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Correction:

On the paper " On the group of automorphisms of a function field ".

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By Kenkichi IWASAWA and Tsuneo TAMAGAWA

A correction was made to our paper mentioned in the title, as the proof of Lemma 1 was not complete. We give here another correction, which will give more explicitly the bound of the order of the automorphism in question. We use the same notations as in Lemma 1 of the original paper, without mentioning explicitly their meanings. Here we are concerned with the modular case, i. e. the case where the characteristic

$$p \text{ of } k \text{ is } \neq 0 .$$

Obviously we can assume either $\sigma(x)=x+\alpha$ or $\sigma(x)=\alpha x$ with some α in k. In the first case, the order of σ does not exceed pn. Hence we assume $\sigma(x)=\alpha x$. If the divisor of x is of the form $P^{n}Q^{-n}$, where P, Q are completely ramified primes of K over K', the contributions of P and Q to the different of K/K' are P^{n-1} and Q^{n-1} respectively. The original proof of Lemma 1 can be applied to this case and we see that the order of σ is at most $n(2n+2g-2)(2n+2g-3)(2n+2g-4)$. Suppose, now, that either the numerator or the denominator of x, say the latter, contains two different primes P_1, P_2 of K. A suitable power $\tau=\sigma^{l}$, $l\leqq n$, leaves P_1 fixed, and we can find an element y in K such that $\tau(y)=\beta y+\gamma$, $\beta, \gamma \in k$, and that the denominator of y is P_1^r, $r\leqq g+1$ (cf. the proof in p. 139 of the original paper). If $\beta=1$, the order of τ is at most $p(g+1)$ and, consequently, the order of σ is at most $pn(g+1)$. We may therefore assume that $\beta \neq 1$ and $\tau(y)=\beta y$. Let $F(X, Y)=\sum \alpha_{ij} X^{i} Y^{j}$ be an irreducible polynomial over k such that $F(x,y)=0$. Since $\tau(x)=\alpha^{l}x$, $\tau(y)=\beta y$, we have $F(\alpha^{l}X, \beta Y)=\xi F(X, Y)$, $\xi \in k$. Therefore, if $\alpha_{ij} \neq 0$, $\alpha_{st} \neq 0$, $(i,j) \neq (s,t)$, $z=x^{i-s}y^{j-t}$

is invariant under σ and is not contained in k, for the denominator of x contains P_1, P_2, while the denominator of y is P_1^r. Since the degree of F in X is at most $r=[K: k(y)] \leqq g+1$ and the degree of F in Y is at most $n=[K: k(x)]$, we have $|i-s| \leqq g+1$, $|j-t| \leqq n$ and, consequently $[K: k(z)] \leqq 2n(g+1)$. It then follows from $\tau=\sigma^l$, $\tau(z)=z$ that the order of σ is at most $2n^2(g+1)$, which is not greater than $n(2n+2g-2)(2n+2g-3)(2n+2g-4)$.

Thus Lemma 1 is completely proved, a bound of the order of σ being the maximum of $n(2n+2g-2)(2n+2g-3)(2n+2g-4)$ and $pn(g+1)$.