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Expansiveness of homeomorphisms and dimension

Dedicated to Professor Akihiro Okuyama on his 60th birthday

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0. Introduction.

In [8], Mañé proved that if a compact metric space X admits an expansive homeomorphism, then X is finite dimensional. In [6, 7], we introduced the notion of continuum-wise (fully) expansive homeomorphism and investigated the several properties. The class of continuum-wise expansive homeomorphisms is much larger than the class of expansive homeomorphisms. In relation to dimension theory, the following results were proved; (1) if a compact metric space X admits a continuum-wise expansive homeomorphism, then X is finite dimensional [6], and (2) if a continuum X admits a positively continuum-wise fully expansive map, then X is 1-dimensional [7]. In this paper, we define the notion of barriers of a homeomorphism $f: X \to X$ and an index $B(f)$. We are interested in the relation between the index $B(f)$ and the dimension of X. The following theorem is proved; if $f: X \to X$ is a continuum-wise expansive homeomorphism of a compact metric space X, then $\dim X \leqq B(f) \leqq 2 \cdot \dim X < \infty$. As a corollary, if $f: X \to X$ is a continuum-wise fully expansive homeomorphism, then $\dim X \leqq B(f) \leqq 2$.

1. Definitions and preliminaries.

By a *continuum*, we mean a compact metric connected nondegenerate space. A homeomorphism $f: X \to X$ of a compact metric space X with metric d is *expansive* (e.g., see [1] and [2]) if there is a positive number $c>0$ such that if $x, y \in X$ and $x \neq y$, then there is an integer $n=n(x, y) \in \boldsymbol{Z}$ such that

$$d(f^n(x), f^n(y)) > c .$$

A homeomorphism $f: X \to X$ is *continuum-wise expansive* [6] if there is a positive number $c>0$ such that if A is a nondegenerate subcontinuum of X, then there is an integer $n=n(A) \in \boldsymbol{Z}$ such that

$$\operatorname{diam} f^n(A) > c ,$$

where diam B denotes the diameter of a set B. Clearly, every expansive homeomorphism is continuum-wise expansive, but the converse assertion is not true. There are many important continuum-wise expansive homeomorphisms which are not expansive (see [**6**, **7**]). Those homeomorphisms have frequent applications in topological dynamics, ergodic theory and continuum theory (e.g., see the references).

Note that if $f: X \to X$ and $g: Y \to Y$ are (resp. continuum-wise) expansive homeomorphisms, then the product $f \times g: X \times Y \to X \times Y$ is also (resp. continuum-wise) expansive. Hence this implies that for each natural number n, there is an expansive homeomorphism $f: X \to X$ of a compact metric space X with $\dim X = n$. In [**7**], we introduced the notion of (positively) continuum-wise fully expansive and study several properties of continuum-wise fully expansive homeomorphisms, which are contained in the class of continuum-wise expansive homeomorphisms. A homeomorphism $f: X \to X$ of a continuum X is *continuum-wise fully expansive* [**7**] if for each $\varepsilon > 0$ and $\delta > 0$, there is a natural number $N = N(\varepsilon, \delta) > 0$ such that if A is a subcontinuum of X with diam $A \geqq \delta$, then either (i) $d_H(f^n(A), X) < \varepsilon$ for each $n \geqq N$, or (ii) $d_H(f^{-n}(A), X) < \varepsilon$ for each $n \geqq N$, where

$$d_H(A_1, A_2) = \inf\{\eta > 0 \mid U(A_1, \eta) \supset A_2 \text{ and } U(A_2, \eta) \supset A_1\},$$

for each closed subsets A_1, A_2 of X and $U(A_1, \eta)$ denotes the η-neighborhood of A_1 in X. The distance d_H is called the *Hausdorff metric*. A map $f: X \to X$ is *positively continuum-wise fully expansive* if for each $\varepsilon > 0$ and $\delta > 0$, there is a natural number $N = N(\varepsilon, \delta) > 0$ such that if A is a subcontinuum of X with diam $A \geqq \delta$, then the above condition (i) is satisfied.

Note that there is a 2-dimensional continuum X which admits a continuum-wise fully expansive homeomorphism. In fact, each hyperbolic toral automorphism $f: T \to T$ of the torus T is (continuum-wise fully) expansive. Note that $\dim T = 2$.

For a compact metric space X, put $C(X) = \{A \mid A$ is a nonempty subcontinuum of $X\}$. Then it is well known that $C(X)$ is a compact metric space with the Hausdorff metric d_H.

2. Barriers of a homeomorphism.

Let $f: X \to X$ be a homeomorphism of a compact metric space X. Then $\boldsymbol{B} = \{\boldsymbol{B}^+ ; \boldsymbol{B}^-\}$, where $\boldsymbol{B}^+ = \{B_1^+, \cdots, B_k^+\}$ and $\boldsymbol{B}^- = \{B_{k+1}^-, \cdots, B_m^-\}$ are families of closed subsets of X, is said to be a *family of barriers of* f provided that for each $\eta > 0$ there is a natural number $N = N(\eta) > 0$ such that if $A \in C(X)$ and diam $A \geqq \eta$, then one of the following two conditions holds:

(+) For some $1 \leqq i \leqq k$, $f^N(A) \cap B_i^+ \neq \emptyset \neq f^N(A) \cap (X - B_i^+)$.

(−) For some $k+1 \leqq i \leqq m$, $f^{-N}(A) \cap B_i^- \neq \emptyset \neq f^{-N}(A) \cap (X - B_i^-)$.

Briefly, we write $\boldsymbol{B} = \{\boldsymbol{B}^+ ; \boldsymbol{B}^-\} = \{B_1^+, \cdots, B_k^+ ; B_{k+1}^-, \cdots, B_m^-\}$. Each B_i^+ $(1 \leqq i \leqq k)$ is said to be a *positive barrier of* f and each B_i^- $(k+1 \leqq i \leqq m)$ is said to be a *negative barrier of* f. Put $B(f) = \min\{m \mid$there is a family $\boldsymbol{B} = \{\boldsymbol{B}^+ ; \boldsymbol{B}^-\}$ of barriers of f such that $|\boldsymbol{B}| = |\boldsymbol{B}^+| + |\boldsymbol{B}^-| = m\}$, where $|A|$ denotes the cardinality of a set A. If there is no finite family of barriers of f, we define $B(f) = \infty$.

3. Dimension and families of barriers of continuum-wise expansive homeomorphisms.

Let X be a compact metric space. Then X has *dimension* $\leqq n$, denoted by $\dim X \leqq n$, if for any $\eta > 0$ there is a covering $\mathcal{U}$ of X by open sets with diameter $< \eta$ such that $\operatorname{ord} \mathcal{U} \leqq n+1$, i.e., every point of X belongs to at most $n+1$ sets of $\mathcal{U}$. If $\dim X \leqq n$ and $\dim X \leqq n-1$ is not true, then $\dim X = n$. Let $f: X \to X$ be a homeomorphism of a continuum X. Put $V^s = \{A \mid A$ is a subcontinuum of X such that $\lim_{n \to \infty} \operatorname{diam} f^n(A) = 0\}$ and $V^u = \{A \mid A$ is a subcontinuum of X such that $\lim_{n \to \infty} \operatorname{diam} f^{-n}(A) = 0\}$.

(3.1) PROPOSITION. *Let $f: X \to X$ and $g: Y \to Y$ be homeomorphisms of compact metric spaces, respectively, and let $f \times g: X \times Y \to X \times Y$ be a product of f and g, i.e., $(f \times g)(x, y) = (f(x), g(x))$ for each $x \in X$ and $y \in Y$. Then $B(f \times g) \leqq B(f) + B(g)$.*

PROOF. Let $\{B_1^+, \cdots, B_i^+ ; B_{i+1}^-, \cdots, B_m^-\}$ and $\{C_1^+, \cdots, C_j^+ ; C_{j+1}^-, \cdots, C_n^-\}$ be families of barriers of f and g, respectively. Then $\{B_1^+ \times Y, \cdots, B_i^+ \times Y, X \times C_1^+, \cdots, X \times C_j^+ ; B_{i+1}^- \times Y, \cdots, B_m^- \times Y, X \times C_{j+1}^-, \cdots, X \times C_n^-\}$ is a family of barriers of $f \times g$. By this fact, we see that $B(f \times g) \leqq B(f) + B(g)$.

(3.2) LEMMA ([**6**, Corollary (2.4)]). *Let $f: X \to X$ be a continuum-wise expansive homeomorphism of a compact metric space X. Then there is a positive number $\delta > 0$ such that for each $\eta > 0$ there is a natural number $N = N(\eta) > 0$ such that if $A \in C(X)$ and* $\operatorname{diam} A \geqq \eta$, *then* $\operatorname{diam} f^n(A) \geqq \delta$ *for each $n \geqq N$, or* $\operatorname{diam} f^{-n}(A) \geqq \delta$ *for each $n \geqq N$.*

(3.3) PROPOSITION. *Let $f: X \to X$ be a continuum-wise expansive homeomorphism of a compact metric space X and let $\delta > 0$ be a positive number as in* (3.2). *Then $B(f) \leqq 2 \cdot i(\delta) < \infty$, where for each $\varepsilon > 0$,*

$i(\varepsilon) = \min\{m \mid$ there is a finite family $\boldsymbol{F} = \{F_1, \cdots, F_m\}$ of closed subsets of X such that for each component C of F_i $(1 \leqq i \leqq m)$, $\operatorname{diam} C < \varepsilon$, *and for each component D of $X - \bigcup_{i=1}^m F_i$,* $\operatorname{diam} D < \varepsilon\}$.

PROOF. Let $i(\delta)=m$ and let $\boldsymbol{F}$ be a family of closed subsets of X as in the statement of (3.3). Put $B_i^+=F_i$ $(1\leqq i\leqq m)$ and $B_j^-=F_j$ $(1\leqq j\leqq m)$. Then the family $\{B_1^+, \cdots, B_m^+; B_1^-, \cdots, B_m^-\}$ is a family of barriers of f (see (3.2)). Hence $B(f)\leqq 2\cdot i(\delta)<\infty$.

(3.4) PROPOSITION. *Let X be a compact metric space with* $\dim X=n$. *Then* $\sup\{i(\varepsilon)\mid\varepsilon>0\}=n$.

PROOF. Note that if $\varepsilon_1\geqq\varepsilon_2>0$, then $i(\varepsilon_1)\leqq i(\varepsilon_2)$. For each $\varepsilon>0$, there is a finite open cover $\mathcal{U}$ of X with mesh $\mathcal{U}<\varepsilon$ and ord $\mathcal{U}\leqq n+1$. By [**9**, p. 213], there are families $\mathcal{V}_1, \cdots, \mathcal{V}_{n+1}$ of pairwise disjoint open sets of X, i.e., ord $\mathcal{V}_i\leqq 1$, such that each $\mathcal{V}_i$ shrinks $\mathcal{U}$ $(1\leqq i\leqq n+1)$ and $\bigcup_{i=1}^{n+1}\mathcal{V}_i$ is a cover of X. Taking a shrinking $\mathcal{V}_i'$ of $\mathcal{V}_i$ $(1\leqq i\leqq n+1)$ such that $\bigcup_{i=1}^{n+1}\mathcal{V}_i'=X$ and each element of $\mathcal{V}_i'$ is a closed subset of X. Put $F_i=\bigcup\mathcal{V}_i'$ $(1\leqq i\leqq n)$. Consider the family $\{F_1, \cdots, F_n\}$. Then the family satisfies the desired condition. Hence $i(\varepsilon)\leqq n=\dim X$. Next, we shall show that $\sup\{i(\varepsilon)\mid\varepsilon>0\}\geqq\dim X$. Suppose, on the contrary, that for each $\varepsilon>0$, $i(\varepsilon)<\dim X=n$. Let $\varepsilon>0$ be a given positive number. Then there is a family $\boldsymbol{F}=\{F_1, \cdots, F_m\}$ of closed sets of X such that $m<n$, for each component C of F_i, diam $C<\varepsilon$ and for each component D of $X-\bigcup_{i=1}^m F_i$, diam $D<\varepsilon$. By considering small neighborhoods of components of $F_i\in\boldsymbol{F}$, we can easily see that there is an open cover $\mathcal{U}$ of X with mesh $\mathcal{U}<\varepsilon$ and ord $\mathcal{U}\leqq m+1$. Hence $\dim X\leqq m<n$. This is a contradiction.

(3.5) REMARK. In [8], Mañé proved that if $f: X\to X$ is an expansive homeomorphism of a compact metric space X, then $\dim X<\infty$. In fact, from his proof, we can see (by [**6**, Proposition 2.2]) that $\dim X\leqq(j(\delta))^2-1$, where $j(\varepsilon)=\min\{n\mid$there is a finite closed cover $\boldsymbol{F}=\{F_1, \cdots, F_n\}$ of X with mesh $\boldsymbol{F}<\varepsilon\}$. Clearly $i(\varepsilon)<j(\varepsilon)$.

(3.6) EXAMPLE. Let A be a 2×2 matrix satisfying (i) all entries of A are integers, (ii) $\det(A)=\pm 1$ and (iii) A is hyperbolic, i.e., none of its eigenvalues have absolute one. Then the eigenvalues λ_1, λ_2 are real and we may assume that $|\lambda_1|>1$, $|\lambda_2|<1$. For example, let $A=\begin{pmatrix}2 & 1\\ 1 & 1\end{pmatrix}$. Let $p: \boldsymbol{R}^2\to T$ be the natural covering projection from the plane $\boldsymbol{R}^2$ to the torus $T=S^1\times S^1$, and let $L_A: T\to T$ be the homeomorphism of T induced by A, i.e., $p\cdot A=L_A\cdot p$. Then L_A is called a hyperbolic toral automorphism. The dynamical properties of L_A is well known. By [**7**, (2.4)], L_A is continuum-wise fully expansive. Let $\boldsymbol{V}_i$ $(i=1, 2)$ be the eigenvector space corresponding to λ_i in $\boldsymbol{R}^2$. Consider the subset D in $\boldsymbol{R}^2$ as in the Figure 1. Put $B=p(D)\subset T$, and $B^+=B$ and $B^-=B$. Since $T\supsetneqq B\supset \mathrm{Int}\, B\neq\emptyset$ and L_A is continuum-wise fully expansive, $\{B^+; B^-\}$ is a family of barriers of L_A. Then $B(L_A)=2$ (see (3.7) below).

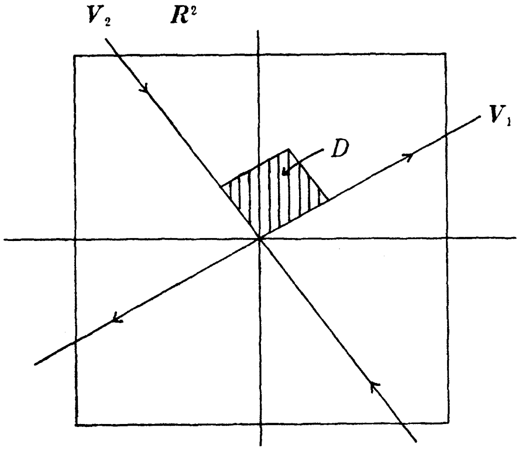

Figure 1.

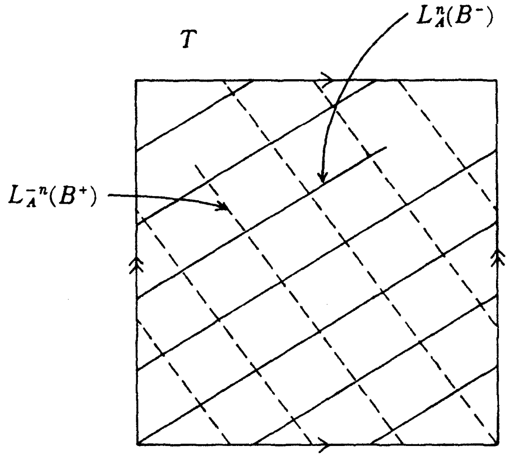

Figure 2.

The following is the main theorem of this paper.

(3.7) THEOREM. *If* $f: X \to X$ *is a homeomorphism of a compact metric space* X, *then* $\dim X \leqq B(f)$. *In particular, if* f *is a continuum-wise expansive homeomorphism, then* $\dim X \leqq B(f) \leqq 2 \cdot i(\delta) \leqq 2 \cdot \dim X < \infty$, *where* δ *is a positive number as in* (3.2).

PROOF. We may assume that $B(f)=m<\infty$. Let $\{\boldsymbol{B}^+;\boldsymbol{B}^-\}=\{B_1^+, \cdots, B_k^+; B_{k+1}^-, \cdots, B_m^-\}$ be a family of barriers of f. For each $n=0, 1, \cdots$, and $i=1, 2, \cdots, m$, consider the following subsets of X;

$$M(i, n) = f^{-n}(B_i^+), \quad \text{for } 1 \leqq i \leqq k, \quad \text{and}$$

$$M(i, n) = f^{n}(B_i^-), \quad \text{for } k+1 \leqq i \leqq m\,.$$

Let $\eta>0$ be a given positive number. Since $\{\boldsymbol{B}^+;\boldsymbol{B}^-\}$ is a family of barriers of f, we can choose a natural number $N=N(\eta)$ such that if $A \in C(X)$ and $\operatorname{diam} A \geqq \eta$, then one of the following two conditions is satisfied;

$(+)$ $f^N(A) \cap B_i^+ \neq \emptyset \neq f^N(A) \cap (X-B_i^+)$ for some $1 \leqq i \leqq k$, or

$(-)$ $f^{-N}(A) \cap B_i^- \neq \emptyset \neq f^{-N}(A) \cap (X-B_i^-)$ for some $k+1 \leqq i \leqq m$.

For each $i=1, 2, \cdots, m$, put $\Lambda_i=\{E \mid E \subset \{1, 2, \cdots, m\} \text{ and } |E|=i\}$. For each $j=1, 2, \cdots, m$, put

$$L_j = \bigcup_{E \in \Lambda_j} (\bigcap \{M(i, N) \mid i \in E\})\,.$$

Note that $L_m \subset L_{m-1} \subset \cdots \subset L_1$ and $L_m=\bigcap_{i=1}^m M(i, N)$, and $L_1=\bigcup_{i=1}^m M(i, N)$. Now, we shall show that there is a family $\mathcal{U}'$ of open sets of X such that $\mathcal{U}'$

covers L_1, i.e., $\cup\mathcal{U}'\supset L_1$, and ord $\mathcal{U}'\leqq m$. First, consider the set L_m. If C is a component of L_m, we see that $f^N(C)\subset B_i^+$ for each $1\leqq i\leqq k$, and $f^{-N}(C)\subset B_i^-$ for each $k+1\leqq i\leqq m$. Since $\{\boldsymbol{B}^+;\boldsymbol{B}^-\}$ is a family of barriers of f, by the choice of N we see that diam $C<\eta$ (see Figure 2). By considering small neighborhoods of components of L_m, we can choose a family $\mathcal{U}_m$ of open sets of X such that $\mathcal{U}_m$ covers L_m, ord $\mathcal{U}_m\leqq 1$ and mesh $\mathcal{U}_m<\eta$. Next, consider the set; $P_{m-1}=L_{m-1}-\cup\mathcal{U}_m$. If C is a component of P_{m-1}, we see that diam $C<\eta$, because that $\{\boldsymbol{B}^+;\boldsymbol{B}^-\}$ is a family of barriers of f. By considering small neighborhoods of components of P_{m-1}, we can choose a family $\mathcal{U}_{m-1}$ of open sets of X such that $\mathcal{U}_{m-1}$ covers P_{m-1}, ord $\mathcal{U}_{m-1}\leqq 1$ and mesh $\mathcal{U}_{m-1}<\eta$. If we continue this procedure, we obtain a sequence $\mathcal{U}_m, \mathcal{U}_{m-1}, \cdots, \mathcal{U}_1$ of families of open sets of X, and a sequence $P_m=L_m, P_{m-1}, \cdots, P_1$ of closed sets of X such that $P_{i-1}=L_{i-1}-\cup(\mathcal{U}_m\cup\mathcal{U}_{m-1}\cup\cdots\cup\mathcal{U}_i)$, $\cup\mathcal{U}_{i-1}\supset P_{i-1}$, and ord $\mathcal{U}_{i-1}\leqq 1$ and mesh $\mathcal{U}_{i-1}<\eta$ $(i=m+1, \cdots, 2)$. Put $\mathcal{U}'=\mathcal{U}_m\cup\mathcal{U}_{m-1}\cup\cdots\cup\mathcal{U}_1$. Then $\mathcal{U}'$ covers L_1, mesh $\mathcal{U}'<\eta$ and ord $\mathcal{U}'\leqq m$. Finally, consider the set $Y=X-\cup\mathcal{U}'$. If C is a component of Y, then diam $C<\eta$. Hence we can choose a family $\mathcal{U}''$ of open sets of X such that ord $\mathcal{U}''\leqq 1$, mesh $\mathcal{U}''<\eta$ and $\mathcal{U}''$ covers Y. Put $\mathcal{U}=\mathcal{U}'\cup\mathcal{U}''$. Then $\mathcal{U}$ is an open cover of X such that ord $\mathcal{U}\leqq m+1$ and mesh $\mathcal{U}<\eta$. Hence we can conclude that $\dim X\leqq m=B(f)$. If $f: X\to X$ is a continuum-wise expansive homeomorphism, by (3.3) and (3.4) $B(f)\leqq 2\cdot i(\delta)\leqq 2\cdot\dim X<\infty$. This completes the proof.

(3.8) COROLLARY. *If a continuum X admits a continuum-wise fully expansive homeomorphism $f: X\to X$, then*

(a) *$B(f)\leqq 2$, and hence X is at most 2-dimensional, and*

(b) *if $A\in V^\sigma$ ($\sigma=s$ and u), then* $\dim A\leqq 1$.

PROOF. (a): Choose a proper closed subset B of X with Int $B\neq\emptyset$. Put $B^+=B$, $B^-=B$. Then the family $\{B^+;B^-\}$ is a family of barriers of f. Hence $\dim X\leqq B(f)\leqq 2$. (b): We prove the case $\sigma=u$. The family $\{B^+;\phi\}$ is a family of "barriers" of $f|A$, where the notion of barriers of the restriction $f|A: A\to X$ of f to A is similarly defined. Hence $\dim A\leqq 1$ (see the proof of (3.7)).

In case of a map $f: X\to X$, we can define the index $B^+(f)$ as follows: Let $f: X\to X$ be a map of a compact metric space X. Suppose that there is a family $\boldsymbol{B}^+=\{B_1^+, \cdots, B_m^+\}$ of *positive barriers* of f, i.e., for each $\eta>0$ there is a natural number N such that if $A\in C(X)$ and diam $A\geqq\eta$, then there is $1\leqq i\leqq m$ such that

$$(+)\quad f^N(A)\cap B_i^+\neq\emptyset\neq f^N(A)\cap(X-B_i^+).$$

Put $B^+(f)=\min\{m\,|\,$there is a family $\boldsymbol{B}^+$ of positive barriers of f such that $|\boldsymbol{B}^+|=m\}$. If there is no finite family of positive barriers of f, we define

$B^+(f)=\infty$.

(3.9) COROLLARY ([7, (4.3)]). *If a continuum X admits a positively continuum-wise fully expansive map, then* $\dim X=B^+(f)=1$.

PROOF. Let B be a proper closed subset of X with $\mathrm{Int}\, B\neq\emptyset$. Then $\{B^+=B\}$ is a family of positive barriers of f. Hence $\dim X\leqq 1$ (see the proof of (3.7)).

(3.10) COROLLARY. *If* $f: X\to X$ *is a positively continuum-wise expansive map of a compact metric space X, then* $\dim X=B^+(f)$.

PROOF. Let $m=\dim X$. By (3.4), we can choose a family $\{B_1, B_2, \cdots, B_m\}$ of positive barriers of f. Hence $\dim X=B^+(f)$ (see the proof of (3.7)).

We have the following problems.

PROBLEM 1. Is it true that if $f: X\to X$ is a continuum-wise fully expansive homeomorphism of a 1-dimensional continuum X, then f or f^{-1} is positively continuum-wise fully expansive?

PROBLEM 2. Is it true that for each continuum-wise expansive homeomorphism $f: X\to X$ of a compact metric space X, $\dim X=B(f)$?

In relation to the above problems, we have the following fact.

(3.11) PROPOSITION ([7, (3.3)]). *Let* $f: X\to X$ *be a continuum-wise fully expansive homeomorphism of a continuum. Then neither f nor* f^{-1} *is positively continuum-wise fully expansive if and only if for each* $x\in X$, *there are two nondegenerate subcontinua A, B of X such that* $x\in A\cap B$, $A\in V^s$ *and* $B\in V^u$. *In particular,* $\dim (A\cap B)=0$ *and* $\dim A=1=\dim B$.

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