

Addendum to my paper "On Veronese manifolds"

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In the present paper we use the same definitions and notations as ones in [1]. Moreover, using the inequalities in [1] and [2], we give an interesting characterization of Veronese manifolds as follows;

THEOREM. *Let M be an n -dimensional compact orientable submanifold which is minimally immersed in an $(n+p)$ -dimensional sphere of constant curvature \tilde{c} . If the immersion is full and the sectional curvatures of M are not smaller than $\frac{n\tilde{c}}{2(n+1)}$, then M is a sphere of constant curvature \tilde{c} or M is a Veronese manifold.*

PROOF. Since M is compact, let c be the minimum of all sectional curvatures of M . In this case, in [2], S. T. Yau gave the following inequality:

$$(1) \quad \sum h_{ij}^\alpha \Delta h_{ij}^\alpha \geq n(a+1)cS - a\tilde{c}nS + aL_N - \frac{1-a}{2}K_N,$$

where a is a constant such that $a+1 \geq 0$. Using the inequality (2.1) $K_N \leq nL_N$ in [1], from (1) we get

$$(2) \quad \sum h_{ij}^\alpha \Delta h_{ij}^\alpha \geq (a+1)cnS - a\tilde{c}nS + \left(\frac{a}{n} - \frac{1-a}{2}\right)K_N,$$

where a is a positive constant.

Now, setting $a = \frac{n}{n+2}$ in (2), we have the inequality

$$(3) \quad \sum h_{ij}^\alpha \Delta h_{ij}^\alpha \geq \frac{2n(n+1)}{n+2} \left\{ c - \frac{n}{2(n+1)}\tilde{c} \right\} S.$$

On the other hand, by definition we know $\frac{1}{2}\Delta S = \sum (h_{ijk}^\alpha)^2 + \sum h_{ij}^\alpha \Delta h_{ij}^\alpha$. This equality, together with (3) and our assumption, implies

$$(4) \quad \frac{1}{2}\Delta S \geq \sum (h_{ijk}^\alpha)^2 + \frac{2n(n+1)}{n+2} \left\{ c - \frac{n}{2(n+1)}\tilde{c} \right\} S \geq 0.$$

Since M is compact and orientable, $\Delta S = 0$ on M . Then all equalities in (1), (2), (3) and (4) hold everywhere on M , that is, the immersion is isotropic,

$$(5) \quad h_{ijk}^\alpha = 0 \quad \text{for all } i, j, k \text{ and } \alpha$$

and

$$(6) \quad S=0 \quad \text{or} \quad c = \frac{n\tilde{c}}{2(n+1)} > 0 \quad \text{on } M.$$

Therefore, as the the same reason in the proof of Theorem 1 in [1], we easily see that M is a sphere of constant curvature \tilde{c} or M is a Veronese manifold of constant curvature $\frac{n\tilde{c}}{2(n+1)}$. Q. E. D.

Bibliography

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- [2] S.T. Yau, Submanifolds with constant mean curvature II, Amer. J. Math., 97 (1975), 76-100.

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