for $i = 0, 1, 2, \cdots$.*

Using this we see that $H_1(\widetilde{N}, \mathbb{R}) \to H_1(N, \mathbb{R})$ is surjective. The rank of $H_1(\widetilde{N}, \mathbb{R})$ is equal to $\sum 2\tilde{g}_i + \tilde{p}$, where $\tilde{g}_1, \tilde{g}_2 \cdots$ are the genera of irreducible components of $\widetilde{V} - \widetilde{X}$ and $\tilde{p}$ is the number of independent loops in the dual graph of $\widetilde{\triangle}$. Let $g_1, g_2, \cdots$ be the genera of irreducible components of $V - X_\ell$ and p the number of loops in the dual graph of $V - X_\ell$. Since $X_u \subset \widetilde{V}$, we have $\sum g_i \geq \sum \tilde{g}_j$ and $p \geq \tilde{p}$. Now by the above lemma $\sum 2\tilde{g}_j + \tilde{p} \geq \sum 2g_i + p$. This easily implies that $\tilde{p} = p$ and $\sum g_i = \sum \tilde{g}_j$. It follows that $\widetilde{X} - X_u$ is a disjoint union of irreducible contractible curves and any singularities of $\widetilde{X}$ which is not contained in X_u is quasi-rational. This completes the proof of Proposition 3. □

We will divide the proof of Theorem 1 according to the value of $\overline{\kappa}(X^0)$.

First we will show that X_u is naturally a Zariski-open subset of $\widetilde{X}$. Since $\widetilde{X}$ is the normalization of X_ℓ in the function field of X_u there is an induced birational morphism $X_u \to \widetilde{X}$. The fibers of this morphism are clearly finite. Hence by Zariski's Main Theorem X_u embeds in $\widetilde{X}$ as a Zariski-open subset via this morphism.

Next, we will dispose off the easy case when $\overline{\kappa}(X^0) = 2$.

Since the morphism $X_u \to X_\ell$ is étale, we get an induced morphism $X_u^0 \to X_\ell^0$. If $\overline{\kappa}(X^0) = 2$, then by a basic result due to Iitaka [4, Theorems 1, 2], the morphism $X_u^0 \to X_\ell^0$ is an isomorphism. Hence in this case $\varphi : X_u \to X_\ell$ is also an isomorphism.

From now onwards we will assume that $\overline{\kappa}(X^0) < 2$.

Case 1. Suppose $\overline{\kappa}(X^0) = -\infty$.

We have $X_u^0 \subset \widetilde{X}^0$.

Case 1.1. Suppose that X_u^0 admits an $\mathbb{A}^1$-fibration $\pi : X_u^0 \to C$. Since $\widetilde{X}$ is affine, π extends to an $\mathbb{A}^1$-fibration $\widetilde{\pi} : \widetilde{X} \to \widetilde{C}$, where $\widetilde{C} \supset C$ as a Zariski-open subset. In [8, Chapter I, §6], it is proved that $\widetilde{X} - X_u$ is a disjoint union of irreducible curves isomorphic to $\mathbb{A}^1$ and $\widetilde{X}$ has at worst cyclic quotient singularities. Further, any irreducible component of $\widetilde{X} - X_u$ contains at most one singular point of $\widetilde{X}$.

Case 1.2. Suppose that X_u^0 does not have an $\mathbb{A}^1$-fibration. In this case by a result of Miyanishi-Tsunoda [9, Chapter 3, Theorem 2.5.4] X_u contains

$\mathbb{C}^2/G$ as a Zariski-open subset, where G is a finite group acting linearly on $\mathbb{C}^2$. There exists a natural $\mathbb{C}^*$-fibration $\pi : (\mathbb{C}^2/G)^0 \to \mathbb{P}^1$. The closure of any fiber of π passes through the singular point of $\mathbb{C}^2/G$. It follows that any such closure is closed in $\widetilde{X}$, so that π extends to a $\mathbb{C}^*$-fibration $\tilde{\pi} : \widetilde{X} - \mathrm{Sing}\ \mathbb{C}^2/G \to \mathbb{P}^1$. Let F be the closure of a general fiber of π. Then $\widetilde{\pi}|_{\widetilde{X}-F}$ is a $\mathbb{C}^*$-fibration to $\mathbb{A}^1$. By Proposition 3 we see that $\widetilde{X} - X_u$ cannot have a $\mathbb{C}^*$ or a union of two $\mathbb{A}^1$'s meeting transversally in one point as a connected component. The result [10, Lemma 2.9] describes the possible singular fibers of a $\mathbb{C}^*$-fibration on a normal affine surface with at most quotient singularities. In order to use this result, we first remark that the proof of Lemma 2.7 in [6] shows that every singular point of $\widetilde{X}$ which lies outside X_u is a quotient singular point. Now by the result in [10] every singular point of $\widetilde{X}$, other than Sing $\mathbb{C}^2/G$, is a cyclic quotient singular point. Further, since X_u contains $\mathbb{C}^2/G$ as a Zariski-open subset and the divisor at infinity for $\mathbb{C}^2/G$ is a tree of $\mathbb{P}^1$'s, we conclude again using [10, Lemma 2.9] that $\widetilde{X} - X_u$ is a disjoint union of $\mathbb{A}^1$'s. Finally, again by the result in [10], we see that any irreducible component of $\widetilde{X} - X_u$ contains at most one singular point of $\widetilde{X}$. This completes the proof of Theorem 1 when $\overline{\kappa}(X^0) = -\infty$.

Case 2. $\overline{\kappa}(X^0) = 1$.

In this case, by the basic structure theorem due to Kawamata [8, Chapter II], there is a morphism $\rho : X^0 \to C$ which is a $\mathbb{C}^*$-fibration induced by the linear system $|\ n(D + K_V)\ |$ with $n \gg 0$, where V is an SNC-completion of X_u^0 and $D = V - X_u^0$.

Since $\varphi : X_u \to X_\ell$ is unramified, φ induces $\varphi^0 : X_u^0 \to X_\ell^0$. It is proved in [6, Lemma 2.4] that φ^0 maps fibers of ρ to fibers of ρ. This implies that there exists an endomorphism $\alpha : C \to C$ such that $\alpha \cdot \rho = \rho \cdot \varphi^0$ and α is an étale endomorphism. Then α is not an automorphism only if $C \cong \mathbb{C}^*$.

We first assume that α is an automorphism. The normalization morphism $\widetilde{\varphi} : \widetilde{X} \to X_\ell$ induces $\widetilde{\varphi}^0 : \widetilde{X}^0 - S \to X_\ell^0 - S^0$, where S, S^0 are finite sets and $\widetilde{\varphi}^0$ is a finite map; a smooth point of $\widetilde{X}^0$ may be mapped to a singular point of X_ℓ. Note that $\overline{\kappa}(\widetilde{X}^0) = \overline{\kappa}(\widetilde{X}^0 - S) = \overline{\kappa}(X^0)$. Hence there exists a $\mathbb{C}^*$-fibration $\widetilde{\rho} : \widetilde{X}^0 \to \widetilde{C}$ such that $\widetilde{\rho}|_{X_u^0} = \rho$. Then $\widetilde{\varphi}\ |_{\widetilde{X}^0 - S} : \widetilde{X}^0 - S \to X_\ell^0 - S^0$ induces a finite morphism $\widetilde{\alpha} : \widetilde{C} \to C$ such that $\widetilde{\alpha}\ |_C = \alpha$. Then $C = \widetilde{C}$ because α is an automorphism and hence $\widetilde{\alpha}$ is birational.

Case 2.1. Suppose that the $\mathbb{C}^*$-fibration $\widetilde{\rho}$ on $\widetilde{X}^0$ extends to a $\mathbb{C}^*$-fibration $\widetilde{p} : \widetilde{X} \to \widetilde{C}$. Since $C = \widetilde{C}$, every connected component of $\widetilde{X} - X_u$ is contained in a fiber of $\tilde{p}$. By Proposition 3, $\widetilde{X} - X_u$ is a disjoint union of contractible irreducible curves. Hence using [10, Lemma 2.9] we conclude that $\widetilde{X} - X_u$ is a disjoint union of $\mathbb{A}^1$'s, $\widetilde{X}$ has at most cyclic singular points and each irreducible component of $\widetilde{X} - X_u$ contains at most one such singular point.

Case 2.2. Suppose that the $\mathbb{C}^*$-fibration $\widetilde{\rho}$ on $\widetilde{X}^0$ does not extend to a $\mathbb{C}^*$-fibration on $\widetilde{X}$. Then the closures of fibers of $\widetilde{\rho}$ in $\widetilde{X}$ have a base point O.

Namely, the closures of the fibers of $\widetilde{\rho}$ form a linear pencil with a base point O. Since $\widetilde{\varphi} : \widetilde{X} \to X_\ell$ maps the fibers of the $\mathbb{C}^*$-fibration $\widetilde{\rho}$ to fibers of the $\mathbb{C}^*$-fibration ρ on X_ℓ^0, the base point O lies on X_u. Hence the complement $\widetilde{X} - X_u$ is contained in a union of fibers of a $\mathbb{C}^*$-fibration, which is obtained by eliminating the base points centered at O. The rest of the argument is the same as in Case 2.1.

The proof when α is not an automorphism is very similar.

Case 3. $\overline{\kappa}(X^0) = 0$.

The proof in this case is more involved.

First we recall some results of T. Fujita [2, §6]. Let Z be a smooth quasi-projective (irreducible) surface with $\overline{\kappa}(Z) = 0$. Let $Z \subset W$ be an SNC completion such that $\Delta := W - Z$ does not contain any extraneous (-1)-curves for Δ. We say that (W, Δ) is an *NC-minimal* completion of Z if in the Zariski-Fujita decomposition $K_W + \Delta \approx P + N, N = \mathrm{Bk}^*(\Delta)$. Since $\overline{\kappa}(Z) = 0$, for some integer n, nP is an integral divisor which is numerically trivial.

Assume now that $(W, \triangle)$ is an NC-minimal completion of Z. Let $\triangle_1, \triangle_2, \cdots$ be the connected components of $\triangle$. Fujita has shown that each $\triangle_i$ is one of the following:

(1) A minimal resolution of a quotient singular point.

(2) A tree of $\mathbb{P}^1$'s with exactly 2 branch points such that the branch points are connected by a (possibly empty) linear chain of $\mathbb{P}^1$'s and each branch point meets exactly two other (-2)-curves.

(3) A simple loop of $\mathbb{P}^1$'s.

(4) A tree of $\mathbb{P}^1$'s with a unique branch point which meets three linear trees defining cyclic quotient singular points at one of their end points. Further, the absolute values d_1, d_2, d_3 of the determinants of the three trees satisfy $\sum_i 1/d_i = 1$.

(5) A tree of five $\mathbb{P}^1$'s with a unique branch point which intersects the other four curves transversally in one point each, and such that the four curves are all (-2)-curves.

(6) A smooth elliptic curve.

Let $\widetilde{X} - X_u = C_1 \cup C_2 \cup \cdots$ be the irreducible decomposition. By Proposition 3, each C_i is contractible and any singularity of $\widetilde{X}$ outside X_u is quasi-rational. Let V be an SNC completion of $\widetilde{X} - \mathrm{Sing} X_u$ obtained by resolving the singularities of X_u and embedding the resulting surface in an SNC completion V. In this process the singularities of $\widetilde{X}$ (and the curves C_i) not lying in X_u are untouched.

Now we assume the following condition (*).

(*) Either C_1 is not smooth or contains a singular point of $\widetilde{X}$ which is not a cyclic quotient singularity.

We will arrive at a contradiction to Fujita's list of NC-minimal completions.

Next we blow up finitely many points of V to obtain a surface $\widetilde{V}$ such that $\triangle_1 := \widetilde{C}_1 \cup E_1, \triangle_2 := \widetilde{C}_2 \cup E_2, \cdots$ are SNC divisors, where $\widetilde{C}_i$ is the closure of the proper transform of C_i in $\widetilde{V}$ and E_i is the union of exceptional irreducible curves which arise by resolving singularities of $\widetilde{X}$ lying on C_i and singularities of C_i itself. Let D_∞ be the connected component of $\widetilde{V} - \widetilde{X}'$ which supports a *big* divisor, where $\widetilde{X}'$ is the Zariski-open subset of $\widetilde{V}$ obtained from $\widetilde{X}$ by blowing up points lying on all the C_i as above and singular points of X_u. We can assume that the only possible extraneous (-1)-curves of $\widetilde{C}_1 \cup E_1$ is $\widetilde{C}_1$. Using $\overline{\kappa}(\widetilde{X}^0) = 0$ we deduce easily that the dual graph of D_∞ is non-linear, even after contracting extraneous (-1)-curves one by one.

Case 3.1. Suppose that X_u^0 has an NC-minimal completion.

Now any irreducible component of D_∞ which is a (-1)-curve and which meets at most two other irreducible components of D_∞ arises while making the divisor $D_\infty \cup \widetilde{C}_1$ a simple normal crossing divisor. Such an irreducible component meets $\widetilde{C}_1$ and two other irreducible components of D_∞.

We assume, as above, that C_1 satisfies (*). If $\widetilde{C}_1 \cup E_1 \cup D_\infty$ has no extraneous (-1)-curve then it is easy to see that $\widetilde{C}_1 \cup E_1 \cup D_\infty$ cannot be one of Fujita's list.

If C_2 also satisfies (*) then blowing down extraneous (-1)-curves in $\widetilde{C}_2 \cup E_2 \cup D_\infty$ successively we get a divisor (taking union with the image of $\widetilde{C}_1 \cup E_1 \cup D_\infty$) with at least three branch points. This will contradict Fujita's list. Hence only C_1 can satisfy (*). Assume that this is the case. We analyse $\widetilde{C}_2 \cup E_2 \cup D_\infty, \widetilde{C}_3 \cup E_3 \cup D_\infty, \cdots$. Now either $\widetilde{C}_2$ meets a twig of D_∞ transversally in one point or a non-twig irreducible component of D_∞. After contracting extraneous (-1)-curves in $\widetilde{C}_2 \cup E_2 \cup D_\infty$ successively as above we reach a new normal crossing divisor. We do this for each of $\widetilde{C}_3 \cup E_3 \cup D_\infty, \cdots$ *before touching* $\widetilde{C}_1 \cup E_1 \cup D_\infty$. Let $\widetilde{D}$ be the image of $\widetilde{C}_2 \cup E_2 \cup \widetilde{C}_3 \cup E_3 \cup \cdots \cup D_\infty$ after all the contractions are made.

We claim that $\overline{\kappa}(\widetilde{V} - (F \cup \widetilde{C}_2 \cup E_2 \cup D_\infty \cup \widetilde{C}_3 \cup E_3 \cup \cdots)) = 0$, where F is $(\widetilde{C}_1 \cup E_1) \cap \widetilde{\varphi}^{-1}(\mathrm{Sing} X_\ell)$ which might be an empty set and is disjoint from $\widetilde{C}_2 \cup E_2 \cup D_\infty \cup \widetilde{C}_3 \cup E_3 \cdots$. This follows from the facts that $\overline{\kappa}(X^0) = 0$, $X_u^0 \subset (\widetilde{V} - (F \cup \widetilde{C}_2 \cup E_2 \cup D_\infty \cup \widetilde{C}_3 \cup E_3 \cup \cdots)$ and there is an induced domiant morphism $\widetilde{V} - (F \cup \widetilde{C}_2 \cup E_2 \cup D_\infty \cup \widetilde{C}_3 \cup E_3 \cup \cdots) \to X_\ell^0$. Using this we infer that the divisor $\widetilde{D}$ still has at least one branch point.

Suppose now that $\widetilde{C}_1$ meets exactly two other irreducible components of $\widetilde{C}_1 \cup E_1 \cup D_\infty$ and it is a (-1)-curve. After contracting $\widetilde{C}_1$ and subsequent extraneous (-1)-curves the image of $\widetilde{C}_1 \cup E_1 \cup D_\infty$ still has at least two branch points. Let this new divisor which is the image of $\widetilde{C}_1 \cup E_1 \cup D_\infty$ be $\widetilde{\widetilde{D}}$. $\widetilde{\widetilde{D}}$ has at least two branch points and if it has two branch points then it has to be of type (2) in Fujita's list of NC-minimal completions.

This in turn implies that $\widetilde{D}$ has the following form. $\widetilde{D}$ has a unique branch point and exactly three linear chains of $\mathbb{P}^1$'s meeting the branch point at one of their end components and two of the chains consist of a single (-2)-curve. Further, $\widetilde{C}_1$ meets the end component of the third linear chain away from the branch point.

Now $\widetilde{D}$ supports a big divisor since D_∞ does. Using the basic theory of Zariski-Fujita decomposition of $K_{\widetilde{V}} + \widetilde{D}$ it is easy to deduce that $\overline{\kappa}(\widetilde{V} - \widetilde{D}) = -\infty$, a contradiction [5, Theorem 1].

Remark 1. It is easy to see from Mumford's result on topology of normal singularities [11] that for a nice tubular neighborhood N of $\widetilde{D}$ the group $\pi_1(\partial N)$ is finite, i.e., the fundamental group at infinity for $\widetilde{V} - \widetilde{D}$ is finite. It is proved in [5, Theorem 1] that for any smooth affine surface with finite fundamental group at infinity the log Kodaira dimension is $-\infty$.

Hence even C_1 cannot satisfy (*). The proof of Case 3.1 is thus complete.

Case 3.2. Suppose that X_u^0 does not have an NC-minimal completion.

By [2, Lemma 6.20], $\widetilde{V}$ contains a (-1)-curve L which is *not* contained in $\widetilde{V} - X_u^0$ and which meets $\widetilde{V} - X_u^0$ in at most two points transversally. If $L \cap (\widetilde{V} - X_u^0)$ has two points then L meets an end (irreducible) component of a connected component of $\widetilde{V} - X_u^0$ which is the minimal resolution of a cyclic quotient singular point of X_u. Since $\widetilde{X}$ is affine we infer that $L \cap ((\widetilde{C}_1 \cup E_1) \cup (\widetilde{C}_2 \cup E_2) \cup (\widetilde{C}_3 \cup E_3) \cup \cdots) = \phi$. Thus either $L \cap X_u^0$ is $\mathbb{A}^1$ or the image of $L - \widetilde{D}$ in X_u is a contractible curve. We contract such curves L successively and in finitely many steps reach an NC-minimal completion of an affine open subset of X_u^0. In this process $(C_1 \cup E_1) \cup (C_2 \cup E_2) \cdots$ is untouched. Finally we reach the situation of Case 1 and get a contradiction as in that case.

We have also proved that X_u^0 and $\widetilde{X}^0$ have the same NC-minimal model. This completes the proof of the case $\overline{\kappa}(X^0) = 0$.

4. Proof of Corollary 2

Assume that X has a singular point p_0 which is not a quotient singular point. Since $\widetilde{X} \to X_\ell$ is a finite morphism and each point of $\widetilde{X}$ which is not in X_u is either smooth or a cyclic quotient singular point, no point in $\widetilde{X} - X_u$ maps to p_0. Set $d = \deg \varphi$. Then $\varphi^{-1}(p_0)$ has d distinct points in X_u. Let N = number of singular points in X which are not quotient singularities. By the above argument X_u contains $d \cdot N$ points which are not quotient singular points. This implies that $d = 1$. Now the map $\varphi : X_u \to X_\ell$ is injective and hence by the theorem of Ax also surjective [1]. This completes the proof of Corollary 2.

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