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Some class of doubly transitive groups of degree n and order $4q(n-1)n$ where q is an odd number

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1. Introduction.

In this paper we shall consider the following situation (*):

(*) *A simple group* $\mathfrak{G}$ *is doubly transitive on* $\Omega=\{1, 2, \cdots, n\}$ *of order* $aq(n-1)n$ *where* $a=2$ *or* 4 *and* q *is an odd number. The stabilizer* $\mathfrak{R}$ *of two points in* Ω *is cyclic and* $\mathfrak{R}\cap A^{-1}\mathfrak{R}A=1$ *or* $\mathfrak{R}$ *for every element* A *in* $\mathfrak{G}$.

Our purpose is to prove the following theorem.

THEOREM. *In our situation* (*) $\mathfrak{G}$ *is isomorphic to the projective special linear group* $PSL(2, 4q+1)$ *or* $PSL(2, 8q+1)$.

REMARK. This theorem was proved by Ito [**9**] and Kimura [**10**] in the case of $q=1$. Thus we assume that $q\geqq 3$ in the following.

The problem of characterization of doubly transitive groups by the structure of the stabilizer of two points was presented by Bender [**1**], Ito [**9**] and Kimura [**11**], [**12**], [**13**].

NOTATION. The stabilizer of points $i, j, \cdots, k$ in $\mathfrak{G}$ is denoted by $\mathfrak{G}_{ij\cdots k}$. On the other hand $\mathfrak{G}_{\{ij\cdots k\}}$ will denote the stabilizer in $\mathfrak{G}$ of a set $\{i, j, \cdots, k\}$ of points. For the subset $\mathfrak{X}$ of $\mathfrak{G}$, $\mathfrak{J}(\mathfrak{X})$ will denote the set of all the fixed points of $\mathfrak{X}$. For the elements $A, B, \cdots$ of $\mathfrak{G}$, $\langle A, B, \cdots\rangle$ is the subgroup of $\mathfrak{G}$ generated by $A, B, \cdots$ and $A\sim B$ means that A is conjugate with B. For a group $\mathfrak{W}$, $Z(\mathfrak{W})$ and $\mathfrak{W}'$ denote respectively the center of $\mathfrak{W}$ and the commutator subgroup of $\mathfrak{W}$. If $\mathfrak{S}$ is a 2-group, $\Omega_1(\mathfrak{S})$ denote the subgroup of $\mathfrak{S}$ generated by all involutions in $\mathfrak{S}$.

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2. The case $a=4$.

Let $\mathfrak{H}$ be the stabilizer of the points 1 and let $\mathfrak{K}$ be the stabilizer of the set of points 1 and 2. Then $\mathfrak{K}$ is of order $4q$ and it is generated by an element K of order $4q$ whose cycle structure has the form $(1)(2)\cdots$. Since $\mathfrak{G}$ is doubly transitive on Ω, it contains an involution I with the cycle structure $(1, 2)\cdots$. Then we have the following decomposition of $\mathfrak{G}$.

$$\mathfrak{G}=\mathfrak{H}\cup\mathfrak{H}I\mathfrak{H} \tag{2.1}$$

Since I is contained in $N_{\mathfrak{G}}(\mathfrak{K})$ it induces an automorphism of $\mathfrak{K}$. If an element $H'IH$ in a coset $\mathfrak{H}IH$, $H\in\mathfrak{H}$, is of order 2, then $I(HH')I=(HH')^{-1}$. Since $HH'=(1)\cdots$ and $I=(1,2)\cdots$, we have $HH'=(1)(2)\cdots$ and hence HH' is contained in $\mathfrak{K}$. Thus the number d of involutions in a coset $\mathfrak{H}IH$ is equal to that of the elements in $\mathfrak{K}$ inverted by I. Put $\langle K'\rangle=\{K\in\mathfrak{K}\ IKI=K^{-1}\}$. Then $\langle K'\rangle$ is of order d and $\langle I, K'\rangle$ is a dihedral group of order $2d$. Now we have

$$I\sim IK'^2\sim IK'^4\sim\cdots \tag{2.2}$$

and

$$IK'\sim IK'^3\sim IK'^5\sim\cdots. \tag{2.3}$$

Let $g(2)$ and $h(2)$ denote the number of involutions in $\mathfrak{G}$ and in $\mathfrak{H}$, respectively. Then the following equality is obtained from (2.1).

$$g(2)=h(2)+d(n-1) \tag{2.4}$$

Put $\mathfrak{I}(\mathfrak{K})=\{1, 2, \cdots, i\}$. By the theorem of Witt [**16**], $N_{\mathfrak{G}}(\mathfrak{K})/\mathfrak{K}$ can be considered as a doubly transitive group on $\mathfrak{I}(\mathfrak{K})$. Since every permutation of $N_{\mathfrak{G}}(\mathfrak{K})/\mathfrak{K}$ distinct from $\mathfrak{K}$ leaves by the definition of $\mathfrak{K}$ at most one point of $\mathfrak{I}(\mathfrak{K})$ fixed, $N_{\mathfrak{G}}(\mathfrak{K})/\mathfrak{K}$ is a complete Frobenius group on $\mathfrak{I}(\mathfrak{K})$.

LEMMA 1. *Let $\mathfrak{G}$ satisfy* (*). *Then $\mathfrak{K}$ is semi-regular on $\Omega-\mathfrak{I}(\mathfrak{K})$.*

PROOF. Assume that K^j fixes a point v in $\Omega-\mathfrak{I}(\mathfrak{K})$. Since $\mathfrak{G}$ is doubly transitive on Ω, there exists an element $W=\begin{pmatrix}1 & 2 & \cdots\\ 1 & v & \cdots\end{pmatrix}$ in $\mathfrak{G}$. Now we have $W^{-1}\mathfrak{K}W=\mathfrak{G}_{1v}$ and $K^j\in\mathfrak{K}\cap W^{-1}\mathfrak{K}W$. It follows from $\mathfrak{K}\neq W^{-1}\mathfrak{K}W$ that K^j must be identity. This proves our lemma.

LEMMA 2. *Let $\mathfrak{G}$ satisfy* (*). *Then $N_{\mathfrak{G}}(\mathfrak{K})\supset C_{\mathfrak{G}}(K^j)$ for $1\leqq j\leqq 4q-1$ and in particular $N_{\mathfrak{G}}(\mathfrak{K})=C_{\mathfrak{G}}(K^{2q})$.*

PROOF. Obviously we have $\mathfrak{G}_{\{12\cdots i\}}\supset C_{\mathfrak{G}}(K^j)$ for $1\leqq j\leqq 4q-1$. Let G be an element in $\mathfrak{G}_{\{12\cdots i\}}$. Then $G^{-1}\mathfrak{K}G\subset\mathfrak{G}_{12\cdots i}=\mathfrak{G}_{1,2}=\mathfrak{K}$. This implies that $G\in N_{\mathfrak{G}}(\mathfrak{K})$ and hence $\mathfrak{G}_{\{12\cdots i\}}\subset N_{\mathfrak{G}}(\mathfrak{K})$. The proof is complete.

Let us assume that n is even. Then applying Lemma 1, it follows from Kantor's theorem [**10**] that $\mathfrak{G}$ is isomorphic to one of the so called Zassenhaus groups. A complete classification of the Zassenhaus groups has been achieved by the combined effort of Zassenhaus [**18**], Feit [**4**], Ito [8] and Suzuki [**15**]. Hence $\mathfrak{G}$ is isomorphic to the projective special linear group $PSL(2, 8q+1)$.

REMARK. In the following we assume that n is odd and prove that there exists no group satisfying (*).

Since $\mathfrak{G}$ is doubly transitive on Ω any involution in $\mathfrak{G}$ which leaves at least two points in Ω fixed is conjugate to K^{2q} and by Lemma 2 the number of such involutions is equal to $|\mathfrak{G}|/|C_{\mathfrak{G}}(K^{2q})|=|\mathfrak{G}|/|N_{\mathfrak{G}}(\mathfrak{K})|=n(n-1)/i(i-1)$. Similarly any involution in $\mathfrak{H}$ which leaves at least two points in Ω fixed is conjugate to K^{2q} in $\mathfrak{H}$ and its number is equal to $|\mathfrak{H}|/|C_{\mathfrak{H}}(K^{2q})|=n-1/i-1$. Because n is odd, every involution fixes at least one point in Ω. Let $h^*(2)$ be the number of involutions in $\mathfrak{H}$ leaving only one point 1 fixed. Since $\mathfrak{H}=\mathfrak{G}_1, \mathfrak{G}_2, \cdots, \mathfrak{G}_n$ are conjugate each other, the following equality is obtained from (2.4).

$$h^*(2)n+n(n-1)/i(i-1)=h^*(2)+n-1/i-1+d(n-1) \tag{2.5}$$

Now we have $n=i\{1+(i-1)(d-h^*(2))\}$ and then

$$|G|=4qi\{1+(i-1)(d-h^*(2))\}\{(d-h^*(2))i+1\}(i-1)\,. \tag{2.6}$$

LEMMA 3. *Let $\mathfrak{G}$ satisfy* (*). *Then $h^*(2)=0$ or $d/2$.*

PROOF. (2.5) implies that

$$n(n-1)/i(i-1)=(d-h^*(2))(n-1)+n-1/i-1\,. \tag{2.7}$$

We have $d>h^*(2)$. Put $I=(1, 2)(a)\cdots$ and $\mathfrak{J}(I)=\{a\}$. Then $a\in\mathfrak{J}(\mathfrak{K})$. The number of elements of the form IK'^{2j} is $d/2$. Thus it follows from (2.2), (2.3), (2.7) that $d-h^*(2)=d$ or $d/2$ because every involution in a coset $\mathfrak{H}IH$ is of the form $H^{-1}(K'^jI)H$. Hence $h^*(2)=0$ or $d/2$. This proves our lemma.

LEMMA 4. *Let $\mathfrak{G}$ satisfy* (*). *Then $IK^qI=K^q$.*

PROOF. Assume by way of contradiction that $IK^qI\neq K^q$. Then we have $IK^qI=K^{-q}$. Lemma 2 yields $N_{\mathfrak{G}}(\mathfrak{K})\supset C_{\mathfrak{G}}(K^q)$ and so $N_{\mathfrak{G}}(\mathfrak{K})=\langle I\rangle C_{\mathfrak{G}}(K^q)$. Since $N_{\mathfrak{G}}(\mathfrak{K})/\mathfrak{K}$ is a Frobenius group of odd degree i, every involution is conjugate each other in $N_{\mathfrak{G}}(\mathfrak{K})/\mathfrak{K}$. Therefore $(C_{\mathfrak{G}}(K^q):\mathfrak{K})$ is odd and $\langle I, K^q\rangle$ is a Sylow 2-subgroup of $N_{\mathfrak{G}}(\mathfrak{K})$. Since $\langle I, K^q\rangle$ is a dihedral group of order 8, d is divisible by 4 and then Lemma 3 implies that $d-h^*(2)$ is divisible by 2. Hence it follows from (2.6) that $\langle I, K^q\rangle$ is a dihedral Sylow 2-subgroup of $\mathfrak{G}$. Now applying the theorem of Gorenstein and Walter [7], $\mathfrak{G}$ is isomorphic to either $PSL(2, r)$ where r is odd or the alternating group A_7. By Lüneburg's theorem [**14**] the former cannot happen. Since A_7 contains no element of order $4q$ for $q\geqq 3$, the latter cannot also happen. Thus we get a contradiction.

LEMMA 5. *Let $\mathfrak{G}$ satisfy* (*). *Then $h^*(2)\neq d/2$.*

PROOF. Assume by way of contradiction that $h^*(2)=d/2$. Since $\mathfrak{G}$ is doubly transitive on Ω we may assume that $\mathfrak{J}(I)=\{a\}$ for some $a\in\Omega$.

Because i is odd $\mathfrak{I}(I)\cap\mathfrak{I}(\mathfrak{K})=\{a\}$. It follows from Lemma 4 that d is not divisible by 4. Now $d-h^*(2)$ is odd and then by (2.6) K^{2q} is a non-central involution. We may assume that I is a central involution of some Sylow 2-subgroup $\mathfrak{S}$ of $\mathfrak{G}$. Since $\mathfrak{G}_a$ is conjugate to $\mathfrak{G}_1=\mathfrak{H}$, the number of involutions in $\mathfrak{G}_a$ which fixes only one point a is equal to $d/2=h^*(2)$. Since $IK'^2, IK'^4, \cdots$ for $K'^{2j}\neq 1$ are not in $C_{\mathfrak{G}}(I)$ it follows from (2.2) that I is the only involution in $C_{\mathfrak{G}}(I)$ which fixes only one point. Now we have $\{G^{-1}IG\,;G\in\mathfrak{G}\}\cap\mathfrak{S}=\{I\}$ and by the Z^*-theorem of Glauberman [6] we have $I\in Z(\mathfrak{G}\ \mathrm{mod}.\ O_{2'}(\mathfrak{G}))$ where $O_{2'}(\mathfrak{G})$ is the maximal normal subgroup of odd order of $\mathfrak{G}$. Thus $\mathfrak{G}$ is non-simple. This contradicts (*). The proof is complete.

LEMMA 6. *Let $\mathfrak{S}$ be a group of order 2^{m+2} containing a cyclic normal subgroup $\mathfrak{V}$ of order 4. Let $\mathfrak{G}$ be a finite group containing $\mathfrak{S}$ as a Sylow 2-subgroup. Assume that all involutions are conjugate in $\mathfrak{G}$.*

(i) *If $\mathfrak{S}/\mathfrak{V}$ is cyclic and $\mathfrak{S}$ is non-abelian, then $|\mathfrak{S}|=8$ and $\mathfrak{S}$ is isomorphic to a dihedral group or a quaternion group.*

(ii) *If $\mathfrak{S}/\mathfrak{V}$ is isomorphic to a generalized quaternion group, then $\mathfrak{G}$ is non-simple.*

PROOF. (i) Put $\mathfrak{V}=\langle V\rangle$ and $\mathfrak{S}/\mathfrak{V}=\langle A\mathfrak{V}\rangle$. Then $V^4=1$ and $A^{2^m}\in\mathfrak{V}$. If $A^{2^m}=V$ or V^{-1}, then $\mathfrak{S}=\langle A\rangle$ is cyclic. This is impossible. Thus $A^{2^m}=1$ or V^2. The group $\mathfrak{S}$ is non-abelian, so that $A^{-1}VA=V^{-1}$ and A^2, V^2 are in $Z(\mathfrak{S})$. Assume that $A^{2^m}=1$. If $m=1$, then $A^2=1$ and $\mathfrak{S}$ is isomorphic to a dihedral group of order 8. If $m>1$, then $A^{2^{m-1}}$ is of order 2 and contained in $Z(\mathfrak{S})$. By our assumption $A^{2^{m-1}}$ is fused with V^2 in $\mathfrak{G}$ and thus Burnside's argument implies that $A^{2^{m-1}}$ is fused with V^2 in $N_{\mathfrak{G}}(\mathfrak{S})$. On the other hand since $\mathfrak{S}'$ is contained in $\mathfrak{V}$ and $\Omega_1(\mathfrak{S}')=\langle V^2\rangle$ is a characteristic subgroup of $\mathfrak{S}$, V^2 is not fused with $A^{2^{m-1}}$ in $N_{\mathfrak{G}}(\mathfrak{S})$. This is a contradiction. Assume that $A^{2^m}=V^2$. Then $\langle A\rangle$ is a cyclic normal subgroup of index 2 in $\mathfrak{S}$. Since $Z(\mathfrak{S})=\langle A^2\rangle$ and $(\mathfrak{S}:Z(\mathfrak{S}))=4$, $\mathfrak{S}$ is isomorphic to a quaternion group or a pseudo semi-dihedral group $\langle X, Y\,; X^{2^{m+1}}=Y^2=1,\ Y^{-1}XY=X^{1+2^m}\rangle$. The latter cannot happen because in this case $\mathfrak{G}$ has a normal 2-complement by Wong's theorem [17] and then the number of conjugacy classes of involutions in $\mathfrak{G}$ is the same as that in $\mathfrak{S}$. Now $|\mathfrak{S}|=8$ and $\mathfrak{S}$ is isomorphic to a quaternion group.

(ii) Put $\mathfrak{V}=\langle V\rangle$ and $\mathfrak{S}=\langle A, B, V\rangle$ with $A^{2^{m-1}}\equiv 1\ (\mathrm{mod}.\ \mathfrak{V})$, $B^{-1}AB\equiv A^{-1}$ $(\mathrm{mod}.\ \mathfrak{V})$, $B^2\equiv A^{2^{m-2}}\ (\mathrm{mod}.\ \mathfrak{V})$, $m\geqq 3$. Put $A^{2^{m-2}}=J$ and so J^2 is in $\mathfrak{V}$. We have $(A^iB^jV^k)^2=J$, JV, JV^2 or JV^3 for $j=1$ or 3. The element A^2 centralizes V, so that J also centralizes V. Thus the order of the elements $A^iB^jV^k$ for $j=1$ or 3 are at most 8. Every involution of $\mathfrak{S}$ is in cosets $\mathfrak{V}$ or $J\mathfrak{V}$ and so the number of involutions in $\mathfrak{S}$ is 3. Now assume by way of contradiction that $\mathfrak{G}$ is simple. If $A^{2^{m-1}}=V$, then $\mathfrak{S}$ contains a cyclic normal subgroup A of index 2 and it follows from the result of Brauer and Suzuki [3], Wong

[**17**] that $\mathfrak{S}$ is isomorphic to a dihedral group or a semi-dihedral group. Clearly they are not our case. If $A^{2^{m-1}}=V^2$, then the exponent of $\mathfrak{S}$ is 2^m. Therefore Fong's theorem [**5**] implies that $\mathfrak{S}$ is isomorphic to the wreath product $Z_4 \sim Z_2$. This is not the case because $Z_4 \sim Z_2$ contains 7 involutions. Assume that $A^{2^{m-1}}=1$ and $m>4$. Now for odd i we have

$$\begin{aligned}(A^iV^j)^{2^{m-2}} &= (A^iV^j)(A^iV^j)\cdots(A^iV^j)(A^iV^j)\\ &= A^i(V^jA^iV^j)A^i\cdots A^i(V^jA^iV^j)=(A^i)^{2^{m-2}}=J^i=J.\end{aligned}$$

Hence J is in $Z(\mathfrak{S})$ and $\langle J\rangle$ is a characteristic subgroup of $\mathfrak{S}$. Since V^2 is in $Z(\mathfrak{S})$ and J is fused with V^2 in $\mathfrak{G}$ by our assumption, Burnside's argument implies that J is fused with V^2 in $N_{\mathfrak{G}}(\mathfrak{S})$. This is a contradiction. Now $m\leqq 4$ and $|\mathfrak{S}|=32$ or 64. Since $\mathfrak{G}$ is simple, we may apply the theorem of Fong [**5**]. If $|\mathfrak{S}|=32$, then $\mathfrak{S}$ is dihedral, semi-dihedral or $Z_4\sim Z_2$. If $|\mathfrak{S}|=64$, then $\mathfrak{S}$ is dihedral, semi-dihedral, the Sylow 2-subgroup of the Mathieu group on 12 symbols or the direct product of four group and semi-dihedral group of order 16. It is easily checked that they are not our case. The proof of Lemma 6 is complete.

By Lemmas 3 and 5 we must have $h^*(2)=0$. Therefore all involutions are fused with K^{2q} in $\mathfrak{G}$. Now let $\mathfrak{S}$ be a Sylow 2-subgroup of $\mathfrak{G}$ contained in $N_{\mathfrak{G}}(\mathfrak{K})$ and containing $\langle I, K^q\rangle$. Since $N_{\mathfrak{G}}(\mathfrak{K})/\mathfrak{K}$ is a Frobenius group of odd degree i, $\mathfrak{S}\mathfrak{K}/\mathfrak{K}\cong\mathfrak{S}/\mathfrak{S}\cap\mathfrak{K}=\mathfrak{S}/\langle K^q\rangle$ is cyclic or a generalized quaternion group. It follows from Lemma 6 that $\mathfrak{S}$ must be abelian because $\langle I, K^q\rangle$ is isomorphic to an abelian group of type $(2, 2^2)$. Let $\mathfrak{S}$ be an abelian group of type $(2^m, 2^2)$. Since $h^*(2)=0$, we have $m=2$. Now the result of Brauer [**2**] implies that $\mathfrak{G}$ is non-simple. In the case $a=4$ the proof of our theorem is complete.

3. The case $a=2$.

If n is even, then Kantor's theorem [**10**] implies that $\mathfrak{G}$ is isomorphic to the Zassenhaus groups. If n is odd, then by the same way as in the case $a=4$ we have $h^*(2)=0$.

LEMMA 7. *Let $\mathfrak{S}$ be a group of order 2^{m+1} and $\mathfrak{V}$ a cyclic normal subgroup of order 2. Let $\mathfrak{G}$ be a finite group containing $\mathfrak{S}$ as a Sylow 2-subgroup. Assume that all involutions are conjugate in $\mathfrak{G}$.*

(i) *If $\mathfrak{S}/\mathfrak{V}$ is cyclic and $\mathfrak{S}$ is non-cyclic, then $\mathfrak{S}$ is isomorphic to a four group.*

(ii) *If $\mathfrak{S}/\mathfrak{V}$ is isomorphic to a generalized quaternion group, then $\mathfrak{G}$ is non-simple.*

PROOF. (i) Now $\mathfrak{V}\subset Z(\mathfrak{S})$, so that $\mathfrak{S}$ is abelian. Since all involutions are conjugate in $\mathfrak{G}$, the result follows immediately from Burnside's argument.

(ii) Put $\mathfrak{V}=\langle V\rangle$ and $\mathfrak{S}=\langle A, B, V\rangle$ with $A^{2^{m-1}}\equiv 1$ (mod $\mathfrak{V}$), $B^{-1}AB\equiv A^{-1}$ (mod. $\mathfrak{V}$), $B^2\equiv A^{2^{m-2}}$ (mod. $\mathfrak{V}$), $m\geqq 3$. Put $A^{2^{m-2}}=J$. We have $(A^iB^jV^k)^2=J$ or JV for $j=1$ or 3. Thus the order of the elements $A^iB^jV^k$ for $j=1$ or 3 are four. Every involution of $\mathfrak{S}$ is in cosets $\mathfrak{V}$ or $J\mathfrak{V}$ and so the number of involutions in $\mathfrak{S}$ is equal to 3. Now assume by way of contradiction that $\mathfrak{G}$ is simple. If $A^{2^{m-1}}=V$, then $\mathfrak{S}$ contains a cyclic normal subgroup $\langle A\rangle$ of index 2. This is impossible. Thus $A^{2^{m-1}}=1$. If $m\geqq 4$, then $\langle J\rangle$ is a characteristic subgroup of $\mathfrak{S}$. Since J is fused with V in $\mathfrak{G}$, Burnside's argument implies that J is fused with V in $N_{\mathfrak{G}}(\mathfrak{S})$. This is impossible. Therefore $m=3$ and so $|\mathfrak{S}|=16$. Fong's theorem [**5**] yields a contradiction.

Let $\mathfrak{S}$ be a Sylow 2-subgroup of $\mathfrak{G}$ contained in $N_{\mathfrak{G}}(\mathfrak{K})$. Applying the theorem of Gorenstein and Walter [**7**], Lemma 7 implies that $\mathfrak{G}$ is isomorphic to either $PSL(2, r)$ where r is odd or the alternating group A_7. By the same way as in the proof of Lemma 4 they are not our case. Thus the proof of our theorem is complete.

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