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On the existence of Yang-Mills connections by conformal changes in higher dimensions

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1. Introduction.

Let (M, g) be a smooth closed Riemannian n-manifold, and P a principal G-bundle over (M, g) with compact Lie group G. Let $\mathcal{C}$ be the space of all smooth connections on $P \rightarrow M$ compatible with the metric on P. The space $\mathcal{C}$ is an affine space. For a connection $A \in \mathcal{C}$, we denote by d_A and δ_A the covariant exterior derivative and its formal adjoint respectively.

The Yang-Mills functional $J: \mathcal{C} \rightarrow \boldsymbol{R}$ is defined by

$$J(A)=\frac{1}{2}\int_{M}|F_{A}|^{2}d\mu_{g}\,, \tag{1}$$

where F_A is the curvature of a connection A, and $d\mu_g$ is the volume element induced from a Riemannian metric g. A is called a *Yang-Mills connection* if A is a critical point of the Yang-Mills functional J, that is, A satisfies the Yang-Mills equation

$$\delta_A F_A = 0\,, \tag{2}$$

which is the Euler-Lagrange equation for the Yang-Mills functional J.

In dimension 4, it is well-known that the Yang-Mills functional is invariant under conformal changes of metric on the base manifold, but in general it is not invariant under this action.

In this paper, using the above fact we will show "some existence" result for Yang-Mills connections in higher dimensions:

THEOREM 1.1. *Let (M, g) be a smooth closed Riemannian manifold of dimension $n \geqq 5$, P a smooth principal G-bundle over (M, g) with compact group G. Then there exists a connection $A_0 \in \mathcal{C}$ and a metric $\tilde{g}$ on M which is conformally equivalent to the original metric g such that A_0 is a Yang-Mills connection on the principal bundle P over $(M, \tilde{g})$.*

The idea of a proof of this Theorem here is essentially due to J. Eells-M. J. Ferreira for harmonic maps ([1]). In section 2, we will state some known facts about the connections on principal fiber bundles, and in section 3, we will give a proof of the above theorem.

2. Preliminaries.

In this section we describe known results concerning connections on principal fiber bundles and fix the notation that will be used in this paper.

Let (M, g) be a smooth closed Riemannian n-manifold and $\pi : P \to M$ a smooth principal G-bundle with compact Lie group G. We denote by Ξ the Lie algebra of G. Let $\pi_{\mathrm{ad}} : G_P \to M$ be the bundle of groups over M, with total space $G_P = P \times_{\mathrm{ad}} G$ and fiber G associated to $\pi : P \to M$ via the adjoint action $\mathrm{ad} : G \to \mathrm{Aut}(G)$ of G onto itself. We denote by $\pi_{\mathrm{Ad}} : \Xi_P \to M$ the vector bundle over M, with total space $\Xi_P = P \times_{\mathrm{Ad}} \Xi$, associated to the principal fiber bundle $\pi : P \to M$ via the adjoint representation $\mathrm{Ad} : G \to \mathrm{Aut}(\Xi)$.

For a given number $1 \leqq p < \infty$, $2p > n$, we define the gauge transformation group $L_2^p \mathcal{G}$ to be the group of principal fiber bundle automorphisms which is naturally isomorphic to the group $L_2^p(G_P)$ of L_2^p-sections of $\pi_{\mathrm{ad}} : G_P \to M$, where L_k^p is a Sobolev space whose derivatives of order less than or equal to k are in L^p. The group $L_2^p \mathcal{G}$ has a smooth Banach manifold structure, and the group operations are smooth (see [7]). The Lie algebra of $L_2^p \mathcal{G}$ may be identified with $L_2^p(\Xi_P)$.

Given $x_0 \in M$, we define $L_2^p \mathcal{G}_0$ as the subgroup of $L_2^p \mathcal{G}$ consisting of those principal fiber bundle automorphisms which fix fiber of $\pi : P \to M$ over x_0.

Let $\mathcal{C}$ be the affine space of smooth connections on P. Taking a base connection $A_0 \in \mathcal{C}$, we have

$$\mathcal{C} = \{A_0 + A \,;\, A \in \Omega^1(\Xi_p)\}, \tag{3}$$

where $\Omega^1(\Xi_P)$ is a Ξ_P-valued 1-form on M. So we can define the L_k^p-Sobolev space $L_k^p \mathcal{C}$ of connection as follows:

$$L_k^p \mathcal{C} = \{A_0 + A \,;\, A \in L_k^p(\Omega^1(\Xi_P))\}, \tag{4}$$

which is independent of the choice of a base connection A_0 because $\mathcal{C}$ is an affine space.

Taking a basis $\{E_a\}$ of Ξ_P, for $\Phi = \sum_a \Phi_a \otimes E_a \in \Omega^k(M) \otimes \Xi_P$, $\Psi = \sum_b \Psi_b \otimes E_b \in \Omega^l(M) \otimes \Xi_P$, we define $[\cdot, \cdot] : \Omega^k(\Xi_P) \times \Omega^l(\Xi_P) \to \Omega^{k+l}(\Xi_P)$ by

$$[\Phi, \Psi] = \sum_{a,b} \Phi_a \wedge \Psi_b \otimes [E_a, E_b], \tag{5}$$

where $[\cdot, \cdot]$ in the right hand side is the natural Lie bracket on Ξ_P induced by Lie algebra Ξ.

If $2p \geqq n$, then the curvature F_A of connection $A \in L_1^p \mathcal{C}$ is locally given by

$$F_A = dA + \frac{1}{2}[A, A] \in L^p(\Omega^2(\Xi_P)), \tag{6}$$

where we use Sobolev embedding theorem $L_1^p \hookrightarrow L^{2p}$. Since we assume G is compact, by using Haar measure on G, there exists an adjoint G-invariant inner product $(\cdot, \cdot)$ on Ξ. Using this inner product $(\cdot, \cdot)$ on Ξ, we can define the fiber metric $(\cdot, \cdot)$ on Ξ_P. So we get a fiber metric $(\cdot, \cdot)$ on $\Omega^k(\Xi_P) \cong \Omega^k(M) \otimes \Xi_P$ using the Riemannian metric g. We use the same notations for these inner products.

The Yang-Mills functional $J: \mathcal{C} \to \boldsymbol{R}$ is defined by

$$J(A) = \frac{1}{2}\int_M |F_A|^2 d\mu_g, \tag{7}$$

where the norm $|\cdot|$ is induced by the above inner product $(\cdot, \cdot)$ on $\Omega^2(\Xi_P)$. The Yang-Mills functional is invariant under the action of gauge transformations, *i.e.* for all $\varphi \in \mathcal{G}$, and $A \in \mathcal{C}$,

$$J(A) = J(\varphi^* A). \tag{8}$$

If $2p > n$, then the gauge transformation group $L_2^p \mathcal{G}$ acts smoothly on the space $L_1^p \mathcal{C}$ of L_1^p-connections on P, sending $A \in L_1^p \mathcal{C}$ to $\varphi^* A \in L_1^p \mathcal{C}$ for $\varphi \in L_2^p \mathcal{G}$. Note that by Sobolev embedding theorem, $L_2^p \hookrightarrow C^0$, the action of gauge transformations does not change the topology of principal fiber bundle provided $2p > n$. But in general this action is not free. If x_0 is some chosen point of M, then the subgroup $L_2^p \mathcal{G}_0$ acts smoothly and freely on $L_1^p \mathcal{C}$. For this action, the following slice theorem holds (see [**5**]):

THEOREM 2.1 (slice theorem). *Let M be a closed manifold of dimension $n \geqq 2$, $\pi: P \to M$ a principal fiber bundle with compact structure group. If p is a real number satisfying $1 < p < \infty$ and $2p > n$, then the orbit space $L_1^p \mathcal{C} / L_2^p \mathcal{G}_0$ admits a unique smooth Banach manifold structure.*

We shall also use the following result, which is used in proving the above slice theorem (see [**5**]):

PROPOSITION 2.2. *Let M, P, and p be as in Theorem 2.1. Then for all $A \in L_1^p \mathcal{C}$, there exists a neighborhood of $\mathcal{V}$ of A in $L_1^p \mathcal{C}$ such that if $A' \in \mathcal{V}$, then there exists a gauge transformation $\varphi \in L_2^p \mathcal{G}$ such that $\varphi^* A \in \mathcal{V}$ and*

$$\delta_A(\varphi^* A' - A) = 0. \tag{9}$$

The following result which we will also use has been proved in [8].

THEOREM 2.3 (Uhlenbeck's weak compactness theorem). *Let M, P, and p as in Theorem 2.1, and $\{A_i\}_{i=1}^\infty \subset L_1^p \mathcal{C}$ a sequence of L_1^p-connections. If L^p-norm* *of curvatures* F_{A_i} *of* A_i *satisfy*

$$\int_M |F_{A_i}|^p d\mu_g \leqq K$$

for some constant K, *then there exists a subsequence of* $\{A_i\}$ *(by renumbering we also use* A_i*), and a sequence of gauge transformations* $\varphi_i \in L_2^p \mathcal{G}$ *such that* $\{\varphi_i^* A_i\}_{i=1}^{\infty}$ *converges weakly in* $L_1^p \mathcal{C}$.

3. Proof of Theorem.

In this section, we will give a proof of Theorem stated in Introduction. The idea of this proof is essentially due to [1].

Let $(M, \tilde{g})$ be a smooth closed Riemannian n-manifold, and P a principal fiber bundle over M. We denote L^p and L_k^p-norm by $\|\cdot\|_p$ and $\|\cdot\|_{k,p}$ respectively. The Yang-Mills functional is

$$J(A;\tilde{g}) = \frac{1}{2}\int_M |F_A|_{\tilde{g}}^2 d\mu_{\tilde{g}}. \tag{10}$$

Note that the norm $|\cdot|$ of curvature F_A depends on the metric $\tilde{g}$. By putting $\tilde{g} = f^{2/(n-4)} g$, $f \in C^\infty(M)$, $f>0$, $J(A;\tilde{g})$ is written on (M, g) as

$$J(A;\tilde{g}) = \frac{1}{2}\int_M f|F_A|^2 d\mu_g. \tag{11}$$

Now we consider a new functional $J_p: \mathcal{C} \to \boldsymbol{R}$ defined by

$$J_p(A) = \frac{1}{p}\int_M (1+|F_A|^2)^{p/2} d\mu_g. \tag{12}$$

If there exists a smooth critical point $A_0 \in \mathcal{C}$ of the functional J_p, then A_0 satisfies the Euler-Lagrange equation for J_p:

$$\delta_{A_0}\{(1+|F_{A_0}|^2)^{p/2-1} F_{A_0}\} = 0. \tag{13}$$

So if we put

$$f = (1+|F_{A_0}|^2)^{p/2-1},$$

then for all $A_t := A_0 + t\phi$, $\phi \in \Omega^1(\Xi_P)$, $t \in \boldsymbol{R}$, and noting that

$$F_{A_t} = F_{A_0} + t d_{A_0}\phi + \frac{1}{2}t^2[\phi, \phi], \tag{14}$$

we get

$$\begin{aligned}\frac{d}{dt}J(A_t;\tilde{g})\Big|_{t=0} &= \frac{1}{2}\int_M (1+|F_{A_0}|^2)^{p/2-1}\frac{d}{dt}|F_{A_t}|^2\Big|_{t=0} d\mu_g \\ &= \int_M (1+|F_{A_0}|^2)^{p/2-1}(F_{A_0}, d_{A_0}\phi) d\mu_g\end{aligned}$$

$$= \int_M (\delta_{A_0}\{(1+|F_{A_0}|^2)^{p/2-1}F_{A_0}\}, \phi)d\mu_g$$
$$= 0 . \tag{15}$$

Therefore A_0 is a Yang-Mills connection on $\pi: P \to (M, \tilde{g})$. So it suffices to show that for a suitable real number p, there exists a critical point $A_0 \in C$ of the functional J_p.

From now on we will show the existence of a smooth critical point of the functional J_p. First we prepare a general ideas of the Palais-Smale condition for the functional J_p. Recall a sequence $\{A_i\}_{i=1}^{\infty} \subset L_1^p C$ is called a *Palais-Smale sequence* for the functional J_p if $J_p(A_i)$ is bounded and $\|\nabla J_{p,A_i}\|^* \to 0$, as $i \to \infty$, where for $B \in \Omega^1(\Xi_P)$,

$$\nabla J_{p,A}(B) = \int_M (1+|F_A|^2)^{p/2-1}(F_A, d_A B)d\mu_g , \tag{16}$$

and $\|\cdot\|^*$ denotes the dual norm of $L_1^p C$, *i.e.*

$$\|\nabla J_{p,A}\|^* = \sup\{|\nabla J_{p,A}(B)| ; B \in L_1^p(\Omega^1(\Xi_P)), \|B\|_{1,p}=1\} . \tag{17}$$

The functional $J_p : L_1^p C \to \boldsymbol{R}$ is also invariant under the gauge transformation group. We conclude that the functional

$$\bar{J}_p : L_1^p C / L_2^p \mathcal{G}_0 \longrightarrow \boldsymbol{R} \tag{18}$$

induced by the functional J_p satisfies the Palais-Smale condition if and only if for every Palais-Smale sequence $\{A_i\}_{i=1}^{\infty} \subset L_1^p C$, there exist a subsequence $\{A_i\}_{i=1}^{\infty}$ and a sequence $\{\varphi_i\}_{i=1}^{\infty} \subset L_2^p \mathcal{G}_0$ such that $\varphi_i^* A_i$ converges strongly in $L_1^p C$ as $i \to \infty$.

The quotient group $L_2^p \mathcal{G} / L_2^p \mathcal{G}_0$ is isomorphic to the compact structure group G. Let $\{\varphi_i\}_{i=1}^{\infty} \subset L_2^p \mathcal{G}$. So after passing to a subsequence if necessary, we can find a strongly convergent sequence $\{\phi_i\}_{i=1}^{\infty} \subset L_2^p \mathcal{G}$ such that $\varphi_i \phi_i \in L_2^p \mathcal{G}_0$ for all $i \in \boldsymbol{N}$. Therefore we conclude that the functional $\bar{J}_p : L_1^p C / L_2^p \mathcal{G}_0 \to \boldsymbol{R}$ satisfies the Palais-Smale condition if and only if every Palais-Smale sequence in $L_1^p C$ has a subsequence which is gauge equivalent to a strongly convergent sequence in $L_1^p C$.

First we assume $2p > n$.

PROPOSITION 3.1. *Let $2p > n$. Then the functional $\bar{J}_p : L_1^p C / L_2^p \mathcal{G}_0 \to \boldsymbol{R}$ satisfies the Palais-Smale condition.*

PROOF. Let $\{A_i\}_{i=1}^{\infty} \subset L_1^p C$ be a Palais-Smale sequence with $J_p(A_i) \leq K$. Then from the boundedness of $J_p(A_i)$, we get L^p-bounds of the curvatures F_{A_i}. By Uhlenbeck's weak compactness theorem, there exist a subsequence $\{A_i\}_{i=1}^{\infty}$ and a sequence of gauge transformations $\{\varphi_i\}_{i=1}^{\infty} \subset L_2^p \mathcal{G}$ such that $\varphi_i^* A_i$ converges weakly to some $A_0 \in L_1^p C$. For simplicity, we denote $\varphi_i^* A_i$ by A_i. Since $\{A_i\}_{i=1}^{\infty} \subset L_1^p C$ is a bounded Palais-Smale sequence, we get for all A_i and A_j,

$$\begin{aligned}(\nabla J_{p,A_i} - \nabla J_{p,A_j})(A_i - A_j) &\leqq |\nabla J_{p,A_i}(A_i - A_j)| + |\nabla J_{p,A_j}(A_i - A_j)| \\ &\leqq \|\nabla J_{p,A_i}\|^* \|A_i - A_j\|_{1,p} + \|\nabla J_{p,A_j}\|^* \|A_i - A_j\|_{1,p} \longrightarrow 0 \qquad (19)\end{aligned}$$

as $i, j \to \infty$.

To obtain Proposition 3.1, we need the following:

LEMMA 3.2. *For all A_1 and $A_2 \in L_1^p C$ satisfying $J_p(A_1), J_p(A_2) \leqq K$, we have*

$$\begin{aligned}&(\nabla J_{p,A_1} - \nabla J_{p,A_2})(A_1 - A_2) \\ &\quad \geqq C\|F_{A_1} - F_{A_2}\|_p^p - C\|F_{A_1} - F_{A_2}\|_p^2 \|A_1 - A_2\|_{2p}^{2p-4} - C(K)\|A_1 - A_2\|_{2p}^2, \qquad (20)\end{aligned}$$

where C is a constant independent of A_1 and A_2, and $C(K)$ depends on the geometry of (M, g) and the bounded constant K.

PROOF. Note that if we write for $0 \leqq t \leqq 1$,

$$A_t = tA_1 + (1-t)A_2 = A_2 + t(A_1 - A_2),$$

then we have

$$F_{A_t} = tF_{A_1} + (1-t)F_{A_2} + \frac{1}{2}(t^2 - t)[A_1 - A_2, A_1 - A_2]. \qquad (21)$$

Using this we get

$$\begin{aligned}&\int_M (1+|F_{A_1}|^2)^{p/2-1}(F_{A_1}, d_{A_1}(A_1 - A_2))d\mu_g \\ &\quad - \int_M (1+|F_{A_2}|^2)^{p/2-1}(F_{A_2}, d_{A_2}(A_1 - A_2))d\mu_g \\ &= \int_M \int_0^1 \frac{d}{dt}((1+|F_{A_t}|^2)^{p/2-1} F_{A_t}, d_{A_t}(A_1 - A_2))dtd\mu_g \\ &= \int_M \int_0^1 (p-2)(1+|F_{A_t}|^2)^{p/2-2} \\ &\quad \times \Big(F_{A_t}, F_{A_1} - F_{A_2} + \Big(t - \frac{1}{2}\Big)[A_1 - A_2, A_1 - A_2]\Big)(F_{A_t}, d_{A_t}(A_1 - A_2))dtd\mu_g \\ &\quad + \int_M \int_0^1 (1+|F_{A_t}|^2)^{p/2-1}(F_{A_1} - F_{A_2}, d_{A_t}(A_1 - A_2))dtd\mu_g \\ &\quad + \int_M \int_0^1 (1+|F_{A_t}|^2)^{p/2-1}\Big(\Big(t - \frac{1}{2}\Big)[A_1 - A_2, A_1 - A_2], d_{A_t}(A_1 - A_2)\Big)dtd\mu_g \\ &\quad + \int_M \int_0^1 (1+|F_{A_t}|^2)^{p/2-1}(F_{A_t}, [A_1 - A_2, A_1 - A_2])dtd\mu_g\end{aligned}$$

$$
\begin{aligned}
&=(p-2)\int_M\int_0^1(1+|F_{A_t}|^2)^{p/2-2}|(F_{A_t},\ d_{A_t}(A_1-A_2)|^2 dtd\mu_g \\
&\quad+\int_M\int_0^1(1+|F_{A_t}|^2)^{p/2-1}(d_{A_t}(A_1-A_2),\ d_{A_t}(A_1-A_2))dtd\mu_g \\
&\quad+\int_M\int_0^1(1+|F_{A_t}|^2)^{p/2-1}(F_{A_t},\ [A_1-A_2,\ A_1-A_2])dtd\mu_g \\
&\geqq \int_M\int_0^1(1+|F_{A_t}|^2)^{p/2-1}(d_{A_t}(A_1-A_2),\ d_{A_t}(A_1-A_2))dtd\mu_g \\
&\quad+\int_M\int_0^1(1+|F_{A_t}|^2)^{p/2-1}(F_{A_t},\ [A_1-A_2,\ A_1-A_2])dtd\mu_g \\
&=\int_M\int_0^1(1+|F_{A_t}|^2)^{p/2-1}\left|F_{A_1}-F_{A_2}+\left(t-\frac{1}{2}\right)[A_1-A_2,\ A_1-A_2]\right|^2 dtd\mu_g \\
&\quad+\int_M\int_0^1(1+|F_{A_t}|^2)^{p/2-1}(F_{A_t},\ [A_1-A_2,\ A_1-A_2])dtd\mu_g \\
&=\int_M\int_0^1(1+|F_{A_t}|^2)^{p/2-1}|F_{A_1}-F_{A_2}|^2 dtd\mu_g \\
&\quad+\int_M\int_0^1(1+|F_{A_t}|^2)^{p/2-1}\left(t-\frac{1}{2}\right)^2|[A_1-A_2,\ A_1-A_2]|^2 dtd\mu_g \\
&\quad+\int_M\int_0^1(1+|F_{A_t}|^2)^{p/2-1}(2t-1)(F_{A_1}-F_{A_2},\ [A_1-A_2,\ A_1-A_2])dtd\mu_g \\
&\quad+\int_M\int_0^1(1+|F_{A_t}|^2)^{p/2-1}(F_{A_t},\ [A_1-A_2,\ A_1-A_2])dtd\mu_g \\
&=:\mathrm{I}+\mathrm{II}+\mathrm{III}+\mathrm{IV}\,. \qquad (22)
\end{aligned}
$$

Note that $\mathrm{II}\geqq 0$, and

$$|\mathrm{III}|,\ |\mathrm{IV}|\leqq C(K)\left(\int_M|A_1-A_2|^{2p}d\mu_g\right)^{1/p}.$$

Moreover for $q\geqq 1$,

$$
\begin{aligned}
(1+|F_{A_t}|^2)^{q/2} &\geqq |F_{A_t}|^q \\
&=\left|F_{A_2}+t(F_{A_1}-F_{A_2})+\frac{1}{2}(t^2-t)[A_1-A_2,\ A_1-A_2]\right|^q \\
&\geqq C|F_{A_2}+t(F_{A_1}-F_{A_2})|^q-C|[A_1-A_2,\ A_1-A_2]|^q\,. \qquad (23)
\end{aligned}
$$

Using the following well known inequality:

$$\int_0^1|x+ty|^q dt\geqq C|y|^q\,, \qquad (24)$$

for all x, $y \in V$, where V is a finite dimensional vector space, we get

$$\int_0^1 (1+|F_{A_t}|^2)^{p/2-1} dt \geqq C|F_{A_1}-F_{A_2}|^{p-2} - C|[A_1-A_2, A_1-A_2]|^{p-2}$$
$$\geqq C|F_{A_1}-F_{A_2}|^{p-2} - C|A_1-A_2|^{2p-4}. \tag{25}$$

So we obtain Lemma 3.2. ■

From Lemma 3.2 and (19), if $\{A_i\}_{i=1}^{\infty} \subset L_1^p \mathcal{C}$ is a Palais-Smale sequence, then there exist a subsequence of $\{A_i\}_{i=1}^{\infty}$ and a sequence $\{\varphi_i\}_{i=1}^{\infty} \subset L_2^p \mathcal{G}$ such that $\varphi_i^* A_i \to A_0$ strongly in $L_1^p \mathcal{C}$. Therefore, the functional J_p satisfies the Palais-Smale condition on $L_1^p \mathcal{C}$ provided $2p > n$. This completes the proof of Proposition 3.1. ■

Assume $p > n$. By the Sobolev embedding theorem $L_1^p \hookrightarrow C^{1-n/p}$, a weak solution $A_0 \in L_1^p \mathcal{C}$ of (13) is Hölder continuous. From Theorem 1.11.1 of Morrey in [**6**], $A_0 \in L_2^p \mathcal{C}$. On account of Theorem 5.6.3 in [**6**] and Proposition 2.2, there exists a gauge transformation $\varphi \in L_2^p \mathcal{G}$ such that $\varphi^* A_0 \in L_2^p \mathcal{C}$ and so Sobolev embedding theorem $L_2^p \hookrightarrow C^{1,1-n/p}$, it has Hölder continuous first derivatives. From this and again using Theorem 5.6.3 in [**6**], we get smoothness of $\varphi^* A_0$ (see [**6**]), which proves Theorem.

REMARKS 3.3. (1) This proof is also valid even in $\dim M=2$ or 3 for the Yang-Mills functional. And in these cases, the regularity theorem has been proved in [**8**]. For details, see [**9**].

(2) In general, the critical point A_0 is neither energy minimizing nor stable critical point for the functional $J(A, \tilde{g})$. Moreover a critical point A_0 and a Riemannian metric $\tilde{g}$ may not be uniquely determined.

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