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Almost periodic models of impulsive Hopfield neural networks

By

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Abstract

In the present paper the problems of existence and uniqueness of almost periodic solutions for impulsive Hopfield neural networks are considered. The impulses are in fixed moments of time and by using the technique of estimation of the Cauchy's matrix new sufficient conditions for the exponential stability of the unique almost periodic solution of such systems are given.

1. Introduction

In the last ten years, many researches have been focused on the study of dynamics of neural network models. Stability of different classes of neural networks, such as Hopfield neural networks, cellular neural networks, bidirectional associative neural networks, Lotka-Volterra neural networks, has been extensively studied and various stability conditions have been obtained for these models of neural networks. See, for example, [1], [2], [4], [5], [15] and the references cited therein.

Since in many neural network models the solutions experience discontinuous jumps at certain moments of the evolution process, the study of impulsive neural network systems has had a great importance lately.

In recent years impulsive differential equations have been intensively researched [6], [8]. Recently, some qualitative properties (oscillation, asymptotic behavior and stability) are investigated in [14].

One of the most important part of qualitative theory of the differential equations is the theory of almost periodic solutions. The main results related to the study of the existence of almost periodic solutions for impulsive dynamical systems are studied in [8], [9]–[13].

In the present paper sufficient conditions for existence and uniqueness of almost periodic solutions for impulsive Hopfield neural networks are obtained. The impulse effects are in fixed moments of time.

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Results related to the study of the existence of almost periodic solutions for Hopfield neural networks without impulses have been obtained in [1], [2], [5], [15].

2. Preliminary notes

Let $\mathbb{R}^n$ be the n-dimensional Euclidean space with elements $x = col(x_1, x_2, \ldots, x_n)$ and norm $|x| = \max_i |x_i|$, $\mathbb{R} = (-\infty, \infty)$, Ω be a domain in $\mathbb{R}^n$, $\Omega \neq \emptyset$.

By UB, $UB = \{\{\tau_k\} : \ \tau_k \in \mathbb{R}, \ \tau_k < \tau_{k+1}, \ k \in \mathbb{Z}\}$ we denote the set of all sequences unbounded and strictly increasing.

We shall investigate the problem of existence of almost periodic solutions of the system of impulsive Hopfield neural networks of the form

$$(1) \qquad \begin{cases} \dot{x}_i(t) = \sum_{j=1}^{n} a_{ij}(t) x_j(t) + \sum_{j=1}^{n} \alpha_{ij}(t) f_j(x_j(t)) + \gamma_i(t), \ t \neq \tau_k, \\ \Delta x(t) = A_k x(t) + I_k(x(t)) + p_k, \ t = \tau_k, \ k \in \mathbb{Z}, \end{cases}$$

where

(i) $t \in \mathbb{R}$, $a_{ij}(.), \alpha_{ij}(.)$, $f_j(.)$, $\gamma_i(.) \in C(\mathbb{R}, \mathbb{R})$, $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, n$;

(ii) $A_k \in \mathbb{R}^{n \times n}$, $I_k(.) \in C(\Omega, \mathbb{R}^n)$, $p_k \in \mathbb{R}^n$, $k \in \mathbb{Z}$;

(iii) $\Delta x(t) = x(t+0) - x(t-0)$, $\{\tau_k\} \in UB$.

Let $PC(J, \mathbb{R}^n)$, $J \subset \mathbb{R}$ is the space of all piecewise continuous functions $x : J \to \mathbb{R}^n$ with points of discontinuity at first kind τ_k at $x(t)$ which it is left continuous, i.e. the following relations hold

$$x(\tau_k - 0) = x(\tau_k), \ x(\tau_k + 0) = x(\tau_k) + \Delta x(\tau_k), \ k \in \mathbb{Z}.$$

Recall [4] it follows that the solution $x(t) = x(t; t_0, x_0)$ of (1) is from $PC(J, \mathbb{R}^n)$ and we adopt the following definitions for almost periodicity.

Definition 1 ([8]). The set of sequences $\{\tau_k^j\}$, $\tau_k^j = \tau_{k+j} - \tau_k$, $k \in \mathbb{Z}$, $j \in \mathbb{Z}$, $\{\tau_k\} \in UB$ is said to be *uniformly almost periodic* if for arbitrary $\varepsilon > 0$ there exists relatively dense set of $\varepsilon-$almost periods common for any sequences.

Definition 2 ([8]). The function $\varphi \in PC(\mathbb{R}, \mathbb{R}^n)$ is said to be *almost periodic*, if:

a) the set of sequences $\{\tau_k^j\}$, $\tau_k^j = \tau_{k+j} - \tau_k$, $k \in \mathbb{Z}$, $j \in \mathbb{Z}$, $\{\tau_k\} \in UB$ is uniformly almost periodic;

b) for any $\varepsilon > 0$ there exists a real number $\delta > 0$ such that if the points t' and t'' belong to one and the same interval of continuity of $\varphi(t)$ and satisfy the inequality $|t' - t''| < \delta$, then $|\varphi(t') - \varphi(t'')| < \varepsilon$;

c) for any $\varepsilon > 0$ there exists a relatively dense set T such that if $\tau \in T$, then $|\varphi(t+\tau) - \varphi(t)| < \varepsilon$ for all $t \in \mathbb{R}$ satisfying the condition $|t - \tau_k| > \varepsilon$, $k \in \mathbb{Z}$. The elements of T are called $\varepsilon -$ *almost periods*.

Together with system (1) we consider the linear system

$$\begin{cases} \dot{x}(t) = A(t)x(t), \ t \neq \tau_k, \\ \Delta x(t) = A_k x(t), \ t = \tau_k, \ k \in \mathbb{Z}, \end{cases} \tag{2}$$

where $t \in \mathbb{R}$, $A(t) = (a_{ij}(t)), \ i = 1, 2, \ldots n, \ j = 1, 2, \ldots, n$.

Introduce the following conditions:

H1. $A(t) \in C(\mathbb{R}, \mathbb{R}^n)$ and it is almost periodic in the sense of Bohr.

H2. $det(E + A_k) \neq 0$ and the sequence $\{A_k\}$, $k \in \mathbb{Z}$ is almost periodic, $E \in \mathbb{R}^{n \times n}$.

H3. The set of sequences $\{\tau_k^j\}$, $\tau_k^j = \tau_{k+j} - \tau_k, \ k \in \mathbb{Z}, \ j \in \mathbb{Z}, \ \{\tau_k\} \in UB$ is uniformly almost periodic and there exists $\theta > 0$ such that $inf_k\{\tau_k^1\} = \theta > 0$.

Recall [6], [8] it follows that if $U_k(t, s,)$ is the Cauchy matrix for the system

$$\dot{x}(t) = A(t)x(t), \ \tau_{k-1} < t \leq \tau_k, \ \{\tau_k\} \in UB$$

then the Cauchy matrix of the system (2) is in the form

$$W(t,s) = \begin{cases} U_k(t,s), \ \tau_{k-1} < s \leq t \leq \tau_k, \\ U_{k+1}(t, \tau_k + 0)(E + A_k)U_k(t,s), \ \tau_{k-1} < s \leq \tau_k < t \leq \tau_{k+1}, \\ U_{k+1}(t, \tau_k + 0)(E + A_k)U_k(\tau_k, \tau_k + 0) \ldots (E + A_i)U_i(\tau_i, s), \\ \tau_{i-1} < s \leq \tau_i < \tau_k < t \leq \tau_{k+1}. \end{cases}$$

and the solutions of (2) can to write in the form

$$x(t; t_0, x_0) = W(t, t_0)x_0.$$

Introduce the following conditions:

H4. The functions $f_j(t)$ are almost periodic in the sense of Bohr,

$$0 < \sup_{t \in \mathbb{R}} |f_j(t)| < \infty, \ f_j(0) = 0,$$

and there exists $L_1 > 0$ such that for $t, s \in \mathbb{R}$

$$\max_j |f_j(t) - f_j(s)| < L_1 |t - s|, \ j = 1, 2, \ldots, n.$$

H5. The functions $\alpha_{ij}(t)$ are almost periodic in the sense of Bohr, and

$$0 < \sup_{t \in \mathbb{R}} |\alpha_{ij}(t)| = \overline{\alpha}_{ij} < \infty.$$

H6. The functions $\gamma_i(t),\ i=1,2,\dots,n$ are almost periodic in the sense of Bohr, the sequence $\{p_k\},\ k\in\mathbb{Z}$ is almost periodic and there exists $C_0>0$ such that

$$\max\left\{\max_i|\gamma_i(t)|,\max_k|p_k|\right\}\le C_0.$$

H7. The sequence of functions $I_k(x)$ is almost periodic uniformly with respect to $x\in\Omega$ and there exists $L_2>0$ such that

$$|I_k(x)-I_k(y)|\le L_2|x-y|$$

for $k\in\mathbb{Z},\ x,y\in\Omega$.

We need the following lemmas.

Lemma 1 ([8]). *Let the following conditions be fulfilled*:

1. *Conditions* H1-H3 *are fulfilled.*
2. *For the Cauchy matrix* $W(t,s)$ *of the system* (2) *there exist positive constants* K *and* λ *such that*

$$|W(t,s)|\le Ke^{-\lambda(t-s)},\ t\ge s,\quad t,s\in\mathbb{R}.$$

Then for any $\varepsilon>0,\ t\in\mathbb{R},\ s\in\mathbb{R},\ t\ge s,\ |t-\tau_k|>\varepsilon,\ |s-\tau_k|>\varepsilon,\ k\in\mathbb{Z}$ *there exists a relatively dense set* T *of* ε*-almost periods of the matrix* $A(t)$ *and a positive constant* Γ *such that for* $\tau\in T$ *it follows*

$$|W(t+\tau,s+\tau)-W(t,s)|\le\varepsilon\Gamma e^{-\frac{\lambda}{2}(t-s)}.$$

We note that inequalities which are used in proof of the main results are connect with the properties of matrix $W(t,s)$ for a system (2). Now we will consider some special cases in which these properties are accomplished.

Definition 3 ([9]). The matrix $A(t)$ is said to be *column dominant with parameter* λ *on* $[a,b],a>0$ if

$$a_{ii}(t)+\sum_{j\ne i}|a_{ji}(t)|\le-\lambda<0$$

for each $i=1,\dots,n$ and $t\in[a,b]$.

Lemma 2. *Let the following conditions be fulfilled*:

1. *The conditions* H1-H3 *are fulfilled.*
2. *The matrix-valued function* $A(t)$ *is column dominant with parameter* $\lambda>0$ *for* $t\in R$.

Then for the Cauchy's matrix $W(t,s)$ it follows

$$|W(t,s)| \leq Ke^{-\lambda(t-s)},$$

where $t \in R,\ s \in R,\ t \geq s,\ K$ is a positive constant.

Proof. The proof of Lemma 2 is analogous to the proof of Lemma 2 in [9]. □

Now from Lemma 2 we have the following lemma.

Lemma 3. *Let the following conditions be fulfilled:*

1. *For the matrix $A(t) = diag[-a_1(t), -a_2(t), \dots, -a_1(t)]$ it follows that: $a_i(t) i = 1, 2, \dots, n$ is almost periodic function in the sense of Bohr and*

$$\lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} a_i(t)dt > 0,\ i = 1, 2, \dots, n.$$

2. *The conditions* H2 *and* H3 *are fulfilled.*

Then for the Cauchy's matrix $W(t,s)$ it follows

$$|W(t,s)| \leq Ke^{-\lambda(t-s)},$$

where $t \in \mathbb{R},\ s \in \mathbb{R},\ t \geq s,\ K,\ \lambda$ are positive constants.

Lemma 4 ([8])**.** *Let the following conditions be fulfilled:*

1. *There exists a constant $\lambda > 0$ such that for $t \in \mathbb{R}$ the eigenvalues $\lambda_i,\ i = 1, 2, \dots, n$ of the almost periodic function in the sense of Bohr matrix $A(t)$ satisfy the conditons*

$$Re\lambda_i(t) < -\lambda.$$

2. *The conditions* H2 *and* H3 *be fulfilled.*

Then for the Cauchy's matrix $W(t,s)$ it follows

$$|W(t,s)| \leq Ke^{-\lambda(t-s)},$$

where $t \in \mathbb{R},\ s \in \mathbb{R},\ t \geq s,\ K$ is positive constant.

Lemma 5 ([8])**.** *Let the conditions* H1-H6 *be fulfilled. Then for each $\varepsilon > 0$ there exist $\varepsilon_1,\ 0 < \varepsilon_1 < \varepsilon$ and relatively dense sets T of real numbers and Q of whole numbers, such that the following relations are fulfilled:*

(*a*) $|A(t+\tau) - A(t)| < \varepsilon,\ t \in \mathbb{R},\ \tau \in T;$

(*b*) $|\alpha_{ij}(t+\tau) - \alpha_{ij}(t)| < \varepsilon,\ t \in \mathbb{R},\ \tau \in T,\ |t - \tau_k| > \varepsilon,\ k \in \mathbb{Z},$ $i, j = 1, 2, \dots, n;$

(*c*) $|f_j(t+\tau) - f_j(t)| < \varepsilon,\ t \in \mathbb{R},\ \tau \in T,\ |t - \tau_k| > \varepsilon,\ k \in \mathbb{Z},\ j = 1, 2, \dots, n;$

(d) $|A_{k+q} - A_k| < \varepsilon,\ q \in Q,\ k \in \mathbb{Z};$
(e) $|\gamma_j(t+\tau) - \gamma_j(t)| < \varepsilon,\ t \in \mathbb{R},\ \tau \in T,\ |t-\tau_k| > \varepsilon,\ k \in \mathbb{Z},\ j = 1, 2, \ldots, n;$
(f) $|p_{k+q} - p_k| < \varepsilon,\ q \in Q,\ k \in \mathbb{Z};$
(g) $|\tau_{k+q} - \tau| < \varepsilon_1,\ q \in Q,\ \tau \in T,\ k \in \mathbb{Z}.$

Lemma 6 ([8]). *Let the set of sequences* $\{\tau_k^j\},\ \tau_k^j = \tau_{k+j} - \tau_k,\ k \in \mathbb{Z},\ j \in \mathbb{Z},\ \{\tau_k\} \in UB$ *be uniformly almost periodic. Then for each* $P > 0$ *there exists a positive integer* N *such that on each interval of length* P*, have no more than* N *elements of the sequence* $\{\tau_k\}$*, i.e.,*

$$i(s,t) \le N(t-s) + N,$$

where $i(s,t)$ *is the number of points* τ_k *in the interval* (s,t)*.*

3. Main results

Theorem 1. *Let the following conditions be fulfilled*:

1. *Conditions* H1-H7 *are fulfilled.*
2. *For the Cauchy matrix* $W(t,s)$ *of the system* (2) *there exist positive constants* K *and* λ *such that*

$$|W(t,s)| \le Ke^{-\lambda(t-s)},\ t \ge s, \quad t, s \in \mathbb{R}.$$

3. *The number*

$$r = K\left\{\max_i \lambda^{-1} L_1 \sum_{j=1}^n \overline{\alpha}_{ij} + \frac{L_2}{1-e^{-\lambda}}\right\} < 1.$$

Then:

1) *There exists a unique almost periodic solution* $x(t)$ *of* (1)*.*
2) *If the following inequalities hold*

$$1 + KL_2 < e,\ \lambda - KL_1 \max_i \sum_{j=1}^n \overline{\alpha}_{ij} - N\ln(1 + KL_2) > 0$$

then the solution $x(t)$ *is exponentially stable.*

Proof of assertion 1. We denote with $D,\ D \subset PC(\mathbb{R}, \mathbb{R}^n)$ the set of all almost periodic functions $\varphi(t)$ satisfying the inequality $\|\varphi\| < \overline{K}$, where

$$\|\varphi\| = \sup_{t\in\mathbb{R}} |\varphi(t)|,\ \overline{K} = KC_0\left(\frac{1}{\lambda} + \frac{1}{1-e^{-\lambda}}\right).$$

Let

$$\varphi_0 = \int_{-\infty}^t W(t,s)\gamma(s)ds + \sum_{t_k<t} W(t,\tau_k)p_k.$$

Then

$$\|\varphi_0\| = \sup_{t\in\mathbb{R}}\left\{\max_i\left(\int_{-\infty}^t |W(t,s)\|\gamma_i(s)|ds\right) + \sum_{\tau_k<t}|W(t,\tau_k)\|p_k|\right\}$$

(3)
$$\le \sup_{t\in\mathbb{R}}\left\{\max_i\left(\int_{-\infty}^t Ke^{-\lambda(t-s)}|\gamma_i(s)|ds\right) + \sum_{\tau_k<t}Ke^{-\lambda(t-\tau_k)}|p_k|\right\}$$
$$\le K\left(\frac{C_0}{\lambda}+\frac{C_0}{1-e^{-\lambda}}\right)=\overline{K}.$$

Set

$$F(t,x) = col\big\{F_1(t,x), F_2(t,x), \dots, F_n(t,x)\big\},$$

where

$$F_i(t,x) = \sum_{j=1}^n \alpha_{ij}(t)f_j(x_j),\ i = 1,2,\dots,n$$

and $\gamma(t) = col(\gamma_1(t), \gamma_2(t), \dots, \gamma_n(t))$. Now we define in D an operator S,

(4) $$S\varphi = \int_{-\infty}^t W(t,s)\,[F(s,\varphi(s))+\gamma(s)]\,ds + \sum_{\tau_k<t} W(t,\tau_k)[I_k(\varphi(\tau_k),)+p_k]$$

and subset $D^*,\ D^*\subset D$,

$$D^* = \left\{\varphi\in D : \|\varphi-\varphi_0\| \le \frac{r\overline{K}}{1-r}\right\}.$$

Consequently, for arbitrary $\varphi\in D^*$ from (3) and (4) it follows

$$\|\varphi\| \le \|\varphi-\varphi_0\| + \|\varphi_0\| \le \frac{r\overline{K}}{1-r}+\overline{K} = \frac{\overline{K}}{1-r}.$$

Now we are proving that S is self-mapping from D^* to D^*. For $\varphi\in D^*$ it follows

(5)
$$\|S\varphi-\varphi_0\| = \sup_{t\in\mathbb{R}}\left\{\max_i\left(\int_{-\infty}^t |W(t,s)|\left[\sum_{j=1}^n|\alpha_{ij}(s)\|f_j(\varphi_j(s))|\right]ds\right)\right.$$
$$\left. + \sum_{\tau_k<t}|W(t,\tau_k)\|I_k(\varphi(\tau_k))|\right\}$$
$$\le \left\{\max_i\left(\int_{-\infty}^t Ke^{-\lambda(t-s)}\sum_{j=1}^n\overline{\alpha}_{ij}L_1ds\right)+\sum_{\tau_k<t}Ke^{-\lambda(t-\tau_k)}L_2\right\}\|\varphi\|$$
$$\le K\left\{\max_i \lambda^{-1}L_1\sum_{j=1}^n\overline{\alpha}_{ij}+\frac{L_2}{1-e^{-\lambda}}\right\}\|\varphi\| = r\|\varphi\| \le \frac{r\overline{K}}{1-r}.$$

Let $\tau \in T$, $q \in Q$ where the sets T and Q are determined in Lemma 5.
Then

(6)
$$\|S\varphi(t+\tau) - S\varphi(t)\|$$
$$\leq \sup_{t\in\mathbb{R}} \Bigg\{ \max_i \Bigg(\int_{-\infty}^{t} |W(t+\tau, s+\tau) - W(t,s)\| \sum_{j=1}^{n} \alpha_{ij}(s+\tau) f_j(\varphi_j(s+\tau))| ds$$
$$+ \int_{-\infty}^{t} |W(t,s)\| \sum_{j=1}^{n} \alpha_{ij}(s+\tau) f_j(\varphi_j(s+\tau)) - \sum_{j=1}^{n} \alpha_{ij}(s) f_j(\varphi_j(s))| ds \Bigg)$$
$$+ \sum_{\tau_k<t} |W(t+\tau, \tau_{k+q}) - W(t,\tau_k)\| I_{k+q}(\varphi(\tau_{k+q}))|$$
$$+ \sum_{\tau_k<t} |W(t,\tau_k)\| I_{k+q}(\varphi(\tau_{k+q}) - I_k(\varphi(\tau_k))| \Bigg\} \leq \varepsilon C_1,$$

where
$$C_1 = \frac{L_1}{\lambda}\left(\max_i \sum_{j=1}^{n} (2\Gamma + K)\overline{\alpha}_{ij} + K \right) + \frac{L_2 \Gamma N}{1 - e^{-\lambda}}.$$

Consequently after (5) and (6) we obtain that $S\varphi \in D^*$.
Let $\varphi \in D^*$, $\psi \in D^*$.
Then

$$\|S\varphi - S\psi\|$$
$$\leq \sup_{t\in\mathbb{R}} \Bigg\{ \max_i \Bigg(\int_{-\infty}^{t} |W(t,s)| \left[\sum_{j=1}^{n} |\alpha_{ij}(s)\| f_j(\varphi_j(s)) - f_j(\psi_j(s))| \right] ds \Bigg)$$
$$+ \sum_{\tau_k<t} |W(t,\tau_k)\| I_k(\varphi(\tau_k)) - I_k(\psi(\tau_k)\| \Bigg\} \tag{7}$$
$$\leq K\left(\max_i \lambda^{-1} L_1 \sum_{j=1}^{n} \overline{\alpha}_{ij} + \frac{L_2}{1-e^{-\lambda}} \right) \|\varphi - \psi\| = r\|\varphi - \psi\|.$$

Then from (7) it follows that S is contracting operator in D^* and there exists unique almost periodic solution of (1). □

Proof of assertion 2. Let $y(t)$ be arbitrary solution of (1). Then from (3) we obtain

$$y(t) - x(t) = W(t,t_0)(y(t_0) - x(t_0)) + \int_{t_0}^{t} W(t,s)[F(s,y(s)) - F(s,x(s))]ds$$
$$+ \sum_{t_0<\tau_k<t} W(t,\tau_k)[I_k(y(\tau_k) - I_k(x(\tau_k))].$$

Hence

$$\begin{aligned}|y(t)-x(t)| &\leq Ke^{-\lambda(t-t_0)}|y(t_0)-x(t_0)| \\ &+ \max_i\left(\int_{t_0}^{t} Ke^{-\lambda(t-s)}L_1\sum_{j=1}^{n}\overline{\alpha}_{ij}|y_i(s)-x_i(s)|ds\right) \\ &+ \sum_{t_0<\tau_k<t} Ke^{-\lambda(t-\tau_k)}L_2|y(\tau_k)-x(\tau_k)|.\end{aligned}$$

Set $u(t)=|y(t)-x(t)|e^{\lambda t}$ and from Gronwall-Bellman's lemma [8] we have

$$|y(t)-x(t)| \leq K|y(t_0)-x(t_0)|(1+KL_2)^{i(t_0,t)}\exp\left(-\lambda+KL_1\max_i\sum_{j=1}^{n}\overline{\alpha}_{ij}\right)(t-t_0).$$

Thus Theorem 1 is complete. □

Example 1. Now consider the classical model of Impulsive Hopfield neural networks

$$(8)\quad \begin{cases} \dot{x}_i(t) = -\dfrac{1}{R_i}x_i(t)+\displaystyle\sum_{j=1}^{n}\alpha_{ij}f_j(x_j(t))+\gamma_i(t),\ t\neq\tau_k,\ i=1,2,\dots,n, \\ \Delta x(t) = Gx(t)+I_k(x(t))+p_k,\ t=\tau_k,\ k\in\mathbb{Z}, \end{cases}$$

where

(i) $t\in\mathbb{R},\ R_i>0,\ \alpha_{ij}\in\mathbb{R},\ i=1,2,\dots,n,\ j=1,2,\dots,n;$
(ii) $\gamma_i(t)\in C(\mathbb{R},\mathbb{R}),\ i=1,2,\dots,n,\ I_k(x)\in C(\Omega,\mathbb{R}^n),\ \{\tau_k\}\in UB,\ k\in\mathbb{Z};$
(iii) $G=diag[g_i],\ g_i\in\mathbb{R}, i=1,2,\dots,n,\ p_k\in\mathbb{R}^n.$

Corollary 1. *Let the following conditions be fulfilled*:

1. *Conditions* H3, H4, H6 *and* H7 *are fulfilled.*
2. *The folloing inequalities hold*

$$\lambda=\min_i\frac{1}{R_i}-N\max_i\ln(1+|g_i|)>0,$$

$$r=\exp\{N\max_i\ln(1+|g_i|)\}\left\{\max_i\lambda^{-1}L_1\sum_{j=1}^{n}\alpha_{ij}+\frac{L_2}{1-e^{-\lambda}}\right\}<1.$$

Then:

1. *There exists a unique almost periodic solution* $x(t)$ *of* (8).

2. *If the following inequalities hold*

$$1+\exp\left\{N\max_i \ln(1+|g_i|)\right\}L_2 < e,$$

$$\lambda-\exp\left(N\max_i \ln(1+|g_i|)\right)L_1\sum_{j=1}^{n}\alpha_{ij}$$

$$\{\}-N\ln\left(1+\{N\max_i \ln(1+|g_i|)\}L_2\right) > 0$$

then the solution $x(t)$ is exponentially stable.

Proof. Let

$$\left\{\begin{array}{l}\dot{x}_i(t)=-\dfrac{1}{R_i}x_i(t),\ t\neq\tau_k,\\ \Delta x(t)=Gx(t),\ t=t_k,\ k\in\mathbb{Z}.\end{array}\right.\tag{9}$$

is the linear part of (8).

Recall [8] the matrix $W(t,s)$ of (9) is in the form

$$W(t,s)=e^{A(t-s)}(E+G)^{i(s,t)},\ A=diag\left[-\frac{1}{R_i}\right],\ i=1,2,\dots,n.$$

Then

$$|W(t,s)|\le e^{N\max_i \ln(1+|g_i|)}e^{-\lambda(t-s)},$$

$t>s,\quad t,s\in\mathbb{R}$ and the proof follows from Theorem 1. □

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